AY230 – Solutions #3

(1) HII Temperature: Because the recombination coefficient to bound levels of hydrogen $\sigma_n^{rec}(v)$ decreases with increasing electron velocity v, the electrons that are the most likely to recombine have energies less than $3kT/2$. Let the mean energy of electrons that recombine to all bound levels of hydrogen be

$$\bar{E} = f\frac{3}{2}kT = \frac{\sum_{n=1}^{\infty} < \sigma_n v \frac{1}{2} m v^2 >_{max}}{\sum_{n=1}^{\infty} < \sigma_n v >_{max}} \tag{1}$$

Evaluate $f(T)$ for temperatures of $T = 5000, 10000, 15000$, and $20000\,\mathrm{K}$ (an approximate solution is encouraged).

Solution: We can express the numerator as follows:

$$\sum_{n=1}^{\infty} < \sigma_n v \frac{1}{2} m v^2 >_{max} = \frac{mA}{\sqrt{\pi L^3}} \beta \sum_{n=1}^{\infty} \frac{\beta}{n^3} \left\{ 1 - \frac{\beta}{n^2} e^{\beta/n^2} E_1\left(\frac{\beta}{n^2}\right) \right\} \tag{2}$$

$$= \frac{mA}{\sqrt{\pi L^3}} \beta \chi(\beta) \tag{3}$$

with

$$\chi(\beta) \approx \frac{1}{2}\left[0.735 + \ln\beta + \frac{1}{(3\beta)}\right] \tag{4}$$

For the denominator, we can use our expression for the Case A recombination coefficient:

$$\alpha_A(T) = \frac{2A}{\sqrt{\pi L}} \beta \phi(\beta) \tag{5}$$

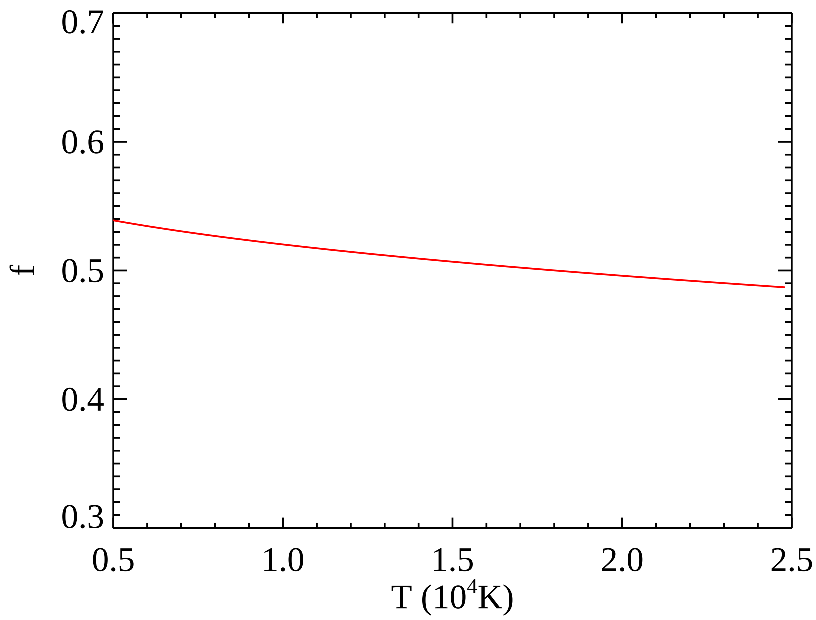
$$\approx \frac{2A}{\sqrt{\pi L}} \beta \frac{1}{2}\left[1.735 + \ln\beta + \frac{1}{6\beta}\right] \tag{6}$$

Therefore, we have

$$\bar{E} = \frac{m}{2}\frac{1}{L}\frac{0.735 + \ln\beta + 1/(3\beta)}{1.735 + \ln\beta + 1/(6\beta)} \tag{7}$$

where, $L \equiv m/2kT$ and $\beta \equiv h\nu_0/kT$.

