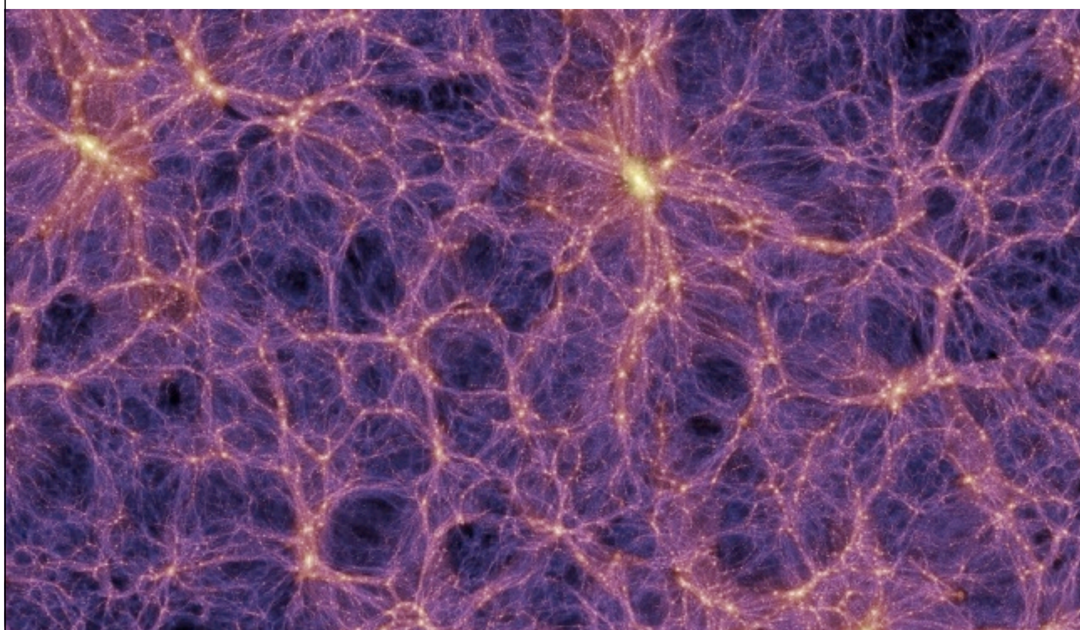
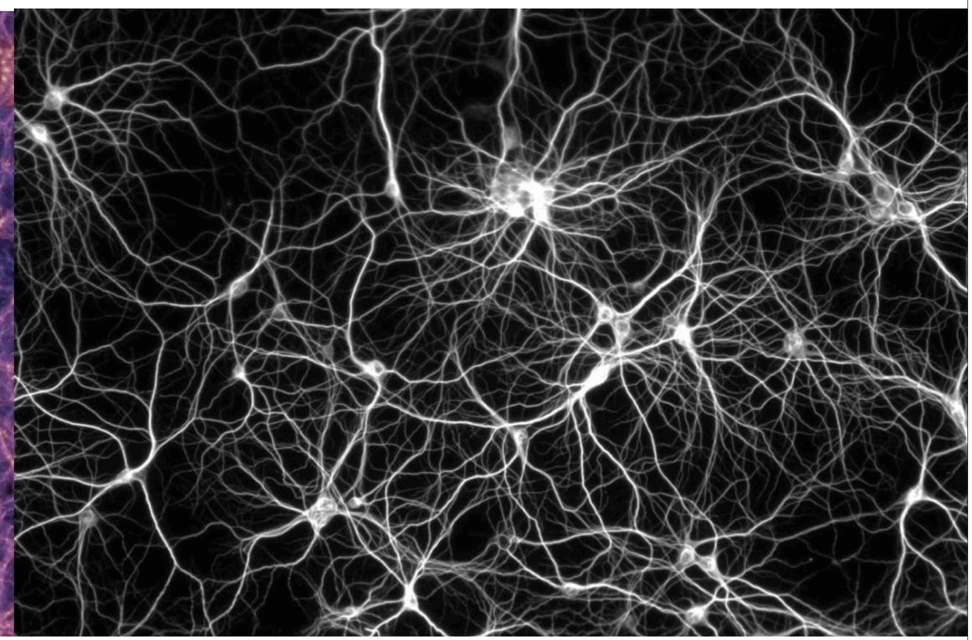
Energy Scales of Physical Phenomena

energy scales: pressure

Pressure exerted by the a system of particles as the rate of momentum transferred (per unit area) from particles of energy ϵ:

$$P = \frac{1}{3}\int_O^\infty n(\epsilon)p(\epsilon)v(\epsilon)d\epsilon = \begin{cases} \frac{2}{3} < n\epsilon > & \text{if } mc^2 \gg \epsilon \\ \frac{1}{3} < n\epsilon > & \text{if } mc^2 \ll \epsilon \end{cases}$$

