AY230 – Solutions #4

(1) The optical depth for an absorption line is related to the column density of gas N by:

$$\tau_\lambda = \frac{\pi e^2}{m_e c^2} f_{ij}\, \lambda_{ij}^2\, N\, \phi_\lambda \tag{1}$$

where f_{ij} is the oscillator strength, λ_{ij} is the wavelength of the transition and ϕ_λ is the line profile.

(a) Derive a 'complete' expression for the optical depth in velocity space τ_v of the Lyβ transition for a slab of gas with velocity dispersion σ and column density $N_{\rm HI}$.

Solution:

Because the system has a finite velocity dispersion, the line broadening will be the convolution of the Doppler profile and the Lorentzian due to natural broadening (i.e. the Voigt profile).

To express the Voigt profile in terms of velocity, we note:

$$\frac{\nu - \nu_0}{c} = \frac{v}{c} \tag{2}$$

$$\frac{dv}{c} = \frac{d\nu}{\nu_0} \tag{3}$$

$$\Delta v_D = \Delta \nu_D \frac{c}{\nu_0} = \sigma\sqrt{2} \tag{4}$$

$$\tag{5}$$

We also relate our line-profile in λ space to one in velocity space via:

$$\phi_\lambda d\lambda = \phi_\nu d\nu \tag{6}$$

$$\phi_\lambda = \phi_\nu \frac{c}{\lambda^2} \tag{7}$$

$$\phi_\nu d\nu = \phi(v) dv \tag{8}$$

$$\phi_\nu = \phi(v)\frac{c}{\nu_0} = \phi(v)\lambda \tag{9}$$

$$\phi_\lambda = \phi(v)\frac{c}{\lambda} \tag{10}$$

Taking the Voigt profile, the line-profile in velocity space is:

$$\phi_V(v) = \frac{1}{\sigma\sqrt{2\pi}} H(a,u) \tag{11}$$

$$\text{where } u = \frac{v}{\sigma\sqrt{2}} \tag{12}$$

$$\text{and } a = \frac{\gamma c}{4\pi\nu_0\sigma\sqrt{2}} \tag{13}$$

For γ, we have two terms: $3 \rightarrow 2$, $A_{3p\rightarrow 2s} = 2.25 \times 10^7$ s^{-1} and $3 \rightarrow 1$, $A_{3p\rightarrow 1s} = 1.67 \times 10^8$ s^{-1}. This gives $\gamma = 1.897 \times 10^8$ s^{-1}.

Finally, evaluating the optical depth equation by replacing ϕ_λ, and inserting $f_{ij} = 0.07912$ and $\lambda_{ij} = 1025.7223$Å, we have:

$$\tau(v) = \frac{\pi e^2}{m_e c} f_{ij}\, \lambda_{ij}\, N\, \phi_V(v) \tag{14}$$

$$= 2.65 \times 10^{-15} \lambda_{ij} f_{ij} N \phi_V(v) \qquad \{\text{with } \lambda \text{ in Å and } \phi(\text{v}) \text{ in s/km}\} \tag{15}$$

$$= 2.15 \times 10^{-13}\, N_{\rm HI}\, \phi_V(v) \tag{16}$$

(b) Consider a distant source with continuum I^*_ν that lies behind the slab. Overplot the observed intensity I_ν relative to the continuum for a series of Lyβ transitions with $N_{\rm HI} = 10^{13}, 10^{15}, 10^{18}$ and $10^{21}\,\rm cm^{-2}$ and $\sigma = 30$ km s^{-1}. Discuss.

