

Planetary Radii across Five Orders of Magnitude in Mass and Stellar Insolation: Application to Transits

J. J. Fortney^{1,2,3}, M. S. Marley¹, J. W. Barnes⁴

ABSTRACT

To aid in the physical interpretation of planetary radii constrained through observations of transiting planets, or eventually direct detections, we compute model radii of pure hydrogen-helium, water, rock, and iron planets, along with various mixtures. Masses ranging from 0.01 Earth masses to 10 Jupiter masses at orbital distances of 0.02 to 10 AU are considered. For hydrogen-helium rich planets, our models are the first to couple planetary evolution to stellar irradiation over a wide range of orbital separations (0.02 to 10 AU) through a non-gray radiative-convective equilibrium atmosphere model. Stellar irradiation retards the contraction of giant planets, but its effect is not a simple function of the irradiation level: a planet at ~ 1 AU contracts as slowly as a planet at ~ 0.1 AU. For all hydrogen-helium planets, we show the effect on radii due to cores of up to 90% of the total planet mass, comparable to that of Uranus and Neptune. If “hot Neptunes” have maintained their original masses and are not remnants of more massive planets, radii of ~ 0.30 - $0.45 R_J$ are expected. We confirm the assertion of Guillot that very old giant planets under modest stellar irradiation (like that received by Jupiter and Saturn) develop isothermal atmospheric radiative zones once the planet’s intrinsic flux drops to a small fraction of the incident flux. Water ice planets are $\sim 40 - 50\%$ larger than rocky planets, independent of mass. Finally, we provide tables of planetary radii at various ages and compositions, and for ice-rock-iron planets we fit our results to analytic functions, which will

¹Space Science and Astrobiology Division, NASA Ames Research Center, MS 245-3, Moffett Field, CA 94035; jfortney@arc.nasa.gov, mark.s.marley@nasa.gov

²Spitzer Fellow

³Carl Sagan Center, SETI Institute, 515 North Whisman Road, Mountain View, CA 94043

⁴Lunar and Planetary Laboratory and Department of Planetary Sciences, University of Arizona, Tucson, AZ, 85721; jbarnes@c3po.lpl.arizona.edu

allow for quick composition estimates, given masses and radii, or mass estimates, given only planetary radii. These results will assist in the interpretation of observations for both the current transiting planet surveys as well as upcoming space missions, including COROT and Kepler.

Subject headings: planetary systems, binaries: eclipsing

1. Introduction

We are still near the start of a radical shift in the field of planetary sciences that is being triggered by the discovery of planets around other stars. These exoplanets now number over 200, with masses as small as $\sim 5\text{-}7 M_{\oplus}$ (Rivera et al. 2005; Beaulieu et al. 2006). Comparative planetology, which once included only our solar system’s planets and moons, now includes sub-Neptune to super-Jupiter-mass planets in other solar systems.

Currently the most important class of exoplanets are those that transit the disk of their parent stars, allowing for a determination of planetary radii. The 14 confirmed transiting planets observed to date are all more massive than Saturn, have orbital periods of only a few days, and orbit stars bright enough such that radial velocities can be determined, allowing for a calculation of planetary masses and bulk densities (see Charbonneau et al. 2006). A planetary mass and radius allows us a window into planetary composition (Guillot 2005). The 14 transiting planets are all gas giants (See Guillot et al. 1996; Guillot & Showman 2002; Bodenheimer et al. 2003; Burrows et al. 2003; Baraffe et al. 2005; Laughlin et al. 2005; Arras & Bildsten 2006 for calculations of structure and contraction of many planets) although one planet, HD 149026b, appears to be $\sim 2/3$ heavy elements by mass (Sato et al. 2005; Fortney et al. 2006; Ikoma et al. 2006). If we can begin to understand how the transiting planet mass-radius relations change as a function of orbital distance, stellar mass, stellar metallicity, or UV flux, we will start to understand the fundamentals of planetary formation, migration, and evolution. Tentatively, some work in this general direction is beginning (Guillot et al. 2006).

The transit method of planet detection is biased towards finding planets that orbit relatively close to their parent stars. This means that radial velocity followup will be possible for some planets as the stellar “wobble” signal is larger for shorter period orbits. However, for transiting planets that are low-mass, or that orbit very distant stars, stellar radial velocity measurements may not be possible. For planets at larger orbital distances, radial velocity observations may take years. Therefore, for the foreseeable future a measurement of planetary radii will be our only window into the structure of these planets. Estimates of

masses will still be important, however, for useful constraints on planet formation theories. This will have to involve some degree of presumption regarding the composition of planets. Orbital distances may give some clues as to a likely composition, but our experience over the past decade with Pegasi planets (or “hot Jupiters”) has shown us the danger of assuming certain types of planets cannot exist at unexpected orbital distances. Here we compute planetary radii as function of mass, composition, and stellar irradiation to help in these future planetary mass and composition estimates.

In §2 we give a brief overview of the COROT and Kepler missions. In §3 we describe the scope of the calculations and our aims in this study. Later, §4 describes our methods while §5 and §6 give our results for ice-rock-iron planets and gaseous planets, respectively. Finally, §7 is our conclusions and suggestions for future work.

2. Upcoming Space Missions

The French/European COROT mission, set to launch in December 2006, and American Kepler mission, set to launch in November 2008, will revolutionize the study of exoplanets. COROT will monitor 12,000 stars in each of 5 different fields, each for 150 continuous days (Bordé et al. 2003). Planets as small as $2 R_{\oplus}$ should be detectable around solar-type stars (Moutou & The COROT/Exoplanet Team 2006). The mission lifetime is expected to be at least 2.5 years. The Kepler mission will continuously monitor one patch of sky, monitoring over 100,000 main sequence stars (Basri et al. 2005). The expected mission lifetime is at least 4 years. Detection of sub-Earth size planets is the mission’s goal, with detection of planets with radii as small as 1 Mercury radius possible around M stars. With these missions, perhaps hundreds of planets will be discovered with masses ranging from sub-Mercury to many times that of Jupiter. Of course while planets close to their parent stars will preferentially be found, due to their shorter orbital periods and greater likelihood to transit, planetary transits will be detected at all orbital separations. In general, the detection of three successive transits will be necessary for a confirmed detection, which will limit confirmed planetary-radius objects to ~ 1.5 AU. It is important to remember that COROT and Kepler will not determine planetary masses, only radii, so until followup radial velocity or astrometry work is done, planetary radii will be our only window into the structure of these new planets.

3. Focus & Scope of Calculations

In the next few years transiting planets will be detected that range from many Jupiter masses ($1 M_J = 317.89 M_\oplus$), to perhaps as small as $0.01 M_\oplus$. Radii for planets must therefore be understood over more than five orders of magnitude in mass. Discoveries to date have been surprising, including planets that are larger than standard models can explain (such as HD 209458b and HAT-P-1b), Saturn-mass planets that have four Neptune-masses worth of heavy elements (such as HD 149026b), Neptune-mass planets that are quite hot and the perhaps not ice-rich (such as HD 68930b), and ~ 5 - $10 M_\oplus$ planets that some are calling “super-Earths” (such as Gliese 876d). Therefore, we think it is useful to take as broad a view as possible. We make few assumptions regarding composition, and we calculate radii of pure hydrogen-helium planets as small as $10 M_\oplus$, water and rock dominated planets up to $1000 M_\oplus$ ($\sim 3 M_J$), along with many compositions in between. Since we have the theoretical tools at our disposal, excluding some compositions a priori is unnecessary at this point.

We use state-of-the-art equations of state (EOS) for iron (Sesame 2140, Lyon & Johnson 1992), olivine (for generic “rock”) (ANEOS, Thompson 1990), water (ANEOS, Thompson 1990), helium (Saumon et al. 1995), and hydrogen (Saumon et al. 1995). Although detailed models for the thermal evolution of super terrestrial planets have been published (Valencia et al. 2006; Ehrenreich et al. 2006), issues discussed in such detailed models, such as lithospheric thickness and interior temperature structure will remain unobservable, and have only a small effect on planetary radii (temperature affects may reach $\sim 4\%$ in radius for the highly irradiated “Super Mercuries” of Valencia et al. 2006) although they may affect tectonic activity, and hence, habitability. Here we are interested in the most readily observed quantity: planetary radii. Even for precisely determined light curves, in all cases planetary radii will remain somewhat imprecise due to uncertainties in parent star radii, which can often reach 10%.