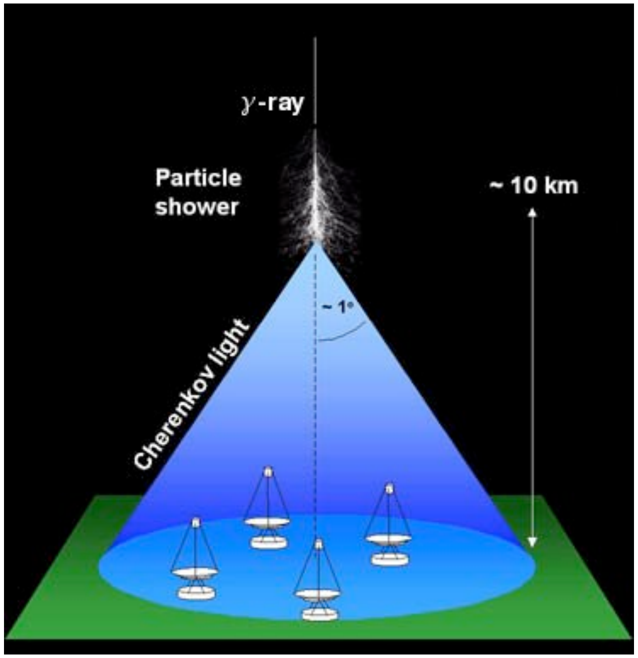
energy scales: rest-mass

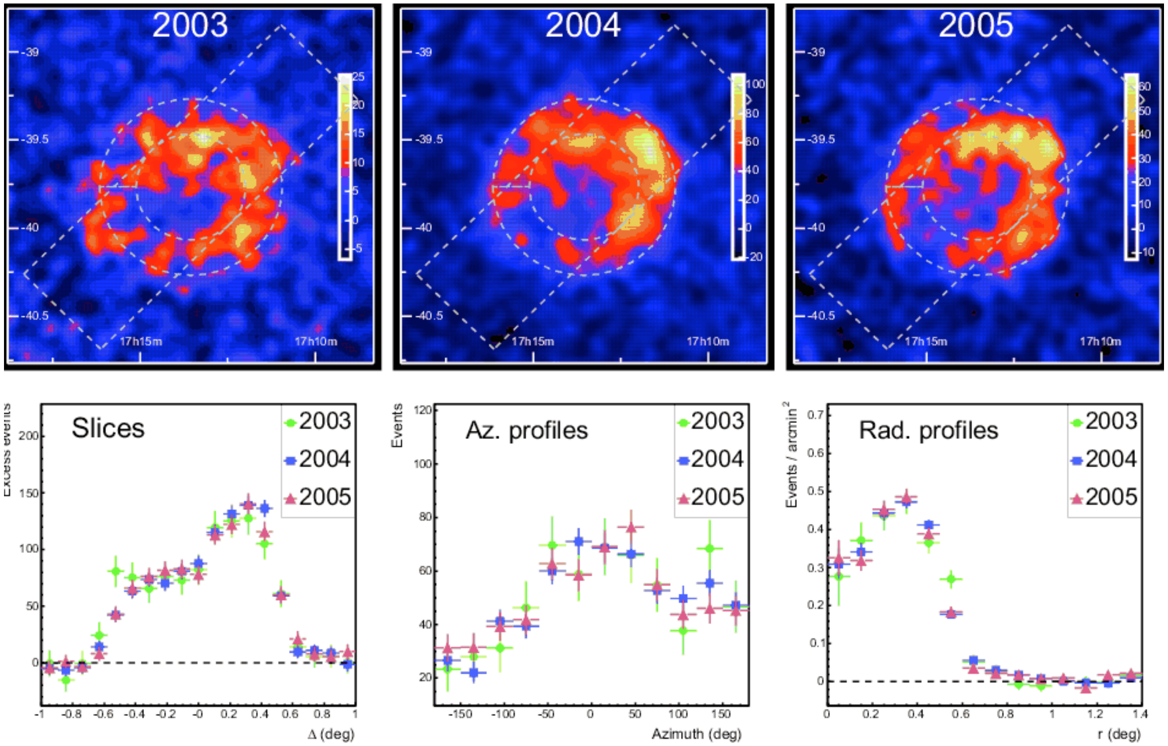
We can associate the rest mass energy mc^2 with each particle of mass m.

$$mc^2 = \begin{cases} m_e c^2 \approx 0.5\text{MeV} \\ m_p c^2 \approx 1\text{GeV} \end{cases}$$

If the particles of the system have internal structure (molecular, atomic, nuclear, etc) then we get further energy scales that are characteristic of the interactions. The simplest is the atomic binding energy of atoms and molecules, which arises from the electromagnetic coupling between particles.

energy scales: rest-mass





energy scales: atomic

Size and energy of the ground state of a hydrogen atom:

$$a_0 = \frac{\hbar^2}{m_e q^2} \approx 5.2 \times 10^{-9} \text{ cm}$$

$$\epsilon_a = \frac{m_e q^4}{2\hbar^2} = \frac{1}{2}\alpha^2 m_e c^2 \approx 13.6 \text{ eV}$$

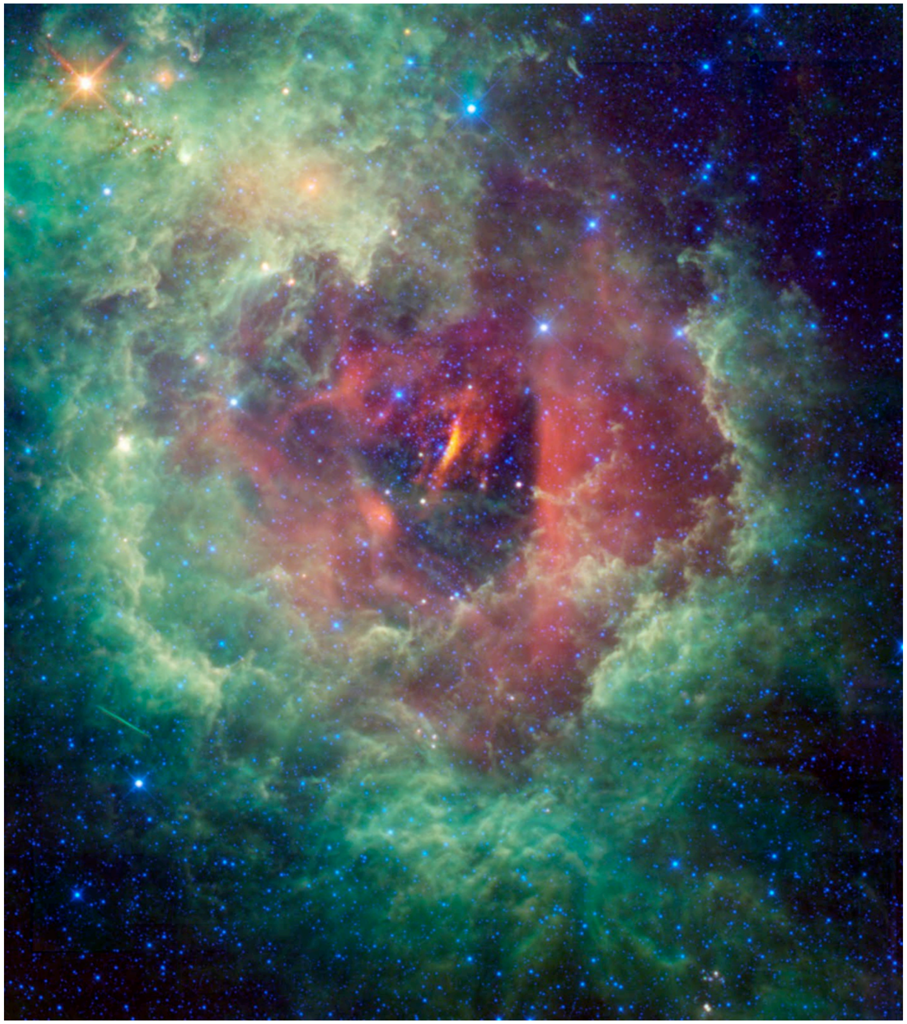
$\alpha = q^2/(\hbar c)$ is the fine structure constant.