Evaluating, I find



(2) Diagnosing the properties of star-forming galaxy: On the website, you will find a gzipped ASCII spectrum listing the wavelengths (Å) and flux values (units are $10^{-15}\mathrm{erg\,s^{-1}\,cm^{-2}\,Å^{-1}}$) of a galaxy. Assume that the galaxy is 450 Mpc away (ignore redshift in the following).

(a) Produce a nice plot of the spectrum identifying all emission lines

(b) Measure the fluxes for these lines. Tabulate.

(c) Estimate the extinction in this galaxy

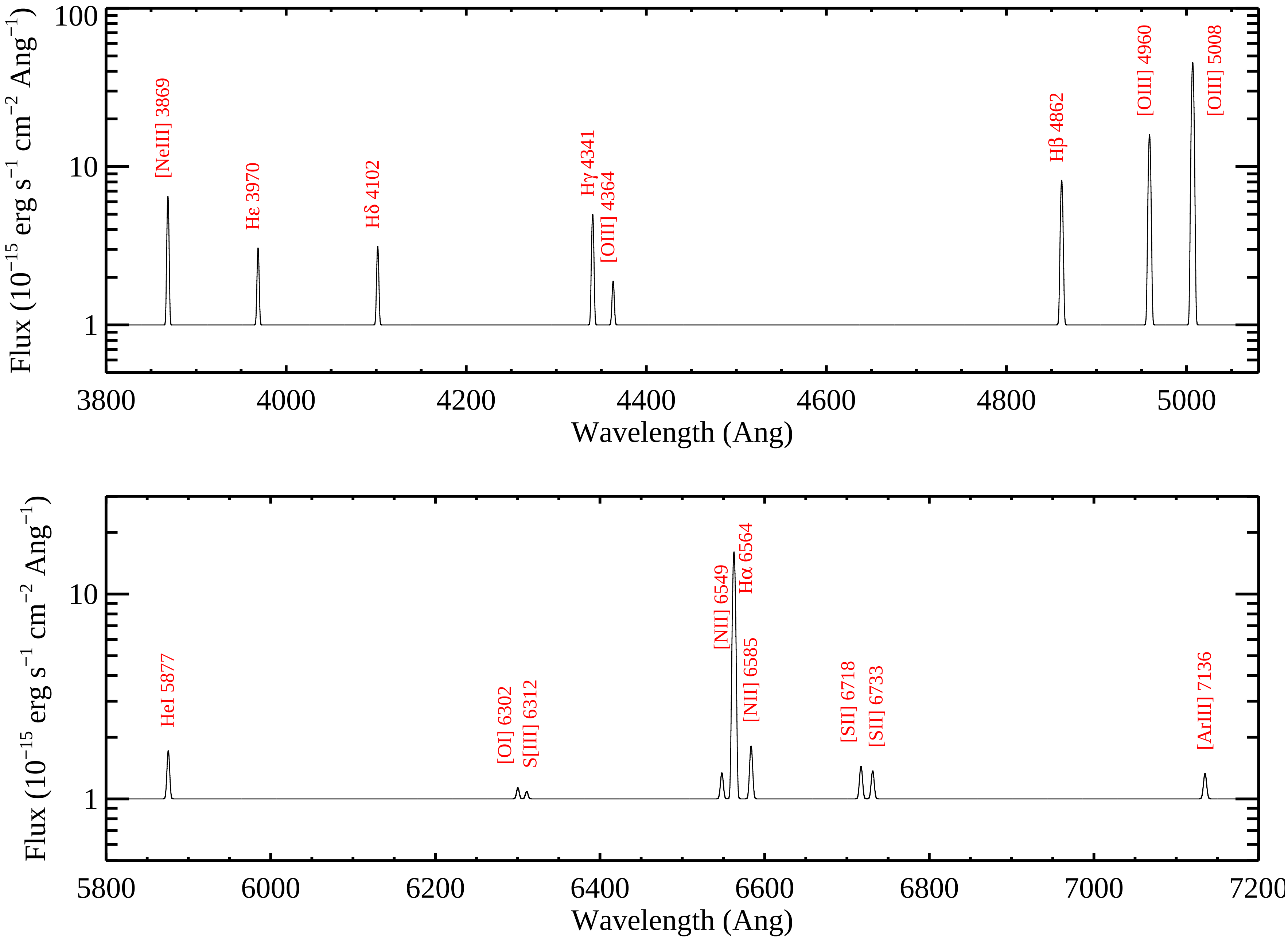
(d) Estimate the temperature of the HII regions in this galaxy