For hydrogen/helium planets, irradiation from a parent star is a significant energy source, and must be accounted for. We accurately incorporate stellar heating into our evolution (contraction) calculations through self-consistent non-gray model atmospheres under irradiation from 0.02 to 10 AU. We are the first to investigate the evolution and contraction of hydrogen/helium planets over such a wide range of orbital separations.

Other authors have previously tabulated radii for some subset of the compositions here. Zepolsky & Salpeter (1969) is perhaps the best known, in which they calculated the radius of zero-temperature spheres of H, He, C, Fe, and Mg for masses from $0.3 M_\oplus$ to $10 M_\odot$. They use Thomas-Fermi-Dirac EOSs that are accurate at very high pressure but somewhat less so the lower masses of our solar system’s planets. A similar tabulation appears in Stevenson (1982), who calculated radii from 1 to $1000 M_\oplus$ for cold and warm H, H/He, ice, and rock. Valencia

et al. (2006) have recently investigated the radii of Earth-like and Mercury-like planets up to $10 M_{\oplus}$. Saumon et al. (1996) have calculated planetary radii of giant planets with and without cores. Guillot et al. (1996) also investigated the radii of 51 Peg b-like planets, with compositions ranging from H/He to rock. More recently Bodenheimer et al. (2003) have calculated radii of H/He planets with and without cores, at various orbital separations, but these models lack realistic atmospheric boundary conditions and equations of state for core materials. Our results for water, rock, and iron bodies do not deviate significantly from these other works, other than we also compute radii for mixed compositions over a wider range of masses. However, our calculations for the radii of H/He rich planets are an improvement over other works, as we explicitly and accurately include irradiation from a parent star. In addition, we include the effects on planetary radii across a wide range of core masses.

4. Methods

4.1. Equations of State

First we will look in a bit more detail into our equations of state before elaborating on the construction of the planet models. The pressure, (P), can be expressed as:

$$P = P_0 + P_T \quad (1)$$

where P_0 is the pressure at zero-temperature and P_T is the thermal pressure. For the degenerate interiors of massive planets, thermal effects are quite small for high atomic number species. For instance, zero-temperature equations of state for rock are thought to be accurate to within $\sim 1\text{-}2\%$ for Uranus and Neptune interior models (Hubbard & Macfarlane 1980; Hubbard 1984), so we make no thermal corrections here. For water, thermal pressure can be important at the $\sim 10\%$ level. Hubbard & Macfarlane (1980) find a relation for the P_T of water that is suitable for the interior pressure-temperature (P - T) profiles of Uranus and Neptune. This relation,

$$P_T = 3.59 \times 10^{-5} \rho T, \quad (2)$$

where P_T is in Mbar, ρ is in g cm^{-3} , and T in K, is relevant for “typical planetary interior conditions.” For $\rho \sim 4 \text{ g cm}^{-3}$ and $T \sim 5000 \text{ K}$, $P_T \sim 0.7 \text{ Mbar}$, or 10% of the total pressure, P . For planets composed of some fraction of water, we assume that interior temperatures within the ice follow the Uranus/Neptune adiabat of Guillot (2005), which reaches $\sim 550 \text{ K}$ at 1 kar and 4000 K at 1 Mbar , and add the thermal pressure correction of Equation (2) at every P_0 . This is our EOS of “warm ice.” For the time being we will ignore the additional complexity of including the EOS of methane and ammonia, solar system “ices” that are not as abundant as water. In Figure 1 we show our equations of state for cold ice, warm ice,

rock, and iron. For hydrogen/helium envelopes, we compute internal adiabats with a helium mass fraction $Y = 0.28$ and do not include heavy elements. A description of this detailed H/He EOS can be found in Saumon et al. (1995).

4.2. Planetary Structure and Evolution

The structure of the spherically symmetric model planets follow the relations set out below. Equations (3) and (4) define mass conservation and hydrostatic stability, respectively. For the planets for which we follow evolution (contraction), which are those with a H/He envelope, we also follow energy conservation, as defined by Equation (5).

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \quad (3)$$

$$\frac{\partial P}{\partial m} = \frac{-Gm}{4\pi r^4} \quad (4)$$

$$\frac{\partial L}{\partial m} = -T \frac{\partial S}{\partial t}, \quad (5)$$

where r is the radius of a mass shell, m mass of a given shell, ρ the local mass density, P the pressure, G the gravitational constant, L the planet’s intrinsic luminosity, T the temperature, S the specific entropy, and t the time.

For planets composed only of ice, rock, or iron, we do not utilize Equation (5), as we assume a constant radius with age. Given the small thermal component of the pressure for these materials, and the expected uncertainty in radius measurements, this assumption is valid. For planets where hydrogen and helium make up an appreciable mass fraction, following the thermal evolution and contraction of these planets is essential. We note that in these models we do not include additional interior energy sources for the hot Jupiters. We also neglect helium phase separation, which will add ~ 1000 km in radius to cold giant planets at Gyr ages (Fortney & Hubbard 2004). Recall that for the planet with the most precisely determined radius HD 209458b, the 1σ radius uncertainty is still 1.9% (~ 1800 km), due to uncertainties in the stellar parameters (Knutson et al. 2006).

Our evolution code for the calculation of the cooling and contraction of adiabatic giant planets is well-tested. It has been used to produce evolutionary models of Jupiter and Saturn (Fortney & Hubbard 2003), cool extrasolar giant planets (Fortney & Hubbard 2004; Marley et al. 2006b), hot Jupiters (Fortney et al. 2006), and it is described in detail in Fortney & Hubbard (2003) and Fortney (2004). For all of these planets, it is the radiative atmosphere that serves as the bottleneck for cooling above the adiabatic H/He envelope.

This is accounted for with our fully non-grey, self-consistent model atmosphere grids. The importance of using detailed atmosphere models for evolutionary calculations of hot Jupiters is discussed in Baraffe et al. (2003) and Marley et al. (2006a).

Below the H/He envelope we assume that heavy elements are found within a distinct core. To model this core we use the EOS of a 50/50 by mass ice/rock mix using the ANEOS zero-temperature water and olivine EOSs (Thompson 1990). The compositions of the cores of Jupiter and Saturn are not known, and given that we model planets that likely formed at a variety of orbital distances, in which different ratios of ice/rock could be accumulated, this simple choice is a reasonable one. We ignore the heat content of the core on the thermal evolution of the planets. This is often done for evolutionary models of Jupiter and Saturn (Hubbard 1977; Saumon et al. 1992; Fortney & Hubbard 2003), as the error involved is small compared to other unknowns. Please see Fortney et al. (2006) for additional discussion on this point.

We also neglect the “transit radius” effect: The apparent radius of a transiting planet is the radius where the slant optical depth through the planet’s atmosphere reaches unity. The corresponding atmospheric pressure can vary across many orders of magnitude, depending on the wavelength (Hubbard et al. 2001; Fortney et al. 2003). Burrows et al. (2003) and Baraffe et al. (2003) have estimated this effect to be $\sim 10\%$ and 5% respectively, for HD 209458b, compared to some reference radius, such as the radiative-convective boundary or the 1 bar level. However, the transit radius effect will be smaller for high gravity planets and, even more importantly, when broad photometric bandpasses are used. The *HST* light curve of Brown et al. (2001) and Charbonneau et al. (2002) was obtained in a narrow wavelength band that overlaps the strong sodium D line absorptions at 589 nm. Across a broad visible wavelength band, for most planets, the transit radius effect would likely be only a few percent. For the models presented here the radii correspond to a pressure of 1 bar.

4.3. Atmosphere Grids for Hydrogen-Helium Planets

Giant planets have been shown to be fully convective, or nearly so, beneath their thin radiative atmospheres (for a review, see Hubbard et al. 2002). The convection is thought to be quite efficient, and hence it is the radiative planetary atmosphere that serves as the bottleneck for escaping radiation and controls the cooling and contraction of the interior (Hubbard 1977). As giant planet atmospheres have a number of atomic and molecular absorbers, including water, ammonia, methane, sodium, and potassium, these atmospheres are far from blackbodies (Burrows et al. 1997; Marley et al. 1999; Sudarsky et al. 2000). A model atmosphere grid, which serves as the upper boundary condition in these evolution calcula-

tions, relates the specific entropy (S) of the planet’s internal adiabat and atmospheric surface gravity (g) to the planet’s effective temperature (T_{eff}). While S and g are calculated from the planetary structure, T_{eff} can only be accurately determined from a non-gray planetary atmosphere code.