The wavelength corresponding to ϵ_a is $\lambda = \frac{hc}{\epsilon_a} = \frac{2\hbar}{\alpha^2 m_e c} \approx 10^3 \text{A}$

and lies in the UV. When the atoms of size a_0 are closely packed:

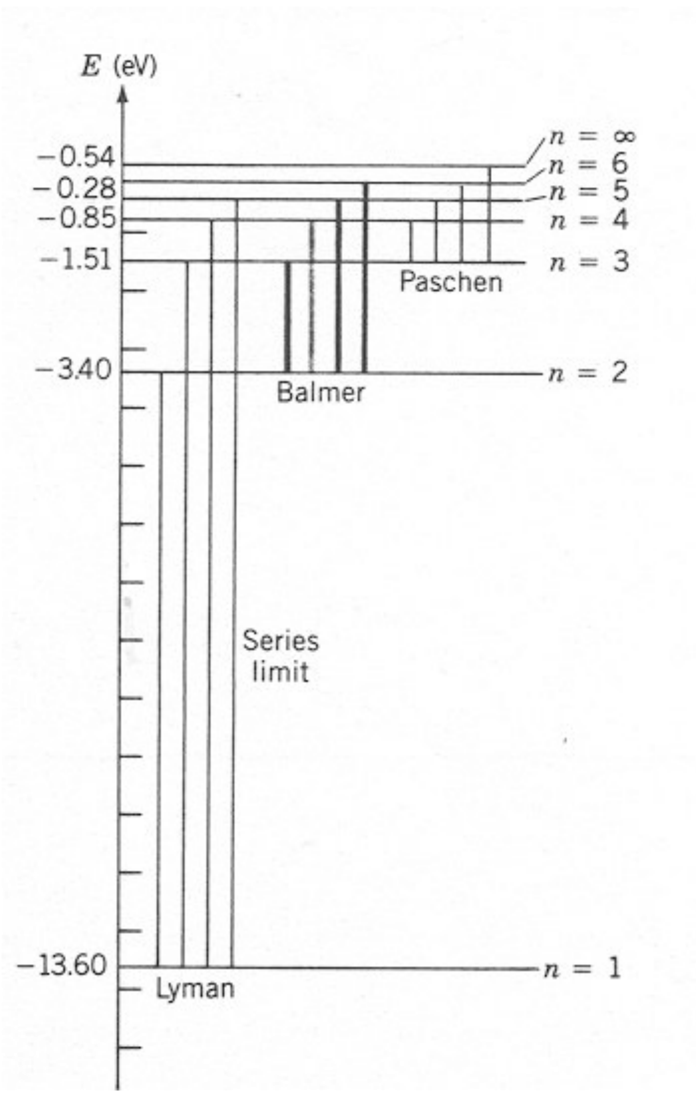
$$n_{\text{solid}} \approx (2a_0)^{-3} \approx 10^{24} \text{ cm}^{-3}.$$