Solution:

Unfortunately, the Voigt profile cannot be evaluated analytically. Therefore, we must resort to numerical integration. Luckily, we only have to calculate the profile once (for $\sigma = 30$km/s) and then plot the intensity functions for each $N_{\rm HI}$ value

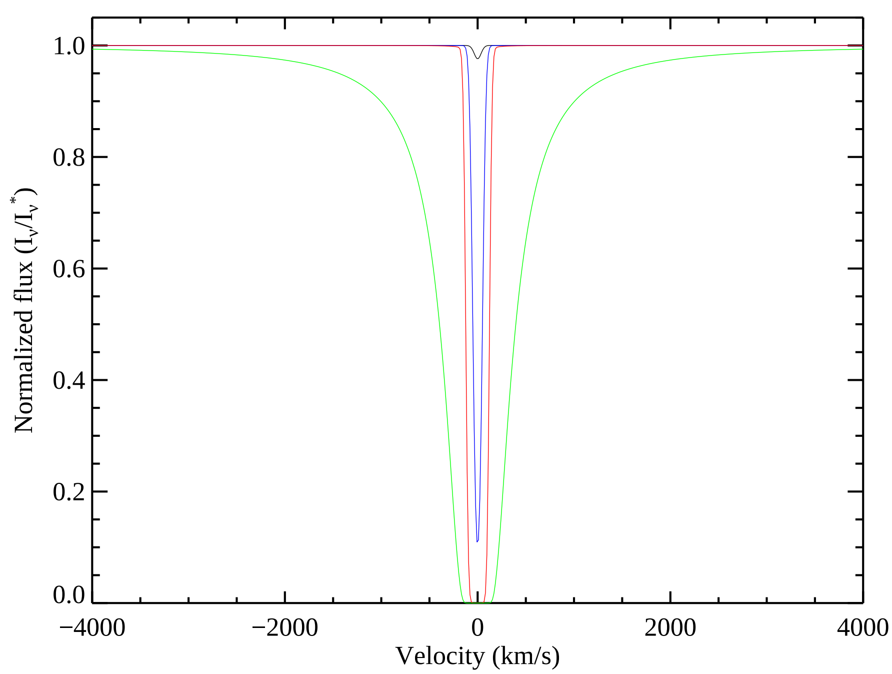


Figure 1: Normalized intensity vs velocity for a Lyβ profile with a range of $N_{\rm HI}$ values: $10^{13}, 10^{15}, 10^{18}, 10^{21}\,\rm cm^{-2}$.

The differences are quite dramatic! For low $N_{\rm HI}$, we only see the effect of Doppler broadening, i.e. the Lorentzian is absent. At large column density, however, the broadening is entirely due to Natural broadening (the width significantly exceeds the Doppler motions).

(c) Plot the equivalent widths W_λ for the Lyβ absorption for $N_{\rm HI}$ ranging from 10^{13} to $10^{21}\,{\rm cm}^{-2}$ for velocity dispersions of $\sigma = 10$, 30, and 100 km s^{-1}. Discuss.

Solution:

The approach is a simple extension of part (b).