(e) Estimate the density of the HII regions in this galaxy

(f) Use several methods to estimate the metallicity of this galaxy

(g) Estimate the SFR of the galaxy

Solution:



Flux (10^−15 erg s^−1 cm^−2 Ang^−1)
100
10
1
[NeIII] 3869
Hε 3970
Hδ 4102
Hγ 4341
[OIII] 4364
Hβ 4862
[OIII] 4960
[OIII] 5008
3800
4000
4200
4400
4600
4800
5000
Wavelength (Ang)
Flux (10^−15 erg s^−1 cm^−2 Ang^−1)
10
1
HeI 5877
[OI] 6302
S[III] 6312
[NII] 6549
Hα 6564
[NII] 6585
[SII] 6718
[SII] 6733
[ArIII] 7136
5800
6000
6200
6400
6600
6800
7000
7200
Wavelength (Ang)

(a) Here is my 'nice' plot. I do not consider a simple IRAF screen shot/plot to be 'nice'.

(b) One could fit Gaussians to each line or do a simple boxcar integration to measure the flux. There is only one subtlety: Do not include the continuum, which we attribute to the underlying stellar population. Doing the simple boxcar, I get (accurate to ≈ 0.1Å):

Table 1: LINE FLUXES

Ion	Line (Å)	f (10^{-15} ergs s^{-1} cm^{-2})
[NeIII]	3869	13.28
Hϵ	3970	5.12
Hδ	4102	5.47
Hγ	4341	10.85
[OIII]	4364	2.43
Hβ	4862	21.96
[OIII]	4960	46.35
[OIII]	5008	139.51
HeI	5877	2.65
[OI]	6302	0.52
[SIII]	6312	0.35
[NII]	6549	1.39
Hα	6564	61.71
[NII]	6585	3.33
[SII]	6718	1.86
[SII]	6733	1.56
[ArIII]	7136	1.47

(c) One method for estimating the extinction is to compare the relative strengths of the Balmer lines (e.g. the Hα to Hβ flux). For an HII region, the relative emission of the Balmer series is a relatively insensitive function of temperature. For a galaxy, one is observing the flux-weighted average of many HII regions yet this still tends toward the idealized case.

If we assume the Case B approximation, then we can use the tabulated Balmer emissivities as tabulated by Osterbrock. We will find below that the gas temperature is approximately 15,000K. This implies an Hα/Hβ emissivity ratio of $j_{3,2}/j_{4,2} \approx 2.81$. Our observed fluxes are $f(\mathrm{H}\alpha)/f(\mathrm{H}\beta) = 2.81$ which implies no reddening. This is highly unusual. I suspect that someone provided us with a de-reddened spectrum! :)

(d) As we discussed in class, the [OIII] lines provide an excellent temperature diagnostic. The point is that the upper levels of O^{++} are populated by collisions and that the relative excitation is sensitive to T_e. Adopting the equation from Osterbrock (similar to the one in our notes),

$$\frac{j_{4959}+j_{5007}}{j_{4363}} = \frac{7.9 \times \exp\left[\,(3.3\times 10^4)/T\,\right]}{1+4.5\times 10^{-3}(n_e/T^{\frac{1}{2}})} \tag{8}$$

For our tabulated fluxes, the LHS evaluates to:

$$\frac{j_{4959}+j_{5007}}{j_{4363}} = 76.5 \tag{9}$$

Ignoring the term related to n_e which is a small correction for modest values of n_e, we find $T_e = 14,500$K.