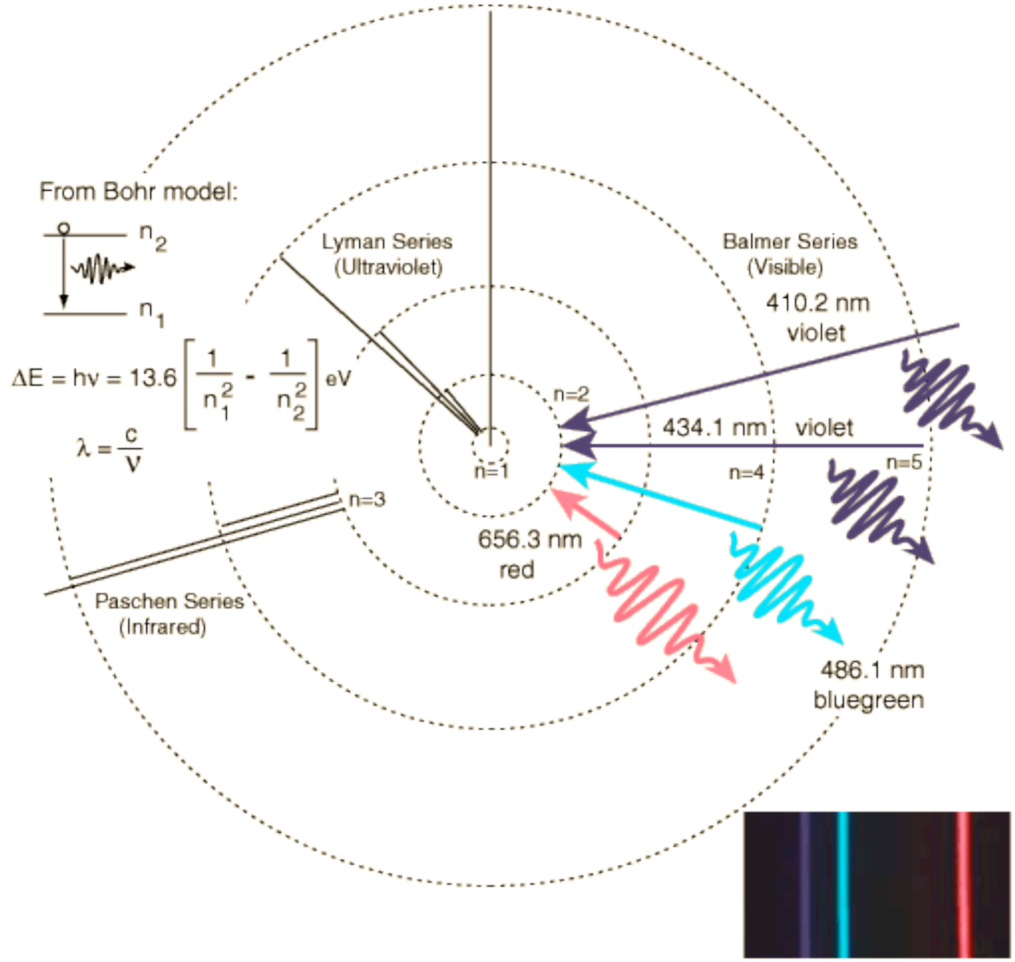
energy scales: atomic



NGC 2237, the Rosette Nebula in Monoceros

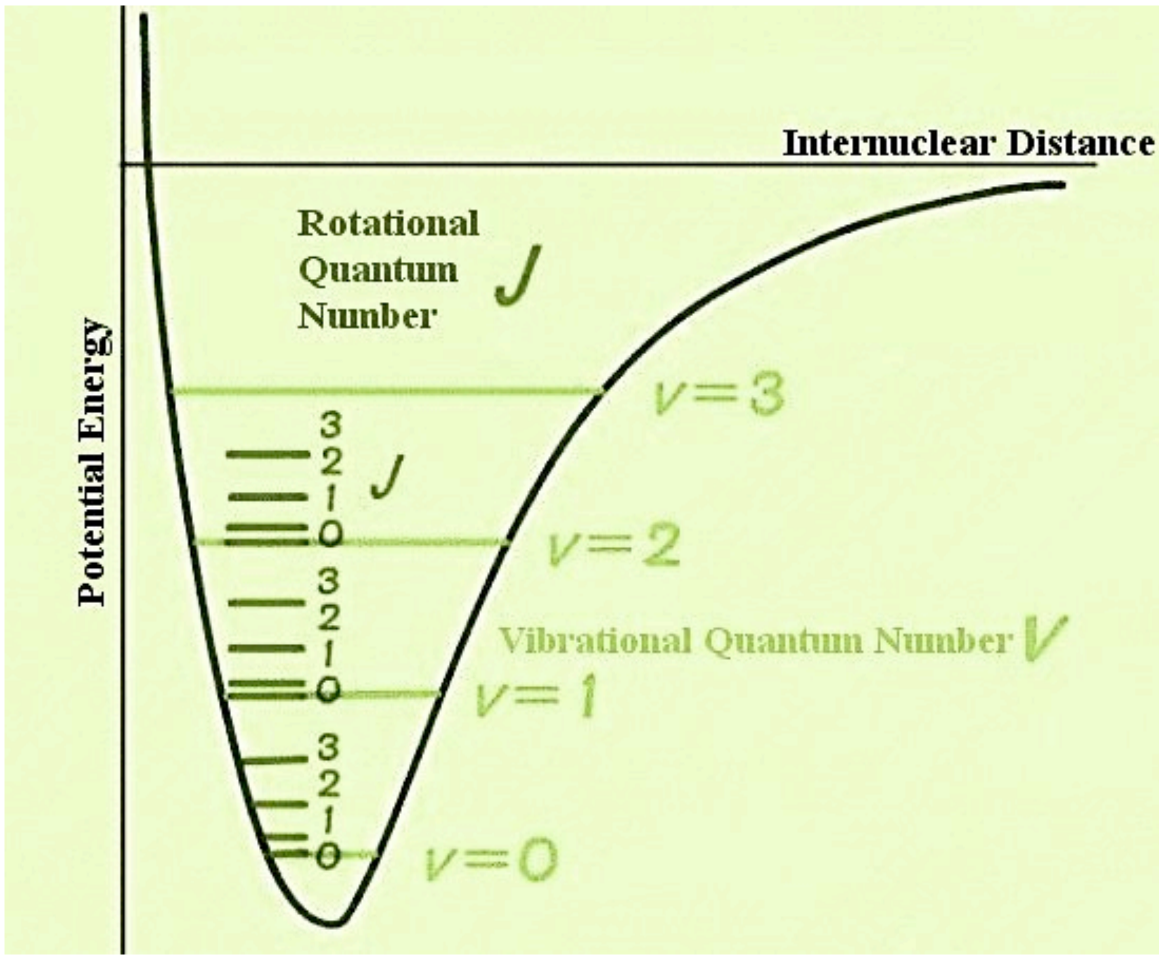
energy scales: atomic





energy scales: molecular

The simplest molecular structure consists of two atoms bound to each other in the form of a diatomic molecule:



energy scales: molecular

In addition to the electronic internal binding energy of the atoms, there are two other contributions to the energy of a diatomic molecule. Vibrational energy levels separated by