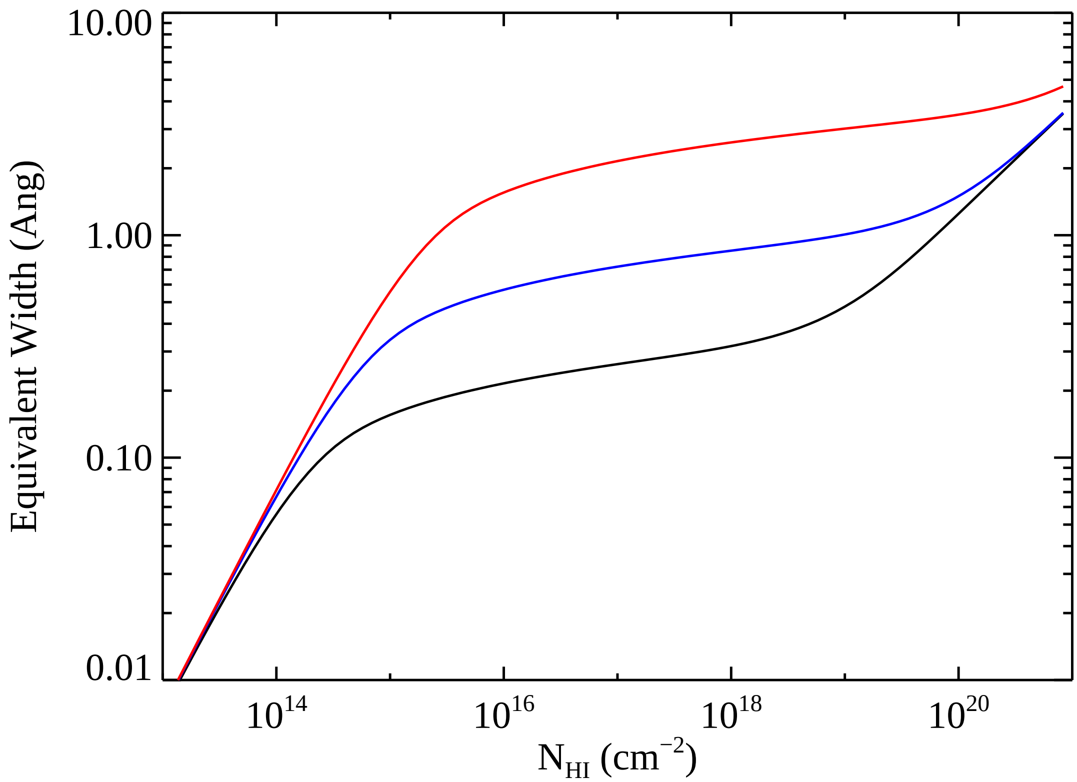


Figure 2: Equivalent width curves of Lyβ vs. $N_{\rm HI}$ for 3 σ values: σ = 10 (black), 30 (blue), and 100 (red) km/s.

The Doppler parameter (velocity dispersion) is only important for the regime where the profile is saturated and yet the Lorentzian wings are still unimportant. Once the Lorentzian dominates, the profile one observes is entirely independent of σ.

(d) For one choice of σ, plot $d(W_\lambda/\lambda)/dN$ vs. N. Discuss.

Solution:

This problem was supposed to read as above (but did not). My apologies. Here is the fig for $\sigma = 30$ km/s.

