Fred's Rocket Ship

The Problem

Fed up with it all you decide to flee the galaxy, taking off at 1 g acceleration perpendicular to the plane of the galaxy. Describe the subsequent visual appearance of the galaxy over your lifetime.

The Solution

A path with acceleration a in the x direction will be a hyperbola in spacetime (it has to be Lorentz symmetric) given by

In[2]:=

```
path = {Sinh[s]/a, (Cosh[s] -1)/a, 0}
```

Out[2]=

```
 Sinh[s]   -1 + Cosh[s]
{-------, ------------, 0}
    a           a
```

Here the dimensionless proper time s is used, time in seconds multiplied by the acceleration a. For the acceleration of gravity on the earth, s will be measured in years.
The angle seen by a stationary observer at the location of the moving observer at proper time s will be (this is the angle measured from the backward direction)

In[11]:=

```
p1[l_,s_] := ArcTan[l/(Cosh[s]-1)]
```

Due to aberration, this angle will appear to the moving observer to be

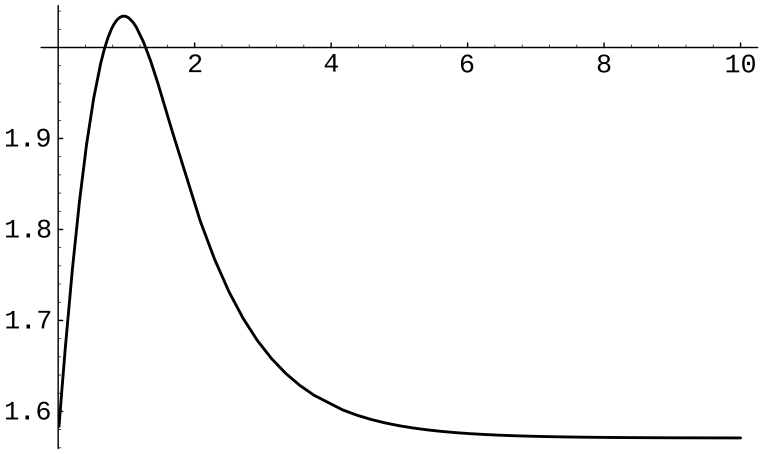
In[12]:=

```
p2[p_, s_] := 2 ArcTan[Exp[s] Tan[p/2]]
```

We can examine various cases numerically

In[29]:=

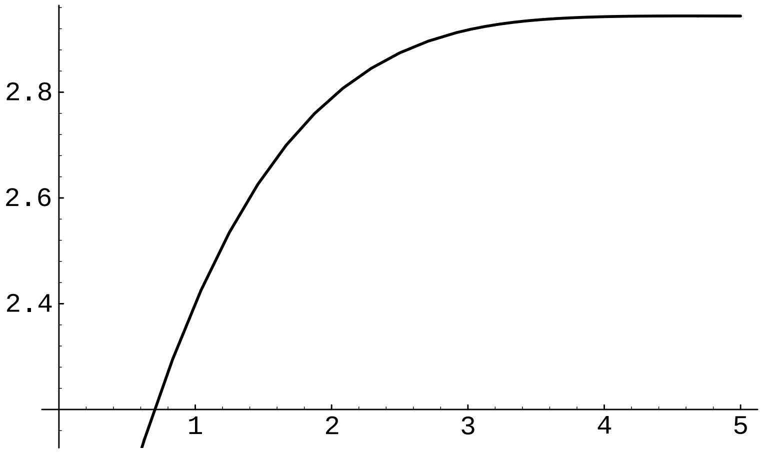
```
Plot[ Evaluate[p2[p1[1.,s],s]], {s,0,10}]
```



Thus we see that an object one light year away from us will first move forward, aberration always wins for early times over geometry, and then move back nearly to right angles.

In[31]:=

```
Plot[ Evaluate[p2[p1[10.,s],s]], {s,0,5}]
```



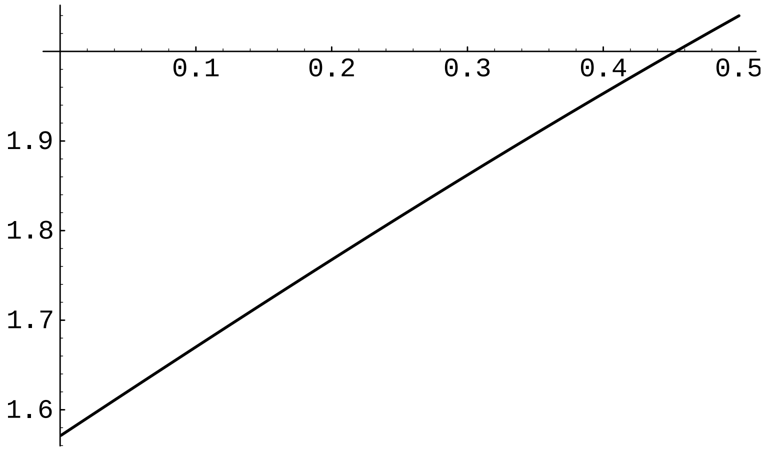
Out[31]=

```
-Graphics-
```

And here we see an object further out, at 10 light years, moving forward and going to a constant angle. What did it do originally?