To compute the boundary condition for the evolution we compute self-consistent radiative-convective equilibrium atmospheric structure models on a large grid of gravities and effective temperatures. Each model computes an atmospheric temperature structure, accounting for deposition and re-radiation of incident starlight and convective transport and emission of internal thermal energy. The model, based on one developed for Titan (McKay et al. 1989), was originally applied to the study of giant planet thermal structure by Marley & McKay (1999) and has also been employed to study the atmospheres of the hot Jupiters (Fortney et al. 2005, 2006) as well as brown dwarfs (Marley et al. 1996; Burrows et al. 1997, M. S. Marley et al. in prep.), albeit without incident radiation. The radiative transfer methods, chemical equilibrium calculations, and molecular and atomic opacities are summarized in the above publications as well as R. S. Freedman & K. Lodders, in prep. We assume solar metallicity atmospheres (?), and while the effect of condensation on atmospheric composition is included in the chemical equilibrium calculation, we neglect the opacity of clouds.

Guillot & Showman (2002), Baraffe et al. (2003), and Marley et al. (2006a) have previously discussed the importance of properly incorporating incident stellar flux into the hot Jupiter atmosphere grids that serve as the upper boundary condition for the evolution of these planets, but it is also necessary at greater orbital separations for old and relatively low-mass planets. This is because these planets have comparatively little internal energy, and their intrinsic effective temperature (T_{int}) quickly falls below its equilibrium temperature (T_{eq}), which is set entirely by absorption of stellar flux. Here, by definition:

$$T_{\text{eff}}^4 = T_{\text{int}}^4 + T_{\text{eq}}^4. \quad (6)$$

Therefore, incident stellar flux dominates over intrinsic flux and a deep atmospheric radiative zone grows, similar to a highly irradiated Pegasi planet. This was previously discussed in Guillot (1999), but we believe this is the first time this effect has been explicitly shown with detailed model atmospheres. Figure 2 shows model atmospheres computed at 0.1 AU (highly irradiated) and 9.5 (modestly irradiated). In both cases once $T_{\text{int}} \ll T_{\text{eq}}$, an isothermal region connects the deep interior adiabat to the upper atmosphere, whose structure is governed only by absorption of stellar flux (see also Hubeny et al. 2003)). Note that the very low T_{int} values in Figure 2 for this Saturn-like planet would only occur after several Hubble times of evolution. For all models we calculate a planet-wide average P - T profile that is representative of the planet as a whole. In practice this means that the incident stellar flux is diluted by a factor of 1/4 (see Marley et al. 2006a).

The common approximation described in Hubbard (1977) for including stellar flux into an atmosphere grid computed for *isolated* model atmospheres is only valid when $T_{\text{int}} \gtrsim T_{\text{eq}}$. Following the Hubbard (1977) prescription to small T_{int} leads to planetary radii that reach an asymptotic value governed by their Bond albedo (Chabrier & Baraffe 2000), and hence overestimates the radii of old, cold planets. This is the first time the evolution of planets that orbit at greater than ~ 0.05 AU have been computed with this proper boundary condition. We have computed model atmosphere grids across a range of surface gravities, for T_{int} from 50-2000 K (and as low as 10 K at low gravity), including the proper solar insolation for orbital distances of 0.02, 0.045, 0.1, 1.0, and 9.5 AU (which is Saturn’s orbital distance).⁵ At Gyr ages, we find Bond albedos of ~ 0.05 -0.1 at distances less than 0.1 AU, and higher values of ~ 0.3 -0.4 from 1-10 AU, where cooler temperatures prevail and sodium and potassium, which absorb strongly in the optical, have condensed into clouds below the visible atmosphere (Sudarsky et al. 2003). We have elected to ignore cloud opacity here, since considerable uncertainties remain concerning their effect on the atmospheric structure and albedos of EGPs. We will pursue this area in more detail in a later paper that focuses on the evolution of Jupiter and Saturn. A separate important issue is the opacity in the deep atmosphere at pressures near 1 kbar. While the temperature structure of highly irradiated atmospheres near $P \sim 1$ kbar is of great importance for understanding giant planet thermal evolution (Guillot & Showman 2002; Arras & Bildsten 2006), the opacities at these pressures remain highly uncertain (de Pater et al. 2005).

Figure 3 shows P - T profiles computed for a Jupiter-like planet from 0.02 to 10 AU from the Sun. The surface gravity g is 25 m s^{-2} and $T_{\text{int}} = 100 \text{ K}$ in all models. These profiles are meant to roughly illustrate the atmospheres of Jupiter-like planets at 4.5 Gyr. $T_{\text{int}} = 100 \text{ K}$ is very close to Jupiter’s current value (Guillot 2005), and we find that in our cooling calculations that the model planets reach a T_{int} of ~ 102 -110 K at 4.5 Gyr, that is only weakly dependent on stellar irradiation.

A deep external radiative zone is found in the most highly irradiated models. For the planets at $\lesssim 0.05$ AU convection does not begin until $P > 1$ kbar. From 0.1 to 2 AU it should be noted that the deep internal adiabat for all models begins at 300+ bar, but there is a 2nd, detached convective zone at pressures close to 1 bar. This detached convective zone grows as stellar distance increases, and by 3 AU the convective zones have merged. Only when these convective zones merge is the interior adiabat cooler as a function of orbital distance. The models from 0.1 to 2 AU have essentially the same internal adiabat, meaning the planets would have the same radius at a given mass. As we will see, a striking consequence of this

⁵Occasionally at higher gravities and low T_{int} extrapolation off the grid was performed as well. This generally only affected highly irradiated core-free low-mass planets.

effect is that stellar irradiation at 2 AU has approximately the same effect on retarding cooling and contraction as at 0.1 AU, even though the incident fluxes vary by a factor of 400!

5. Results: Ice–Rock–Iron Planets

5.1. Planetary Radii

It seems likely that planets with masses within an order of magnitude of the Earth’s mass will be composed primarily of more refractory species, like the planetary ices, rocks, and iron. Within our solar system, objects of similar radius can differ by over a factor of 3 mass, due to compositional differences. As a reminder, a planet of Mercury radius may be detectable with Kepler. This could for instance indicate of mass of $0.055 M_{\oplus}$, like Mercury itself, or a mass of 1/3 this value, like Callisto, which has a radius that differs by only 30 km. With our equations of state, we are able to explore the radii of objects with any possible combination of ice, rock, and iron.

In order to keep this task manageable, we have limited our calculations to several compositions that are quite illustrative. These include pure ice and ice/rock mixtures, which could be described as “water worlds” or “Ocean planets.” Objects similar to these that we have in our solar system, such as the icy satellites of the outer planets, generally have small masses. However, Kuchner (2003) and Léger et al. (2004) have pointed out water-rich objects could reach many Earth masses (perhaps as failed giant planet cores) and migrate inward to smaller orbital distances. We also consider planets composed of pure rock, rock and iron mixtures, and pure iron, more similar to our own terrestrial planets. The ice/rock and rock/iron mixtures are computed in 75/25, 50/50, and 25/75 percentages by mass, with ice always overlaying rock, and rock always overlaying iron.

Our results are shown in Figure 4. Since from the outset we have decided to make few assumptions regarding what is a reasonable planet, we have computed radii from masses of 0.01 to $1000 M_{\oplus}$. For all compositions, the radii initially grow as $M^{1/3}$, but at larger masses, compression effects become important. As a greater fraction of the electrons become pressure ionized, the materials begin to behave more like a Fermi gas, and there is a flattening of the mass-radius curves near $1000 M_{\oplus}$. Eventually the radii shrink as mass increases, with radii falling with $M^{-1/3}$ (see Zapolsky & Salpeter 1969).

At the top left of Figure 4 we also show the size of various levels of uncertainty in planetary mass, as a percentages of a given mass, from 10 to 200%. For instance, if one could determine the mass of a $1 M_{\oplus}$ planet to within 50%, even a radius determination

accurate to within $0.25 R_{\oplus}$ would lead to considerable ambiguity concerning composition, ranging from 50/50 ice/rock to pure iron. The shallow slope of the mass-radius curves below a few M_{\oplus} makes accurate mass determinations especially important for understanding composition. In Table 1 we give the mass and radius for a subset of these planets. We note that from 1-10 M_{\oplus} we find excellent agreement between our models and the detailed “Super-Earth” models of Valencia et al. (2006).