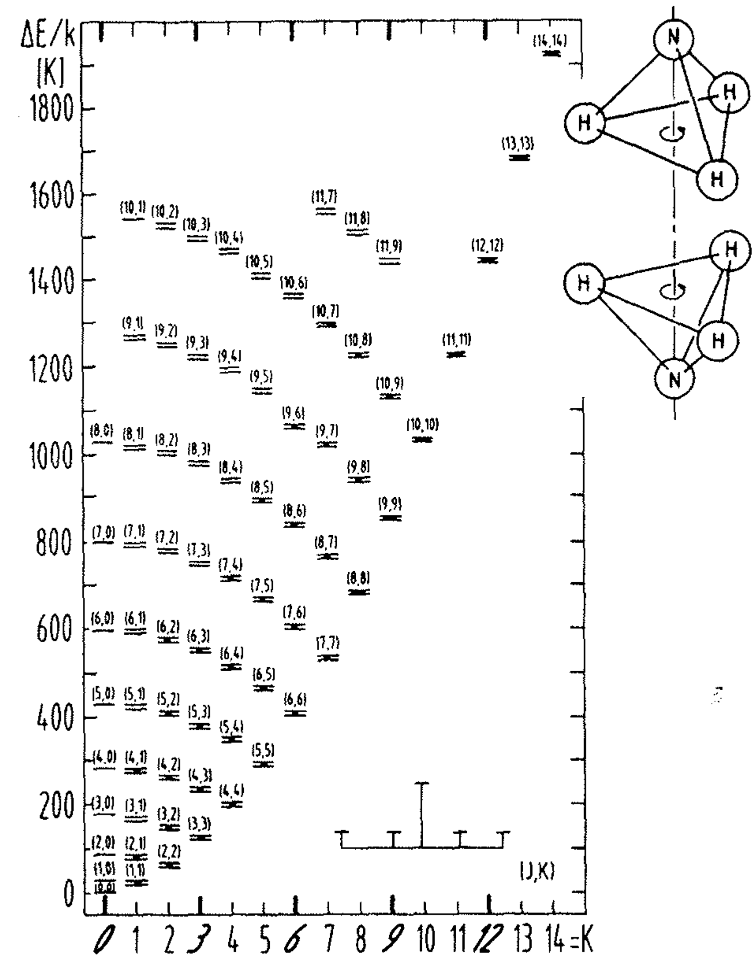
$$E_{\rm vib} = \hbar\omega_{\rm vib} \approx \left(\frac{m_e}{\mu}\right)^{1/2} \epsilon_a \approx 0.25 \text{ eV}.$$

The molecule can also rotate. This will contribute with an energy of approximately

$$E_{\rm vrot} = \frac{J^2}{\mu a_0^2} \approx \left(\frac{m_e}{\mu}\right) \epsilon_a \approx 10^{-2} \text{ eV}.$$

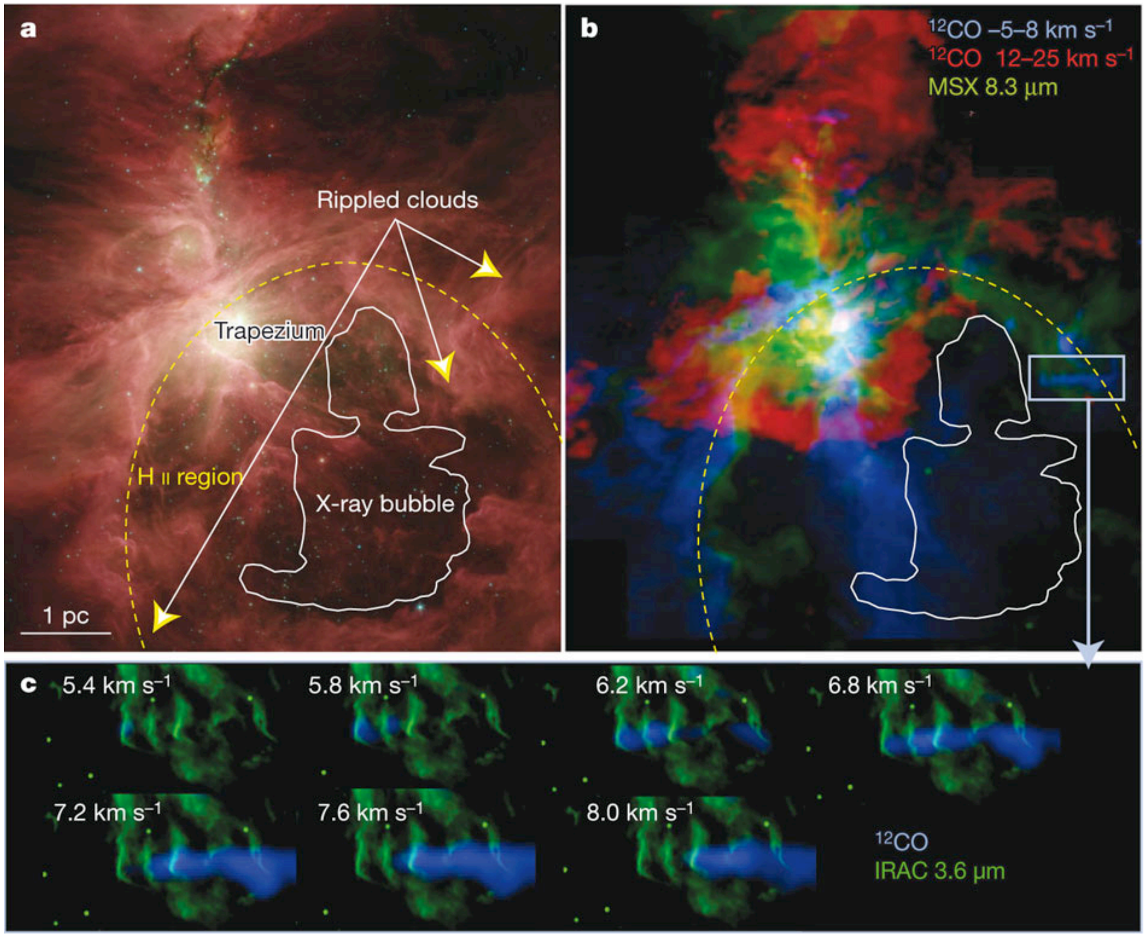
The wavelengths of radiation from vibrational transitions are about 40 times larger than those of electronic transitions

energy scales: molecular



Energy levels of ammonia (NH3) in the lowest vibrational state (Wilson, T. L. et al. 1993, A&A, 276, L29).