In[32]:=

```
Plot[ Evaluate[p2[p1[10.,s],s]], {s,0,.5}]
```



Limits

The cases of interest are rather extreme, neither s nor l is of order unity, but rather more like 50 and 10,000. The limit structure of the above equations is rather complex, and there are five distinguished cases (a distinguished case cannot be derived as an approximation to another distinguished case).

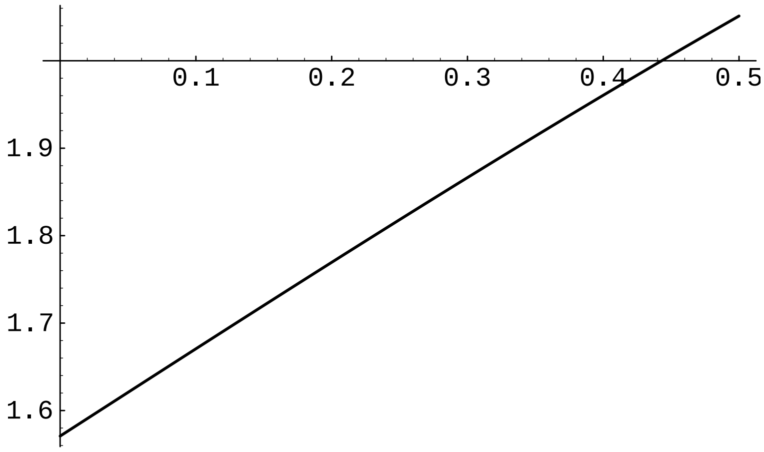
If you have l large and s of order one, then the limit will be:

In[33]:=

```
p2a[s_] := 2 ArcTan[Exp[s]]
```

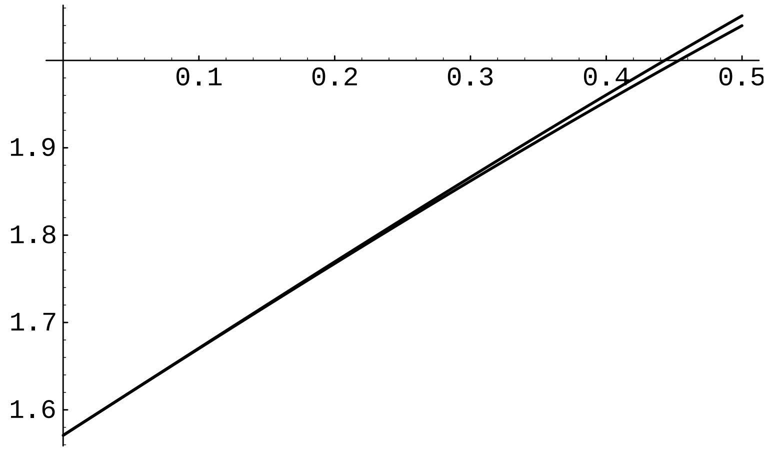
In[34]:=

```
Plot[ Evaluate[p2a[s]], {s,0,.5}]
```



In[35]:=

```
Show[ %34, %32];
```



Another extreme is to have l tiny and s of order unity:

In[36]:=

```
p2b[l_, s_] := Exp[s] l/(Cosh[s]-1)
```

Surprisingly, this is not correct for very small times, and there are two special cases to cover this. For l and s of the same small size we have

In[37]:=

```
p2c[l_, s_] := Pi/2 - s^2/(2 l) + s
```

and for l of size s^2 we have:

In[38]:=

```
p2d[l_, s_] := ArcTan[2 l/s^2]
```

For large times, if we have l of order unity:

```
p2e[l_] := 2 ArcTan[l]
```

and so objects the size of the solar system go to a limiting angle, as it appeared in the numerical plot above. The final distinguished limit is for l of size Exp[s]:

In[39]:=

```
p2f[l_] := Pi - 2 Exp[-s]/Tan[ArcTan[2 l Exp[-s]]/2]
```

In terms of these limits, on the human lifetime of say 50 years, a large galaxy would have a size of Exp[50] light years. So for the actual galaxy, the small object limit of either p2e or p2f is the appropriate one.

Conclusion

The final conclusion is that the galaxy appears to surround you, but leaves a small disk in front of you unfilled, of size

In[40]:=

```
p2g[l_] := Pi - 2/l
```