5.2. Validation in the Solar System

In open circles we have also plotted the masses and radii of solar system planets and moons. This can be used as a means to validate our methods. For instance, detailed models of the Earth’s interior indicate that the Earth is approximately 33% iron by mass with a core-mantle boundary at 3480 km (Dziewonski & Anderson 1981). This is readily recovered from Figure 4, where Earth plots between the 25% and 50% iron curves, but closer to 25%. Our simple Earth model, with a iron/rock boundary at 3480 km yields a planetary radius within 100 km (1.5% smaller) of the actual Earth. Given that our model lacks thermal corrections to EOSs that are found in detailed Earth models and that we ignore lower density species such as nickel and sulphur that are likely mixed with iron into the Earth’s core, we regard this agreement as excellent, and entirely sufficient with regard to the expected radii uncertainties as measured by transit surveys.

Elsewhere in the solar system, one can see that we recover ice/rock or rock/iron ratios of other bodies, which are derived by more complex models. A brief overview of the structure of the terrestrial planets and icy moons is given in de Pater & Lissauer (2001). Earth’s moon is composed almost entirely of rock, with a very small iron core of radius $\lesssim 400$ km. Here, the moon (the leftmost circle) plots on top of the line for pure rock. Mercury is calculated to be $\sim 60\%$ iron by mass, and with our models Mercury falls between the 50/50 (rock/iron) and 25/75 curves, but again, closer to 50/50, which shows excellent agreement. Titan is calculated to be composed of $\sim 35\%$ ices, and again we find excellent agreement, as Titan falls between the 50/50 (ice/rock) and 25/75 curves, slightly closer to 25/75. Uranus and Neptune cannot be composed purely of ice, although it likely makes up the bulk of their masses.

5.3. Applications

In addition to Table 1 we have also fit the radii of these planets to analytic functions that are quadratic in $\log M$ and linear in R . These are Equation (7), for ice/rock planets, and (8), for rock/iron, shown below:

$$R = (0.0592 \text{ } \mathit{imf} + 0.0975)(\log M)^2 + (0.2337 \text{ } \mathit{imf} + 0.4938) \log M + (0.3102 \text{ } \mathit{imf} + 0.7932) \quad (7)$$

$$R = (0.0912 \text{ } \mathit{rmf} + 0.1603)(\log M)^2 + (0.3330 \text{ } \mathit{rmf} + 0.7387) \log M + (0.4639 \text{ } \mathit{rmf} + 1.1193). \quad (8)$$

Here R is in R_{\oplus} and M is in M_{\oplus} , while imf is the ice mass fraction (1.0 for pure ice and 0.0 for pure rock) and rmf is the rock mass fraction (1.0 for pure rock and 0.0 for pure iron). The fits were performed for planetary masses from 0.01 to 100 M_{\oplus} . For ice/rock, the equation is on average accurate to within 2.5%. For rock/iron, 1.5%. Accuracy near 1 M_{\oplus} for both equations is better than 0.5%, although deviations can reach 10% for ice/rock at $\sim 0.01 M_{\oplus}$. If a mass and radius can be determined for a given planet these equations will allow for a quick and reliable composition estimate. However, since radial velocity or astrometric followup for these small planets will be extremely difficult and time consuming, radii may have to suffice as a proxy for mass for some time. Given an assumed composition, such as "Earth-like," one could assign masses to terrestrial-sized transiting planets. The distribution of planetary masses vs. orbital distance and stellar type could then be compared with planet formation model predictions.

6. Results: Hydrogen-Helium Dominated Planets

6.1. Radii of Gas Giants

Planets around the mass of Uranus and Neptune ($\sim 15 M_{\oplus}$) to objects as large as 75 M_J can be described by the same cooling theory. In general, planets with larger cores will have smaller radii, and planets closer to their parent stars will have larger radii at a given age than planets at larger orbital distances. In Figure 5 we plot the contraction of planets from 0.1 to 3 M_J , as a function of age at various orbital distances. We also show the effect of a core of 25 M_{\oplus} , the approximate mass of heavy elements within Jupiter and Saturn (Saumon & Guillot 2004). The first feature to notice is that, independent of mass, the spread in radii between 0.045 and 1 AU is quite small compared to the large radii at 0.02 AU and small radii at 9.5 AU. As expected, the effect of the 25 M_{\oplus} core diminishes with increased planet

mass, as the core becomes a relatively smaller fraction of the planet’s mass. The radii at early ages it quite large, especially for the low-mass planets under intense stellar irradiation. These hot low-gravity planets are potentially susceptible to evaporation (Baraffe et al. 2005; Hubbard et al. 2005).

As discussed in detail by Marley et al. (2006b), the physical properties of giant planets at young ages are quite uncertain. The models presented here do not include a formation mechanism and are arbitrarily large and hot at very young ages. Marley et al. (2006b) have found that in their implementation fo the core-accretion mechanism of giant planet formation (Pollack et al. 1996; Hubickyj et al. 2005) giant planets form with radii than can be several tenths of a Jupiter radius smaller than one computes with an arbitrarily large and hot start. These differences may last ~ 10 Myr for a $1 M_J$ object to hundreds of millions of years for planets of several Jupiter masses. Therefore, determining the radius of young giant planets, has the potential to elucidate their post-formation structure, and give us clues to their formation mechanism. In addition, finding a young hot Jupiter would place constraints on planetary migration times.

If we look at the radii of these same planets from 0.1 to $3 M_J$ at a given age we can see more clearly how stellar insolation effects their radii. Radii as a function of orbital distance, at 4.5 Gyr, are shown in Figure 6. For the planets from 0.3 (Saturn’s mass) to $3 M_J$, notice the nearly flat radius curve between 0.1 and 1 AU. A prediction of these models is that while irradiation effects will have important consequences for planets < 0.1 AU from their parent stars, that these effects will change in character with larger orbital distance. *Planets with the same composition, from 0.1 to ~ 2 AU, should have nearly the same radii.* This is a consequence of the atmospheric temperature structures shown in Figure 3. Although planets with cores are smaller, this does not change the “flattening” of these curves from 0.1 to ~ 1 AU.

We can take a broader look at giant planet radii is well, including the effects of extremely large and small core masses. One view of our full range of 4.5 Gyr-old planets is shown in Figure 7 where radii vs. mass are plotted at 5 orbital separations from 0.02 AU to 9.5 AU. Coreless planets and planets with core masses that are 10%, 50%, and 90% of their *total planet mass* are shown. Note that here we have chosen to compute models with a core mass that is a constant *percentage* of planetary mass, meaning that a 50% core mass for a $10 M_\oplus$ planet is $5 M_\oplus$, and for a $5 M_J$ planet it is $800 M_\oplus$. This plot is meant to show extremes, but is still quite illustrative. Several real planets are shown as well.

In Figure 7 Uranus and Neptune plot very close to the 90% heavy elements curves, as expected from more detailed models (Hubbard et al. 1991; Podolak et al. 1995), although these planets likely contain more ice than rock. Turning to “hot Neptune” planets, a pre-

diction of these models is that the radii of these objects should be similar to that of Uranus and Neptune, *if* hot Neptunes do not suffer the effects of significant mass loss and are $\sim 10\%$ H/He, like Uranus and Neptune. Baraffe et al. (2005) and Baraffe et al. (2006) have shown that if these planets are remnants of much larger original planets, radii may well exceed $1 M_J$. Looking at the solar system’s gas giants, it is clear that for Jupiter⁶ at 5.2 AU, 10% heavy elements by mass is a reasonable estimate as it falls between the 1 AU and 9.5 AU 10% heavy element curves. Saturn, at 9.5 AU, is clearly more enhanced in heavy elements than Jupiter, and from this plot perhaps 20% heavy elements would be estimated. Given that one of the first ten transiting planets found has a core mass of $\sim 60\text{--}80 M_\oplus$, and it orbits a star with a metallicity $2.3\times$ that of the Sun, it is probably realistic to expect the occasion planet around metal rich stars with core masses of $\sim 100 M_\oplus$ or more.