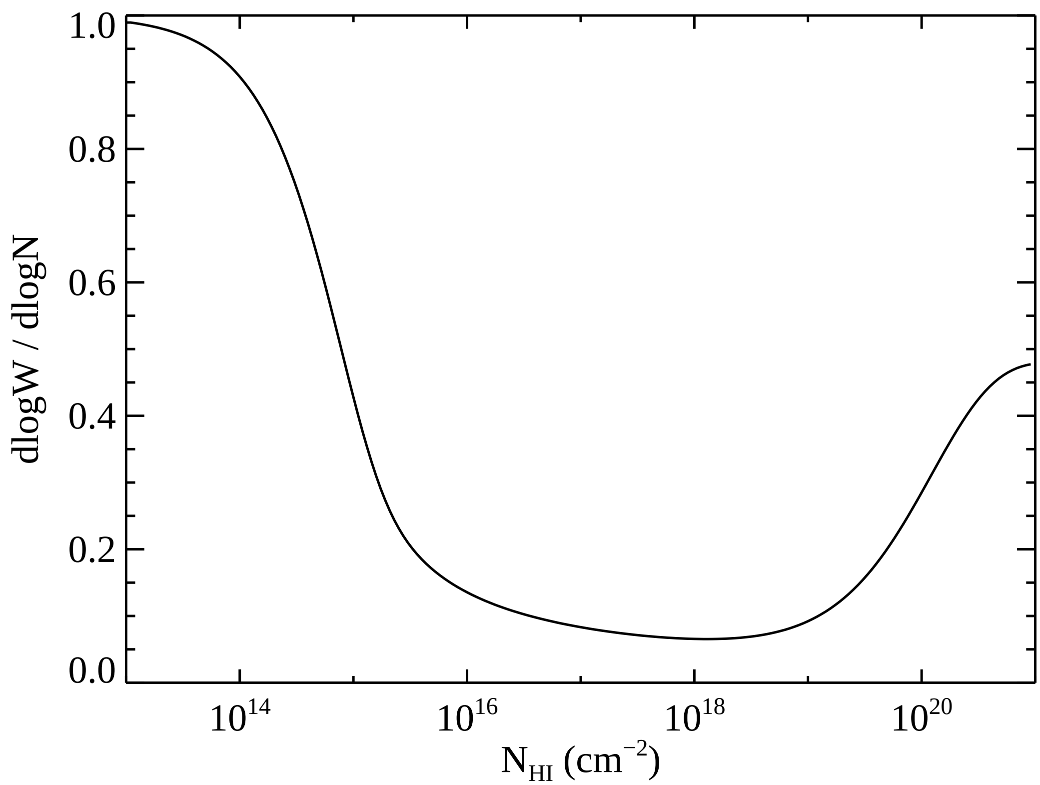


Figure 3: $d\log W/d\log N$ vs. $N_{\rm HI}$ for $\sigma = 30$km/s. This plot reveals the power-law dependence of W on N, i.e. α where $W = N^\alpha$.

As one can see from the Figure in part (c), the equivalent width rises rapidly at small N, flattens at intermediate N and then rises again at large N. This 'curve of growth' is described by 3 regimes:

i. Linear: Low N; $W \propto N$
ii. Flat: Moderate N; W has almost no N dependence
iii. Damped: Large N; $W \propto N^{\frac{1}{2}}$

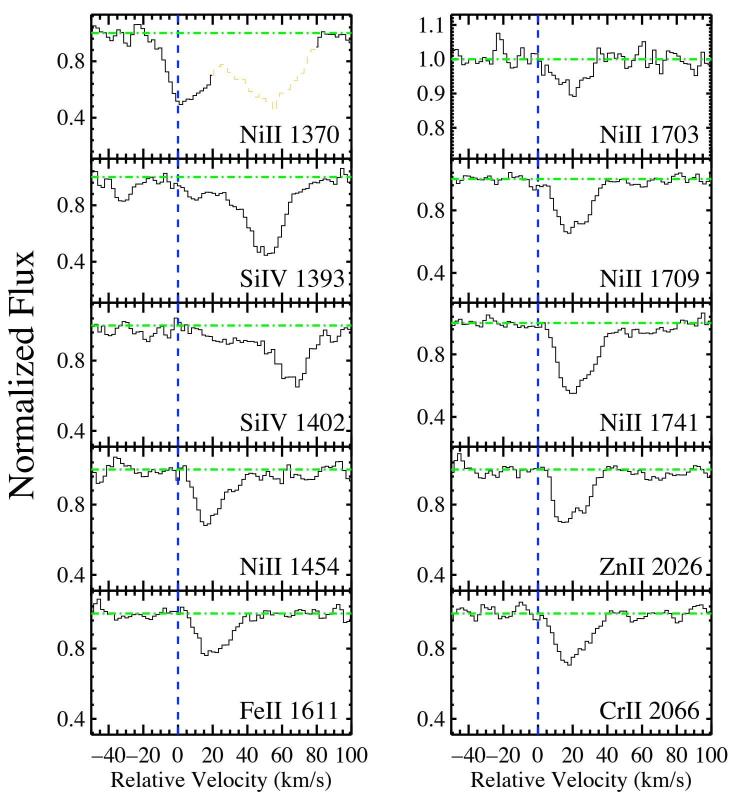
(2) ISM analysis and dust depletion:

(a) On the web site is a normalized spectrum of a high redshift quasar. Along the sightline to this quasar is a damped Lyα system with a redshift of $z \approx 2.309$. Identify and plot the following transitions from this galaxy:

Table 1: Atomic Data

Wave (Å)	Name	f
1370.131	NiII 1370	0.0769
1393.755	SiIV 1393	0.5280
1402.770	SiIV 1402	0.2620
1454.842	NiII 1454	0.0323
1611.2005	FeII 1611	0.00136
1703.405	NiII 1703	0.0060
1709.6042	NiII 1709	0.0324
1741.5531	NiII 1741	0.0427
2026.136	ZnII 2026	0.48900
2066.161	CrII 2066	0.0515

Solution: Here is a velocity plot where $v = 0$ is $z = 2.309$. It is evident that the Ni II 1370 profile is blended with some coincident feature (unrelated to our galaxy!) and should not be included in any of the following analysis.



(b) Explain the physics which leads to the Si IV doublet and then predict the ratio of the oscillator strengths of these two transitions. Measure the reduced equivalent widths (W/λ) of the Si IV transitions and discuss the values.

Solution: The doublet arises due to spin-orbit coupling, analagous to the 2p splitting in the Hydrogen atom. The ratio of the oscillator strengths, is therefore the ratio of the level populations of the split energy levels where $g_J = 2J + 1$. For $J = 3/2$ and $J = 1/2$, we expect a 2:1 ratio which is as observed.

I find: $W/\lambda = 6.5 \times 10^{-5}$ for $\lambda 1393$ and $W/\lambda = 3.9 \times 10^{-5}$ for $\lambda 1402$. Not quite in the 2:1 ratio so $\lambda 1393$ must no longer be on the linear portion of the curve-of-growth.

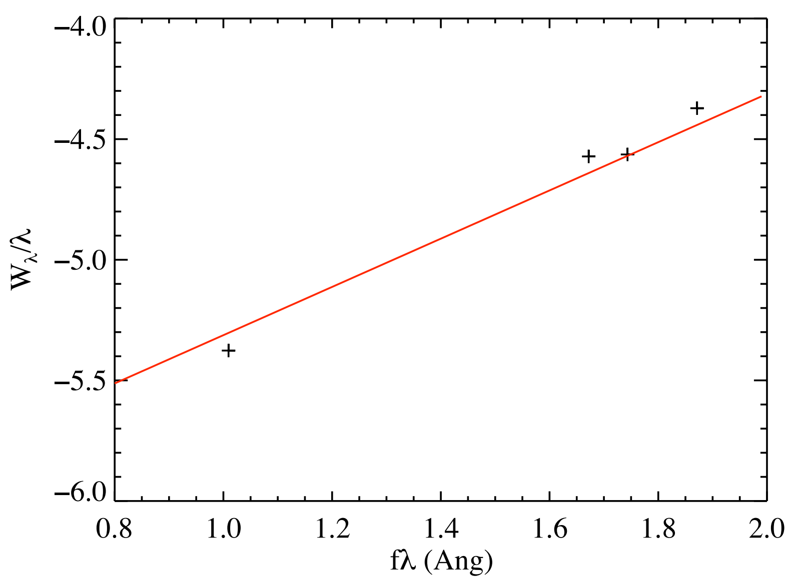
(c) Perform a curve of growth analysis of the Ni II transitions. What is the best 'fit' column density? (You don't have to perform a reduced χ^2 analysis. A simple fit by eye is fine). Present a plot of $\log W/\lambda$ vs. $\log(f\lambda)$ for the data and an overplot of the best fit N and b values. You may neglect Natural broadening in your analysis.

Solution: Here are the reduced EW that I measure, ignoring the blended Ni II 1370 transition.

Table 2: LINE EMISSION

Wave (Å)	fλ (Å)	W_λ/λ (10^{-5})
1454.84	46.99	2.68
1703.41	10.22	4.20
1709.60	55.39	2.73
1741.55	74.36	4.24

And here is my plot (log values on each axis), with column density $N = 5.5 \times 10^{13}\,\text{cm}^{-2}$ The results are independent of the b-value because the Ni II lines are on the linear portion of the curve-of-growth.