(e) We discussed the the [OII] lines offer a nice estimate of the gas density of HII regions because the doublet is tightly spaced and its excitation is insensitive to T_e. In the low density limit, one expects their relative emission to be a function of the relative collisional strengths, i.e. $j_{3729}/j_{3726} \approx 1.5$. And at high n_e, the levels will be populated according to their degeneracies and spontaneous emission coefficients, i.e. $j_{3729}/j_{3726} \approx 0.3$. Unfortunately, we did not observe these lines. But, we see from the figure in the notes that [SII] has a similar electronic configuration (S has 8 more protons and electrons than O and hence a few more filled levels but otherwise similar chemistry).

There are differences, of course, because of differences in the spontaneous coefficients such that at high n_e we exepct $j_{6716}/j_{6731} \approx 0.45$. We observe $j_{6716}/j_{6731} = 1.86/1.56 = 1.19$. All of you chose to read off the curve in Osterbrock Fig 5.8 as opposed to performing the following calculation. At the least, you should have taken care to allow for the temperature dependence to the result (take the x-axis in Fig 5.8 as $n_e[10^4/T]^{\frac{1}{2}}$).

I will ignore the $^2P^0$ states which contribute to the problem mainly at high densities. And, I am going to label the remaining three levels as S for the ground-state and $3/2, 5/2$ for the two $^2D^0$ levels. We are interested in the ratio of emissivity from these two upper levels which may be written as:

$$\frac{j_{6716}}{j_{6731}} = \frac{n_{5/2}}{n_{3/2}}\frac{A_{5/2}}{A_{3/2}}\frac{\nu_{5/2}}{\nu_{3/2}} \tag{10}$$

We know the A and ν values (Table 3.13 of Osterbrock) and need to solve for the ratio of the populations. We can do this following detailed balance. Namely, consider the population of the ground-state and one of the excited levels to give two constraints on three unknowns: $n_S, n_{5/2}, n_{3/2}$. Here are the two constraints:

$$\left(\,n_e q_{S,3/2} + n_e q_{S,5/2}\,\right) n_S = \left(\,A_{3/2,S} + n_e q_{3/2,S}\,\right) n_{3/2} + \left(\,A_{5/2,S} + n_e q_{5/2,S}\,\right) n_{5/2} \tag{11}$$

$$\left(\,n_e q_{3/2,S} + n_e q_{3/2,5/2} + A_{3/2,S}\,\right) n_{3/2} = n_e q_{S,3/2} n_S + \left(\,A_{5/2,3/2} + n_e q_{5/2,3/2}\,\right) n_{5/2} \tag{12}$$