We now turn to models computed with a constant core mass (10, 25, 50, and $100 M_\oplus$) as function of age, mass, and stellar irradiation. Figure 8 shows the radii of planets at two ages, 500 Myr and 4.5 Gyr. The contraction of planets by $\sim 0.1\text{--}0.2 R_J$ across this factor of ~ 10 in age is clear. At 4.5 Gyr in this plot, it is again clear that radii at 1 AU are only slightly smaller than at 0.1 AU, while radii at 10 AU are appreciably smaller than at 1 AU. In Tables 2, 3, and 4 we give our calculations of the radii of these planets at ages of 300 Myr, 1 Gyr, and 4.5 Gyr. Looking at radii at 4.5 Gyr, we find generally good agreement with the models of Bodenheimer et al. (2003) for planets $\gtrsim 0.69 M_J$ (within $0.03 R_J$), but at lower masses we predict significantly smaller planets with differences that can approach $0.4 R_J$ for coreless models at 0.02 AU. It is likely that in the very low-mass core-free, highly irradiated corner of phase space that evolution models are perhaps most uncertain. Evaporation effects may be important here as well.

6.2. Applications

Extending our calculations to other parent stars should be done with some care. Marley et al. (1999) have shown that planetary Bond albedos are a function of the spectral type of the primary star. A planet around a later-type star, with a spectrum peaking closer to the infrared, will have a greater fraction of its flux absorbed by a planetary atmosphere, meaning a lower planetary Bond albedo. Although this must surely be accounted for eventually, given current uncertainties in chemistry and cloud formation in these planets, which leads directly

⁶Jupiter itself plots below $1 R_J$ because its mean radius is 2.2% smaller than its equatorial radius at 1 bar, 71492 km, which has become the standard “Jupiter radius.” We note that planetary oblateness will be extremely difficult to determine from transit light curves (Barnes & Fortney 2003).

to uncertainties in their atmospheric absorption and scattering properties, for now we will ignore this effect. However, the incident stellar flux of the currently known transiting planets already differ by a factor of five, so the differing magnitudes of incident fluxes already cannot be ignored. If a planet’s orbital distance from its parent star is d , then the distance from the Sun that the planet would have to be to receive this same flux, d_{\odot} , is given by:

$$d_{\odot} = d \left(\frac{L_{\odot}}{L} \right)^{1/2} \quad (9)$$

or, if one uses a mass-radius relation such as,

$$\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}} \right)^{\eta}, \quad (10)$$

where η may typically be ~ 3.5 , then

$$d_{\odot} = d \left(\frac{M_{\odot}}{M} \right)^{\eta/2}. \quad (11)$$

For instance, HD 209458b, at 0.046 AU from its G0V parent star ($L \sim 1.6 L_{\odot}$), would receive that same level of irradiation at 0.036 AU from the Sun.

Figure 8 shows the mass and radius of three transiting planets. The planet with the lowest mass, HD 149026b, receives an incident flux equivalent to that of a planet at 0.026 AU from the Sun. Clearly the planet has a core of perhaps $70 M_{\oplus}$, which is confirmed by more detailed models (Fortney et al. 2006). The middle diamond is TrES-1, which receives insolation equal to what is received from the Sun at 0.059 AU. Using these models, one would estimate that the planet has a core of perhaps $25 M_{\oplus}$, very similar to the bulk abundance of heavy elements in Jupiter and Saturn (Saumon & Guillot 2004). HD 209458b is clearly anomalous on this plot and must have an additional interior energy source. However, a $0.1 M_J$ planet of pure H/He at 0.02 AU (if such a planet exists) with a similar radius would not require an additional energy source. It should also be pointed out that all the hot Jupiters could have additional interior energy sources of similar magnitude. This would require planets such as TrES-1 and HD 149026b to have even larger cores (Fortney et al. 2006; Guillot et al. 2006).

7. Discussion & Conclusions

In this paper we have taken a broad look at the radii of planets at nearly all possible masses and orbital separations. These calculations should serve as a baseline for comparisons

with transiting planets currently being discovered from the ground and those that will soon be discovered from space. Since our main effort here is to calculate radii over a very large phase space, there are surely particular corners of space where these calculations could be improved. If in the coming years it becomes clear under what conditions planets may have an additional interior energy source, and how this changes as a function of orbital separation, incident stellar flux, and stellar type, this could be included in a new grid of calculations. If evaporation effects are observed for numerous transiting planets, across a wide range of planetary surface gravities and orbital separations, then the effects of appreciable mass loss could be added to these evolution calculations. The work of Baraffe et al. (2006) and Hubbard et al. (2005) is moving in this direction.

Ice-rich terrestrial planets close to their parent stars could have steam atmospheres (Kuchner 2003) which, depending on the atmospheric scale height, could lead to larger radii at a given mass for these objects. If the cooling of Uranus and Neptune are eventually better understood, then the additional energy due to the cooling of their heavy elements interiors could be added to our evolution models of these lower-mass EGPs. Including the EOS of other forms of rock, or other astrophysical ices, such as CH_4 and NH_3 , would be useful when more detailed models are eventually needed. One could also envision more exotic planets, such as those that formed in solar systems where $\text{C/O} > 1$, which could lead to carbon dominated “terrestrial planets” (Kuchner & Seager 2005), which would likely have radii intermediate between pure rock and ice (Zapolsky & Salpeter, 1969; M. Kuchner, personal communication).

As there is in the solar system, there will always be ambiguity in the bulk composition of exoplanets, especially those that do not possess substantial low density gaseous envelopes. For instance, since a mixture of H/He gas and rocks has an EOS that is similar to that of ice, this leads to considerable uncertainty in the composition of Uranus and Neptune. For a terrestrial-mass “ocean planet,” perhaps the best that we can expect will be an understanding a planet’s ice/rock ratio, with a given uncertainty based on the mass and radius measurements, along with the knowledge that the uncertainty could be even larger if the planet has a substantial iron core. Since iron and silicates condense at similar temperatures (?), it is possible that these species will condense out in ratios similar to that found in the Earth, although subsequent collisions (e.g., as experienced by the Moon and probably Mercury) could alter this ratio, especially for smaller bodies. Experience will tell us how to best classify newly discovered terrestrial-type planets, but at this point we would advocate composition classes based on simple ratios of ice/rock and rock/iron. Our analytic fits reproduce the behavior of our models and will allow for mass estimates prior to radial velocity or astrometric follow up work.

A clear prediction from our models for giant planets is that irradiation effects on planetary radii are not a simple function of stellar insolation. At a given planetary mass and composition, we predict a flattening in planetary radius as a function of orbital distance from ~ 0.1 to $\sim 1-2$ AU. However, if unknown additional energy sources are still in play at these orbital distances, then this effect may be more difficult to see.

Given the recent flurry of transit detections it is likely that we are now just seeing the tip of the iceberg of transiting planets. Even before detections from COROT and Kepler are announced it seems likely that we will have a steady stream of new planets to challenge our understanding.

We thank Kevin Zahnle for a helpful discussion at the start of this project. JJF acknowledges the support of a Spitzer Fellowship from NASA and NSF grant AST-0607489 and MSM from the NASA Origins and Planetary Atmospheres Programs.

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Table 1. Ice-Rock-Iron Planetary Radii

Composition	0.010	0.032	0.1	0.32	1.0	3.16	10.0	31.6	100	316
ice	0.38	0.55	0.79	1.12	1.55	2.12	2.87	3.74	4.68	5.43
50/50	0.33	0.48	0.69	0.97	1.36	1.85	2.48	3.23	4.03	4.67
rock	0.25	0.37	0.54	0.77	1.08	1.48	1.97	2.54	3.14	3.64
67/33	0.24	0.34	0.50	0.71	1.00	1.36	1.80	2.31	2.84	3.29
50/50	0.23	0.33	0.48	0.68	0.95	1.30	1.71	2.19	2.69	3.12
iron	0.19	0.27	0.39	0.55	0.77	1.04	1.36	1.72	2.09	2.42

Note. — Radii of planets, in R_{\oplus} . Column headers are planet masses, in M_{\oplus} .