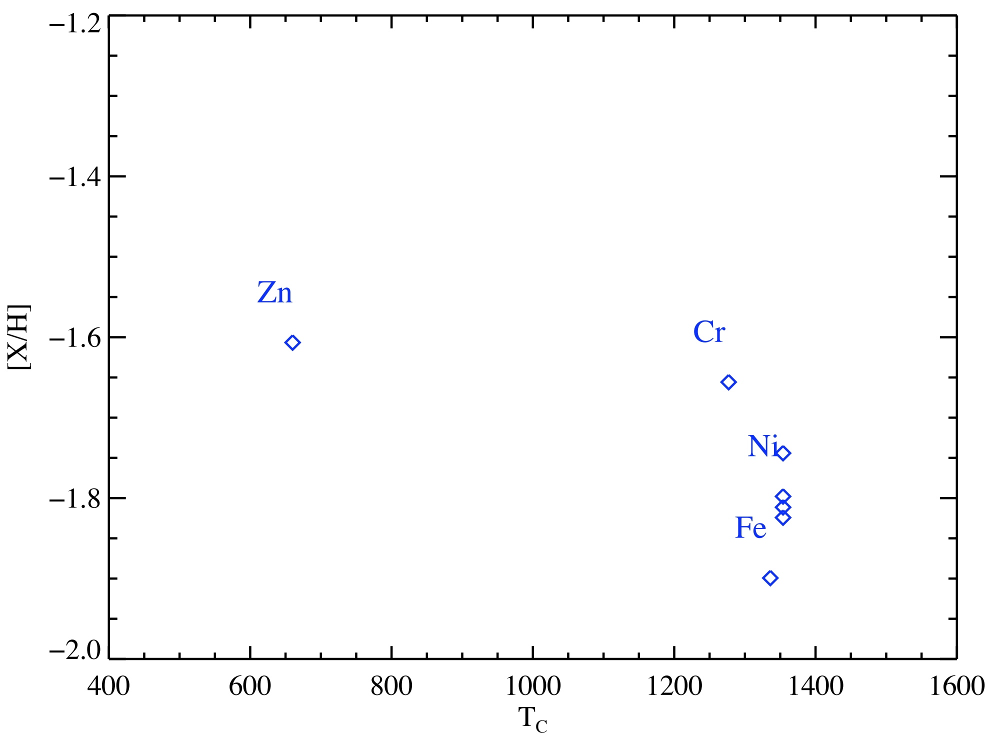


(d) Measure column densities (I suggest the apparent optical depth method of Savage & Sembach 1991) for all of the transitions in Table 1. Plot your values relative to the Solar values (i.e. [X/H]) against the condensation temperature T_C assuming $N_{\rm HI} = 10^{21.37}\,{\rm cm}^{-2}$. Discuss (what do you make of [Si/H]?).

Solution: Here are my numbers for the column densities.

Table 3: LINE EMISSION

Wave (Å)	T_C (K)	log N	[X/H]
1393.76	1311	13.12	−3.8
1402.77	1311	13.14	−3.8
1454.84	1354	13.82	−1.8
1611.20	1336	14.97	−1.9
1703.41	1354	13.79	−1.8
1709.60	1354	13.80	−1.8
1741.55	1354	13.87	−1.7
2026.14	660	12.43	−1.61
2066.16	1277	13.32	−1.65



There is a very modest trend with T_C, much less than observed in the Milky Way. Therefore, the gas is much less dusty than the standard Milky Way sightline. The values are primarily low because the gas has a low metallicity, about 1/30 solar abundance. [Si/H] is abnormally low because Si IV is not the dominant ion in the neutral ISM. See Wolfe et al. 1994 for more details on this galaxy (one of the first Keck papers published).