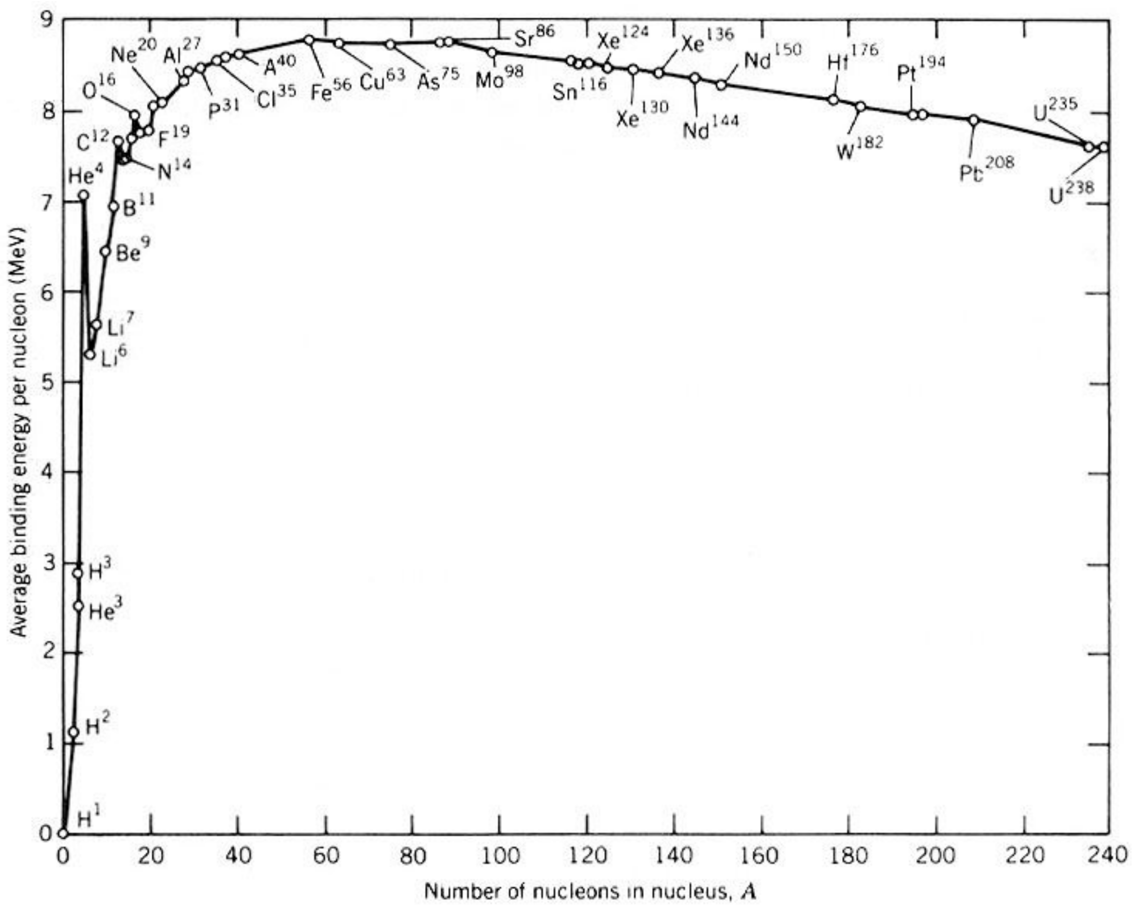
energy scales: molecular



Multi-wavelength overview of the Orion nebula.

energy scales: nuclear

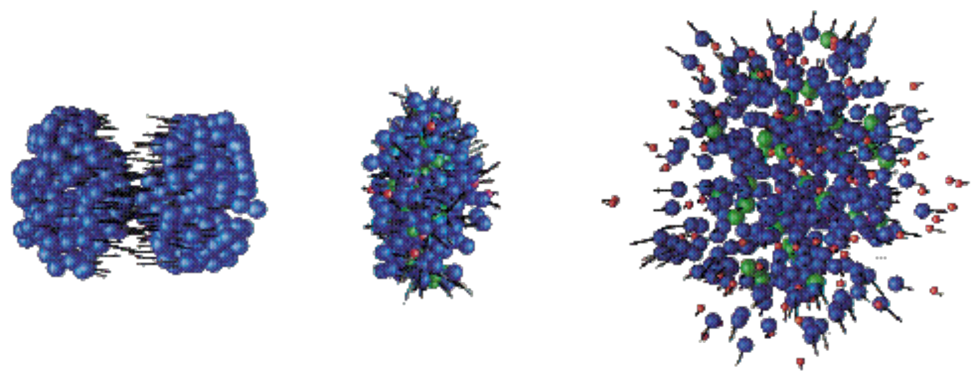
Atomic nuclei are bound by the strong interaction force that produces a binding energy per particle about 8 MeV, which is the characteristic scale for nuclear energy levels



energy scales: nuclear

In the astrophysical context, a more relevant energy scale is the one at which nuclear reactions can be triggered. For two protons to fuse together, while undergoing nuclear reaction, it is necessary that they are brought within the range of attractive nuclear force. This requires overcoming the Coulomb repulsion:

$$\epsilon \approx \frac{q^2}{l} = \frac{\alpha}{2\pi} m_p c^2 \approx 1\ \mathrm{MeV}.$$



order of magnitude

This discussion assumes that the sun is primary supported by pressure of an ideal gas of electrons and ions. Thus, in a star, the total (thermal) kinetic energy of particles is

$$U_{\rm th} \sim kT$$

An estimate of the typical temperature inside the Sun can be estimated by combining the kinetic and gravitational potential energy per particle:

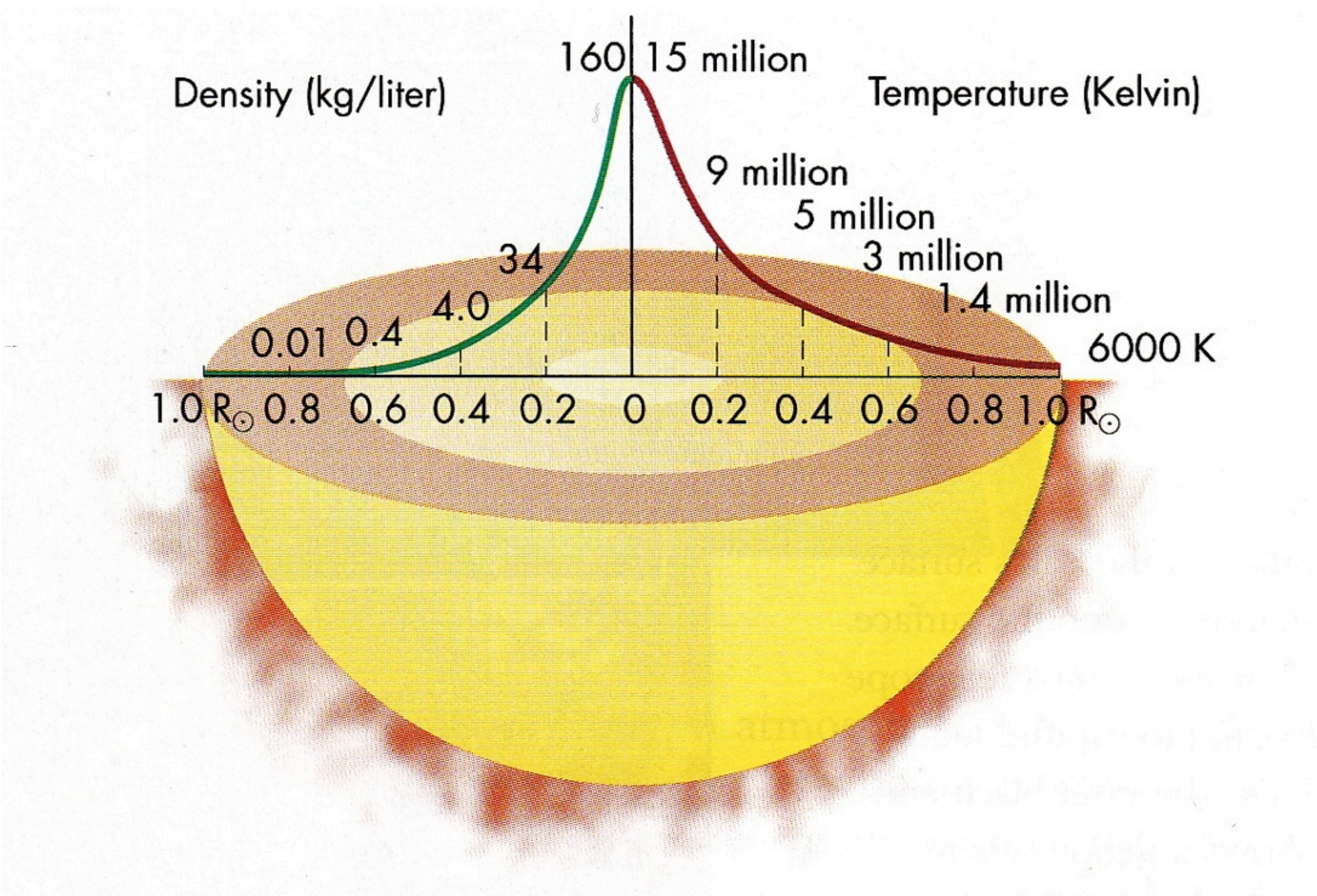
$$kT_i \sim \frac{GM_\odot m_p}{R_\odot}$$

$$T_i \sim 1.8 \times 10^7 \ {\rm K}$$

where *k* is the Boltzman constant:

$$k = 1.38 \times 10^{-23} \ {\rm J/K}$$

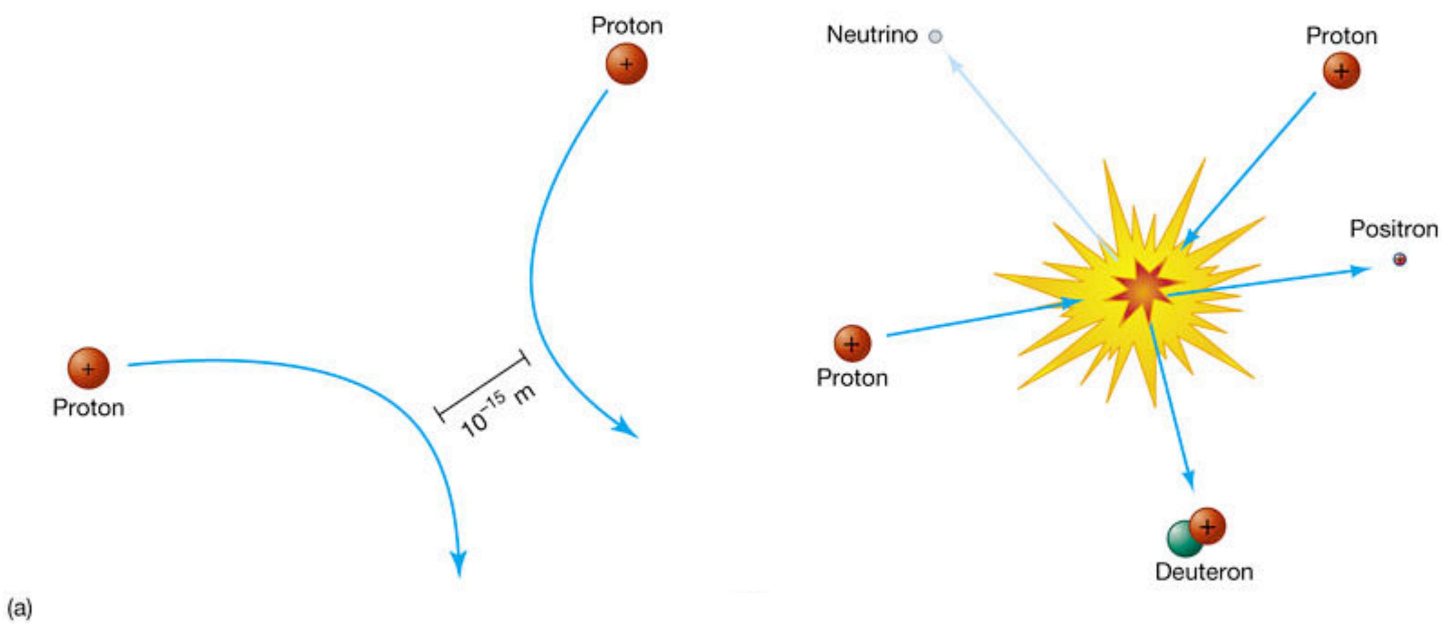
sun's structure



energy scales: nuclear

It is, however, possible for nuclear reactions to occur through quantum-mechanical tunneling when the de Broglie wavelength $\lambda_{\rm deB} = h/(m_p v)$ of the two protons overlap. This occurs when the energy is approximately

$$\epsilon_{\rm nucl} \approx \frac{\alpha^2}{2\pi^2}\, m_p c^2 \approx 1 \text{ keV}.$$



energy scales: gravitational

In the non-relativistic Newtonian theory of gravity, the gravitational binding energy:

$$E_{\rm grav} \approx \frac{GM^2}{R} \approx \frac{Gm_p^2}{R} N^2.$$

The potential energy per particle:

$$\epsilon_g = \frac{E_{\rm grav}}{N} = \frac{Gm_p^2}{R} N = \frac{4\pi}{3}^{1/3} G m_p^2 N^{2/3} n^{1/3}.$$

General relativistic become important when

$$R_{gm} = E_{\rm grav}/E_{\rm mass} \sim 1.$$

energy scales: thermal

The behavior of the system depends on the origin of the momentum distribution of the particles. The familiar situation is the one in which short-range interactions between particles effectively exchange the energy so as to randomize the momentum distribution. When such a system is in steady state, we can assume that the local thermodynamical equilibrium, characterized by a temperature T, exists in the system:

$$\epsilon \approx k_B T$$

$$p \approx mc \left[\frac{2k_B T}{mc^2} + \left(\frac{k_B T}{mc^2} \right)^2 \right]^{1/2} \approx \begin{cases} (2mk_B T)^{1/2} & \text{if } mc^2 \gg k_B T \\ k_B T/c & \text{if } mc^2 \ll k_B T \end{cases}$$

In this case, the momentum and the kinetic energy of the particle vanishes when

$$T \to 0.$$

energy scales: thermal

energy scales: degeneracy

The mean energy of a system of electrons will not vanish at zero temperature because electrons obey the Pauli exclusion principle, which requires that the number of electrons that can occupy any quantum state be two.

Because the uncertainty principle requires

$$\Delta x \Delta p_x \geq h,$$

we can associate the number of quantum states with a phase space volume:

$$p_F = \hbar(3\pi^2 n)^{1/3}.$$

The quantity ϵ_F sets the quantum mechanical scale of the energy

$$\epsilon_F = \sqrt{p_F^2 c^2 + m^2 c^4} - mc^2 \approx \begin{cases} \frac{p_F^2}{2m} = \left(\frac{\hbar^2}{2m}\right)(3\pi^2 n)^{2/3} & \text{if } mc^2 \gg \epsilon_F \\ p_F c = (\hbar c)(3\pi^2 n)^{1/3} & \text{if } mc^2 \ll \epsilon_F \end{cases}$$

energy scales: degeneracy

Quantum-mechanical effects will be dominant if

$$\epsilon_F \gg k_B T \text{ (degenerate)}$$

Electrons have $\epsilon_F \sim m_e c^2$ for

$$n = \left(\frac{\hbar}{m_e c}\right)^{-3} \approx 10^{31} \text{ cm}^{-3}$$
$$\rho \sim n m_p \approx 10^7 \text{ g cm}^{-3}.$$

$\epsilon_F \sim k_B T$ occurs at

$$n T^{-3/2} = \frac{(m k_B)^{3/2}}{\hbar^3} = 3.6 \times 10^{16} (\text{cgs}) \quad \text{for} \quad mc^2 \gg \epsilon_F.$$

energy scales: ionization

The energy scale of the individual particles also characterizes the energy involved in the collisions between particles. If this quantity is larger than the binding energy of the atomic system, the atoms will be ionized and the electrons will be separated:

$$k_B T \geq \epsilon_a.$$

The transition temperature at which nearly half of the atoms are ionized occurs when

$$k_B T \approx \epsilon_a/10 \sim 10^4 \text{ K}.$$

The atoms can be stripped off the atoms in another context. This occurs when the density is so high that the atoms are so close together, with electrons forming a common pool

$$\epsilon_F \geq \epsilon_a.$$

In this case, the electrons will be quantum mechanical.

energy scales: ideal plasma

To treat a plasma as ideal, it is necessary that the Coulomb interaction energy of ions and electrons be negligible. The typical Coulomb potential energy interaction between ions and the electrons in the plasma is given by

$$\epsilon_{\text{Coul}} \approx Zq^2 n^{1/3}.$$

If the high temperature plasma is to be treated as an ideal gas, this energy should be

$$\epsilon_{\text{Coul}} \leq k_B T,$$

which requires

$$nT^{-3} \leq \left(\frac{k_B}{Zq^2}\right)^3 \approx 2.2 \times 10^8 Z^{-3} \text{ (cgs)}.$$

energy scales: ideal plasma

On the other hand, to treat the high density quantum gas as ideal

$$\epsilon_{\rm Coul} \leq \epsilon_F = (\hbar^2/2m)n^{2/3},$$

which requires

$$n \geq 8Z^3 a_0^{-3} \approx 10^{26} Z^3 \ {\rm cm}^{-3}$$

$$\rho \approx n m_p \geq 10^2 \ {\rm g\ cm}^{-3}.$$