Table 2. Giant Planet Radii at 300 Myr

Distance (AU)	Core Mass	0.052 17	0.087 28	0.15 46	0.24 77	0.41 129	0.68 215	1.0 318	1.46 464	2.44 774	4.1 1292	6.8 2154	11.3 3594
0.02	0	*	2.326	1.883	1.656	1.455	1.378	1.342	1.327	1.308	1.311	1.315	1.284
0.02	10	1.102	1.388	1.465	1.422	1.349	1.325	1.311	1.306	1.295	1.304	1.310	1.281
0.02	25	-	0.493	0.945	1.133	1.220	1.253	1.267	1.275	1.276	1.294	1.304	1.277
0.02	50	-	-	-	0.801	1.030	1.144	1.193	1.226	1.245	1.276	1.292	1.270
0.02	100	-	-	-	-	0.669	0.939	1.055	1.128	1.187	1.242	1.270	1.256
0.045	0	2.795	1.522	1.345	1.255	1.240	1.228	1.212	1.206	1.199	1.210	1.203	1.170
0.045	10	0.801	1.012	1.091	1.124	1.168	1.185	1.185	1.188	1.188	1.204	1.199	1.168
0.045	25	-	0.447	0.793	0.968	1.071	1.124	1.147	1.161	1.173	1.195	1.193	1.164
0.045	50	-	-	-	0.719	0.921	1.033	1.084	1.119	1.148	1.179	1.183	1.157
0.045	100	-	-	-	-	0.627	0.863	0.968	1.036	1.101	1.148	1.163	1.146
0.1	0	1.595	1.395	1.270	1.197	1.202	1.198	1.187	1.182	1.178	1.189	1.178	1.144
0.1	10	0.755	0.956	1.035	1.084	1.134	1.157	1.160	1.164	1.168	1.183	1.174	1.142
0.1	25	-	0.438	0.767	0.938	1.042	1.099	1.123	1.138	1.153	1.174	1.169	1.138
0.1	50	-	-	-	0.702	0.899	1.011	1.063	1.098	1.129	1.158	1.159	1.132
0.1	100	-	-	-	-	0.618	0.847	0.950	1.018	1.084	1.128	1.140	1.121
1.0	0	1.504	1.325	1.222	1.169	1.182	1.182	1.173	1.169	1.168	1.179	1.169	1.136
1.0	10	0.727	0.921	1.004	1.063	1.116	1.141	1.146	1.152	1.158	1.173	1.165	1.134
1.0	25	-	0.433	0.754	0.923	1.027	1.085	1.110	1.127	1.143	1.164	1.159	1.130
1.0	50	-	-	-	0.693	0.888	0.999	1.051	1.087	1.120	1.149	1.149	1.124
1.0	100	-	-	-	-	0.613	0.839	0.941	1.009	1.075	1.119	1.131	1.113
9.5	0	0.929	0.951	0.983	1.020	1.070	1.106	1.127	1.146	1.167	1.169	1.156	1.130
9.5	10	0.565	0.733	0.847	0.939	1.016	1.072	1.104	1.131	1.157	1.163	1.152	1.127
9.5	25	-	0.394	0.664	0.826	0.942	1.024	1.073	1.146	1.142	1.153	1.146	1.124
9.5	50	-	-	-	0.635	0.823	0.951	1.020	1.072	1.119	1.137	1.137	1.118
9.5	100	-	-	-	-	0.587	0.810	0.920	0.999	1.072	1.107	1.119	1.107

Note. — Radii of planets, in R_J . Row 1 column headers are planet masses, in M_J , while row 2 is in M_\oplus . The symbol “-” indicates that the planet mass is smaller than the given core mass.

Table 3. Giant Planet Radii at 1 Gyr

Distance (AU)	Core Mass	0.052 17	0.087 28	0.15 46	0.24 77	0.41 129	0.68 215	1.0 318	1.46 464	2.44 774	4.1 1292	6.8 2154	11.3 3594
0.02	0	*	1.770	1.539	1.387	1.309	1.281	1.258	1.248	1.235	1.244	1.240	1.199
0.02	10	0.909	1.150	1.221	1.211	1.228	1.234	1.229	1.229	1.224	1.237	1.235	1.197
0.02	25	-	0.461	0.838	1.022	1.121	1.169	1.189	1.200	1.206	1.228	1.229	1.192
0.02	50	-	-	-	0.746	0.958	1.072	1.122	1.156	1.180	1.211	1.218	1.186
0.02	100	-	-	-	-	0.640	0.888	0.997	1.068	1.130	1.179	1.198	1.173
0.045	0	1.490	1.271	1.183	1.144	1.163	1.167	1.160	1.157	1.156	1.164	1.149	1.107
0.045	10	0.698	0.888	0.975	1.043	1.099	1.127	1.134	1.140	1.147	1.158	1.145	1.105
0.045	25	-	0.426	0.739	0.908	1.012	1.072	1.099	1.115	1.132	1.149	1.140	1.101
0.045	50	-	-	-	0.684	0.877	0.988	1.041	1.077	1.109	1.134	1.130	1.095
0.045	100	-	-	-	-	0.607	0.831	0.932	0.999	1.065	1.105	1.111	1.084
0.1	0	1.298	1.197	1.127	1.105	1.133	1.143	1.139	1.138	1.139	1.147	1.130	1.087
0.1	10	0.665	0.847	0.934	1.012	1.072	1.105	1.114	1.122	1.130	1.141	1.126	1.085
0.1	25	-	0.420	0.719	0.883	0.989	1.051	1.080	1.097	1.116	1.132	1.121	1.081
0.1	50	-	-	-	0.670	0.859	0.970	1.023	1.059	1.094	1.117	1.111	1.076
0.1	100	-	-	-	-	0.600	0.818	0.918	0.984	1.050	1.088	1.093	1.065
1.0	0	1.229	1.148	1.095	1.086	1.118	1.130	1.128	1.127	1.130	1.137	1.121	1.079
1.0	10	0.646	0.823	0.915	0.996	1.058	1.092	1.103	1.111	1.121	1.131	1.117	1.077
1.0	25	-	0.416	0.709	0.871	0.977	1.040	1.069	1.087	1.107	1.123	1.112	1.073
1.0	50	-	-	-	0.663	0.850	0.961	1.014	1.050	1.085	1.108	1.102	1.068
1.0	100	-	-	-	-	0.595	0.811	0.910	0.976	1.042	1.080	1.085	1.057
9.5	0	0.857	0.877	0.910	0.955	1.003	1.044	1.068	1.089	1.113	1.119	1.109	1.074
9.5	10	0.532	0.683	0.791	0.882	0.955	1.013	1.047	1.075	1.104	1.113	1.105	1.072
9.5	25	-	0.386	0.631	0.780	0.888	0.970	1.018	1.089	1.090	1.105	1.100	1.069
9.5	50	-	-	-	0.610	0.784	0.904	0.970	1.021	1.068	1.090	1.091	1.063
9.5	100	-	-	-	-	0.570	0.775	0.878	0.954	1.026	1.063	1.074	1.053

Note. — Radii of planets, in R_J . Row 1 column headers are planet masses, in M_J , while row 2 is in M_\oplus . The symbol “-” indicates that the planet mass is smaller than the given core mass.