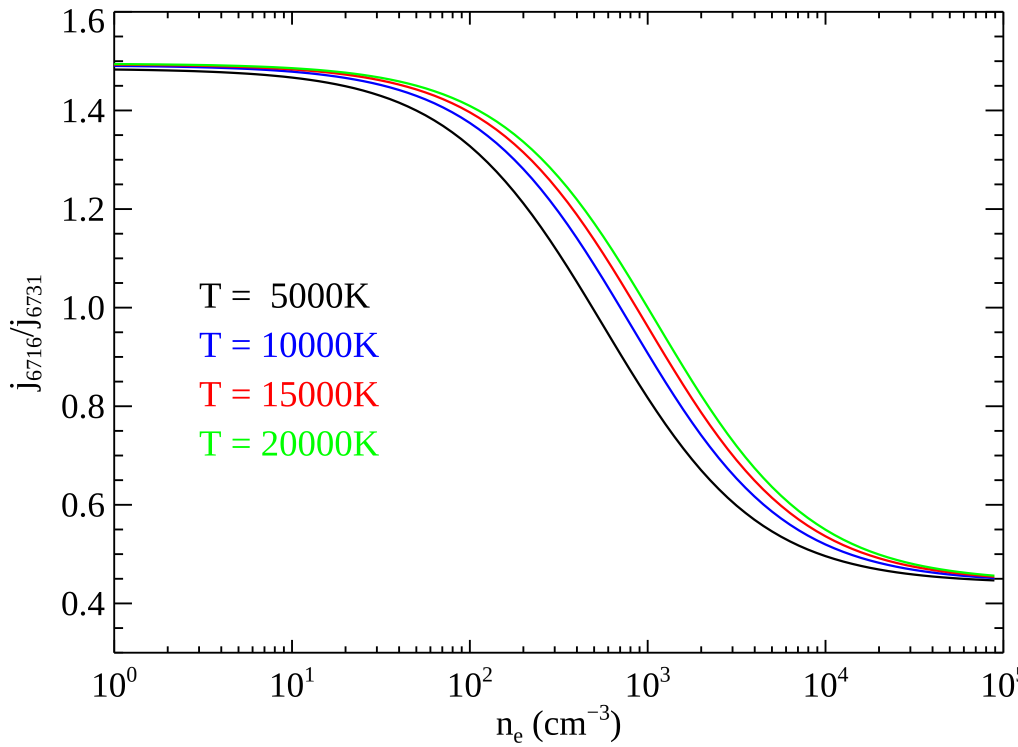
Recall that our q values can be written as

$$q_{ij} = \frac{8.629 \times 10^6}{g_i T^{\frac{1}{2}}} \Omega_{ij} \exp\left(-\Delta E/kT\right) \tag{13}$$

$$q_{ji} = \frac{8.629 \times 10^6}{g_j T^{\frac{1}{2}}} \Omega_{ij} \tag{14}$$

The collisional strengths are given in Table 3.7 of Osterbrock. Note that they give the full level values for the $S \rightarrow D$ collisions and you have to modify these by $(2J+1)/[(2S+1)(2L+1)]$.

Putting this altogether and having a little fun with algebra, I produce the following figure (which differs from Ostebrock primarily because I have ignore the upper P levels):



Solving for our observed flux ratio and taking $T = 15,000$K, I find $n_e = 390\,\mathrm{cm}^{-3}$.

(f) There are several empirical methods for estimating the metallicity that we can apply to our dataset, even absent the [OII] emission lines.

i. The N2 index:

$$12 + \log(\mathrm{O/H}) = 8.90 + 0.57 \times N2 \tag{15}$$

For $N2 = \log([\mathrm{NII}]\lambda6583/\mathrm{H}\alpha) = -1.26$, I recover $12 + \log(\mathrm{O/H}) = 8.18\,\mathrm{dex}$.

ii. The O3N2 index:

$$12 + \log(\mathrm{O/H}) = 8.73 - 0.32 \times O3N2 \tag{16}$$

For $O3N2 = \log\{([\mathrm{OIII}]\lambda5007/\mathrm{H}\beta)/([\mathrm{NII}]\lambda6583/\mathrm{H}\alpha)\} = 2.07$, I get $12 + \log(\mathrm{O/H}) = 8.07\,\mathrm{dex}$.

(g) The 1998 Kennicutt review article gives a number of empirical prescriptions for relating observables to estimates of the star formation rate. A nearly 'gold standard' method is the Hα luminosity and the empirical relation is:

$$SFR = 7.8 \times 10^{-42} L(\mathrm{H}\alpha) \quad M_\odot\,\mathrm{yr}^{-1} \tag{17}$$

We need to convert our observed bolometric flux of Hα emission into a luminosity. For a distance of $D = 450\,\mathrm{Mpc}$, we have

$$L(\mathrm{H}\alpha) = 4\pi D^2 F(\mathrm{H}\alpha) = 1.5 \times 10^{42}\ \mathrm{erg\,s^{-1}} \tag{18}$$

Therefore, we estimate a SFR = 11.7 $M_\odot/\mathrm{yr}$