Table 4. Giant Planet Radii at 4.5 Gyr

Distance (AU)	Core Mass	0.052 17	0.087 28	0.15 46	0.24 77	0.41 129	0.68 215	1.0 318	1.46 464	2.44 774	4.1 1292	6.8 2154	11.3 3594
0.02	0	*	1.355	1.252	1.183	1.190	1.189	1.179	1.174	1.170	1.178	1.164	1.118
0.02	10	0.726	0.934	1.019	1.072	1.123	1.148	1.153	1.157	1.160	1.172	1.160	1.116
0.02	25	-	0.430	0.756	0.928	1.032	1.091	1.116	1.131	1.145	1.163	1.155	1.112
0.02	50	-	-	-	0.695	0.891	1.004	1.056	1.091	1.121	1.148	1.144	1.106
0.02	100	-	-	-	-	0.613	0.841	0.944	1.011	1.076	1.118	1.125	1.095
0.045	0	1.103	1.065	1.038	1.049	1.086	1.105	1.107	1.108	1.113	1.118	1.099	1.053
0.045	10	0.599	0.775	0.878	0.964	1.029	1.069	1.083	1.092	1.104	1.112	1.095	1.050
0.045	25	-	0.403	0.686	0.846	0.952	1.019	1.050	1.069	1.090	1.104	1.090	1.047
0.045	50	-	-	-	0.648	0.831	0.942	0.996	1.033	1.068	1.090	1.081	1.042
0.045	100	-	-	-	-	0.587	0.798	0.896	0.961	1.026	1.062	1.063	1.032
0.1	0	1.068	1.027	1.005	1.024	1.062	1.085	1.090	1.092	1.099	1.104	1.084	1.038
0.1	10	0.592	0.755	0.858	0.942	1.008	1.051	1.067	1.077	1.090	1.098	1.080	1.036
0.1	25	-	0.404	0.675	0.829	0.934	1.002	1.034	1.054	1.077	1.090	1.075	1.033
0.1	50	-	-	-	0.639	0.817	0.928	0.982	1.019	1.055	1.076	1.066	1.027
0.1	100	-	-	-	-	0.582	0.788	0.884	0.949	1.014	1.049	1.049	1.018
1.0	0	1.014	0.993	0.983	1.011	1.050	1.074	1.081	1.084	1.091	1.096	1.075	1.030
1.0	10	0.576	0.738	0.845	0.931	0.997	1.041	1.058	1.068	1.082	1.090	1.072	1.028
1.0	25	-	0.400	0.666	0.820	0.924	0.993	1.026	1.046	1.069	1.082	1.067	1.025
1.0	50	-	-	-	0.633	0.810	0.920	0.974	1.011	1.048	1.068	1.058	1.020
1.0	100	-	-	-	-	0.578	0.782	0.878	0.942	1.007	1.041	1.041	1.010
9.5	0	0.798	0.827	0.866	0.913	0.957	0.994	1.019	1.037	1.056	1.062	1.055	1.023
9.5	10	0.508	0.653	0.759	0.844	0.911	0.966	0.999	1.024	1.048	1.057	1.052	1.021
9.5	25	-	0.378	0.611	0.750	0.849	0.926	0.972	1.037	1.035	1.050	1.047	1.018
9.5	50	-	-	-	0.594	0.754	0.865	0.926	0.973	1.015	1.037	1.039	1.013
9.5	100	-	-	-	-	0.558	0.746	0.842	0.911	0.976	1.012	1.023	1.004

Note. — Radii of planets, in R_J . Row 1 column headers are planet masses, in M_J , while row 2 is in M_\oplus . The symbol “-” indicates that the planet mass is smaller than the given core mass. Given the albedos we calculate (see §4.3) the approximate T_{eq} value at each distance is, 1960 K (0.02 AU), 1300 K (0.045 AU), 875 K (0.1 AU), 260 K (1 AU), and 78 K (9.5 AU). The symbol “-” indicates that the planet mass is smaller than the given core mass.

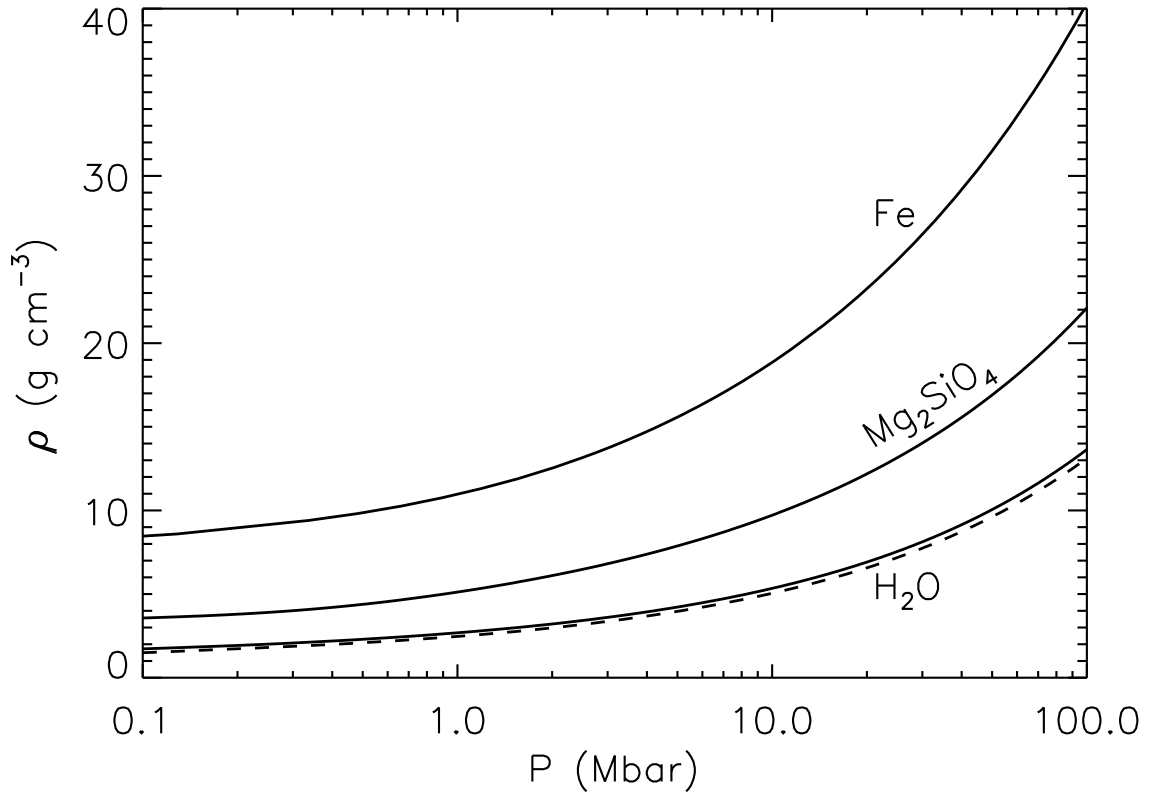


Fig. 1.— Zero-temperature pressure-density relations for iron (Fe), rock (Mg_2SiO_4), and ice (H_2O). For ice, the dashed curve shows our EOS with the thermal correction describe in §4.1.

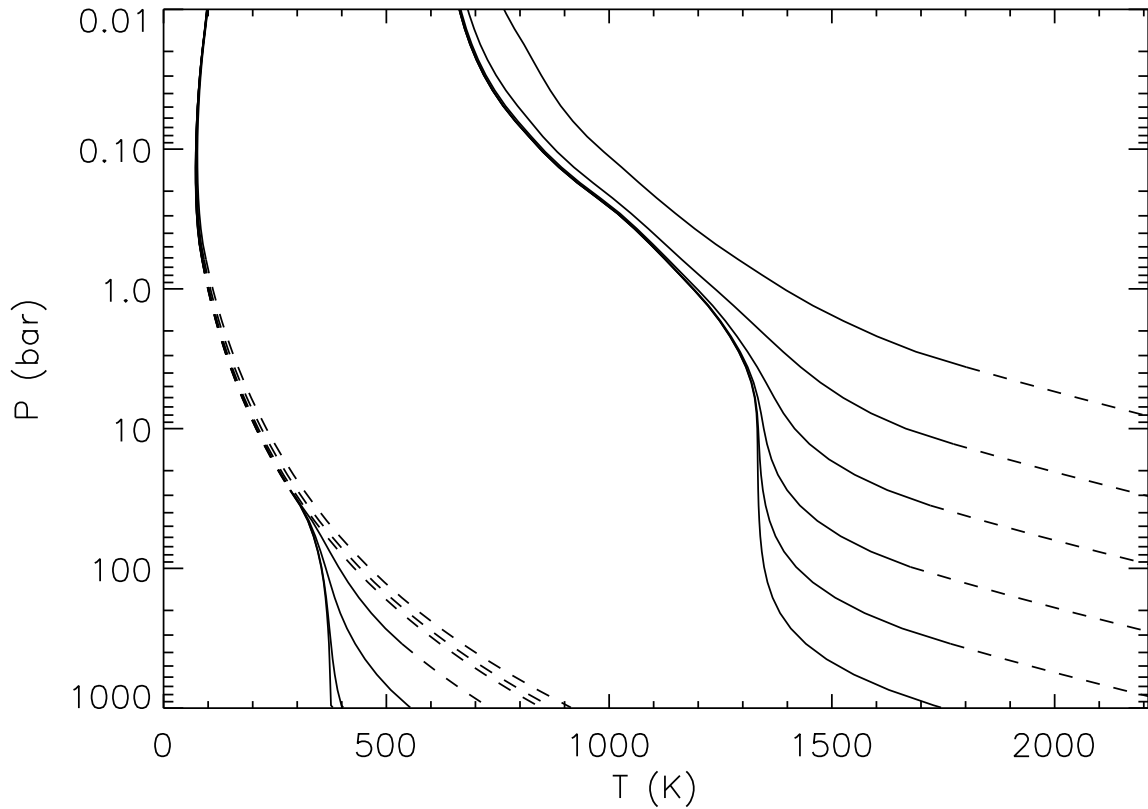


Fig. 2.— A subset of pressure-temperature profiles taken from two of our grids. On the left are profiles for a planet at 9.5 AU with $g=13 \text{ m s}^{-2}$, decreasing in T_{int} , with values of 60, 50, 40, 30, 20, 10 and 3 K. On the right are profiles for a planet at 0.1 AU with $g=40 \text{ m s}^{-2}$, decreasing in T_{eff} , with values of 1000, 630, 400, 250, 160, and 100 K. The solid portions of the profiles are radiative regions and the dashed portions are convective regions.

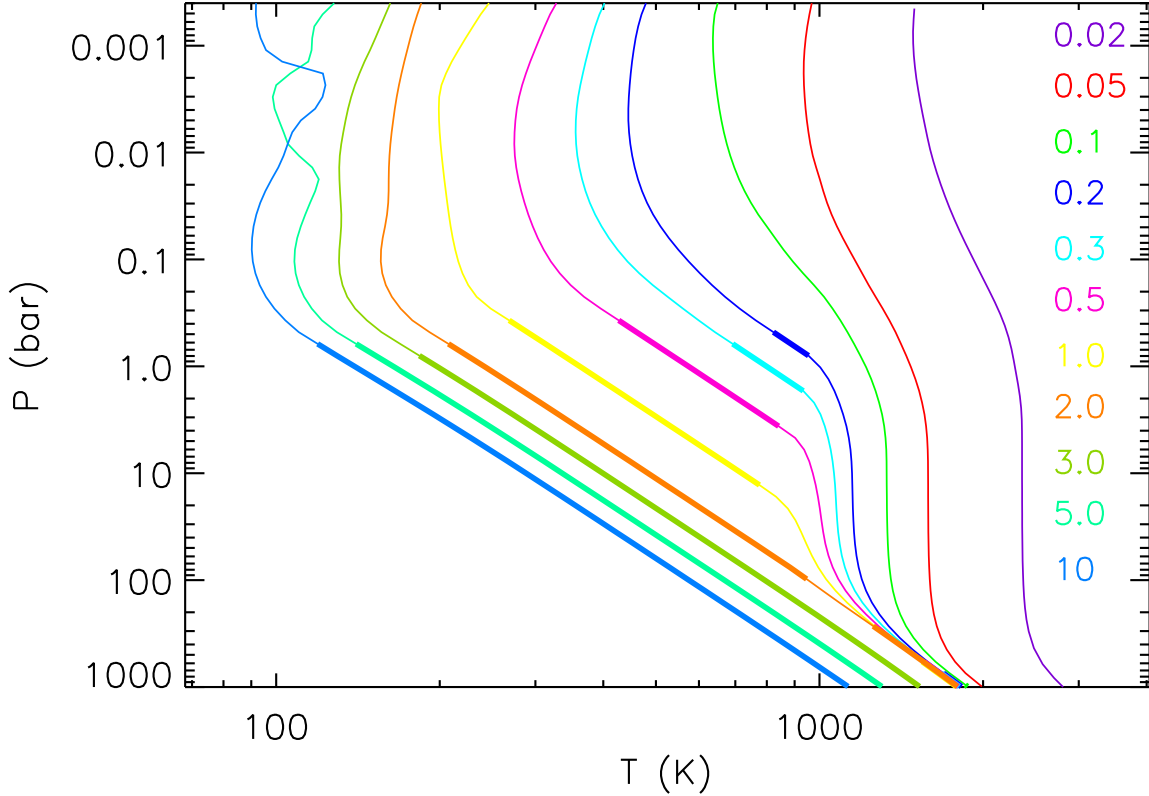


Fig. 3.— Pressure-temperature profiles for ~ 4.5 Gyr-old Jupiter-like planets ($g=25 \text{ m s}^{-2}$, $T_{\text{int}}=100 \text{ K}$) from 0.02 to 10 AU from the Sun. Distance from the Sun in AU is color coded along the right side of the plot. Thick lines are convective regions while thin lines are radiative regions. The profiles at 5 and 10 AU show deviations that arise from numerical noise in the chemical equilibrium table near condensation points, but this has a negligible effect on planetary evolution.

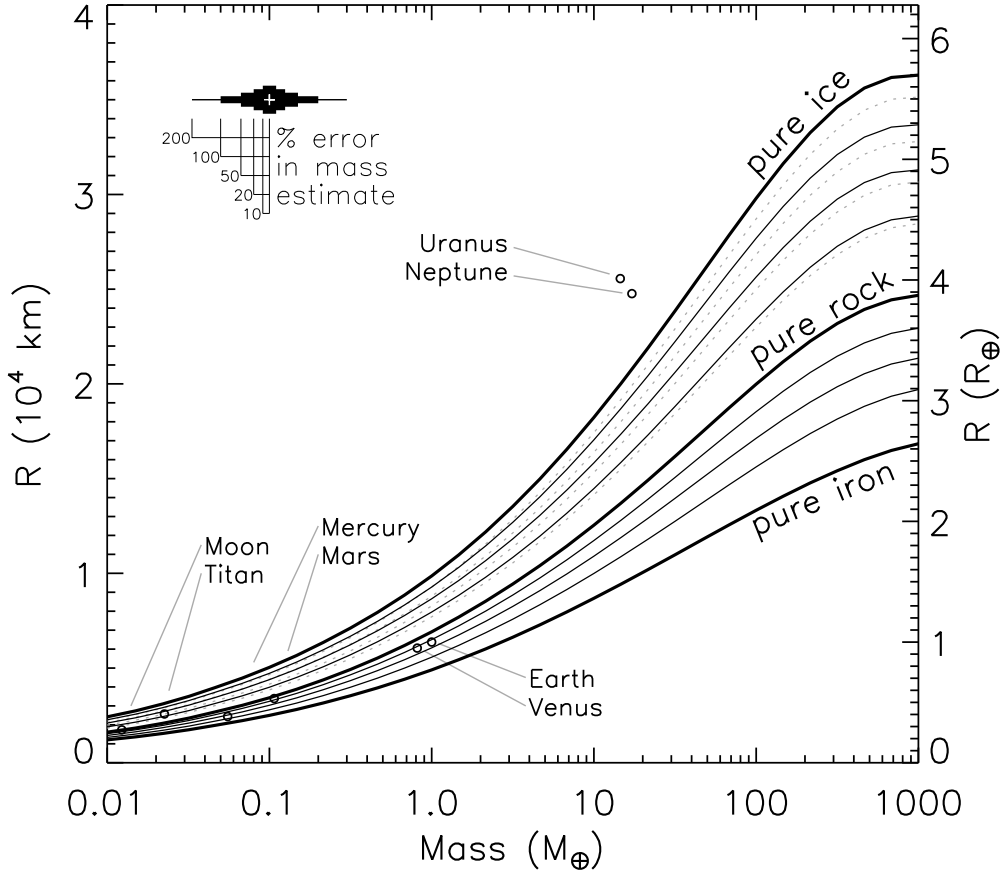


Fig. 4.— Mass (in M_{\oplus}) vs. radius (in km and R_{\oplus}) for planets composed for ice, rock, and iron. The topmost thick black curve is for pure “warm” water ice. (See text.) The middle thick curve is for pure rock (Mg_2SiO_4). The bottommost thick curve is for pure iron (Fe). The three black thin curves between pure ice and pure rock, are from top to bottom, 75% ice/25% rock, 50/50, and 25/75. The inner layer is rock and the outer layer is ice. The gray dotted lines between rock and pure warm ice are the same pure ice and ice/rock curves, but for zero-temperature ice. The three black thin curves between pure rock and iron, are from top to bottom, 75% rock/25% iron, 50/50, and 25/75. The inner layer is iron and the outer layer is rock. Solar system objects are open circles. At the upper left we show the horizontal extent of mass error bars, for any given mass, from 10 to 200%.

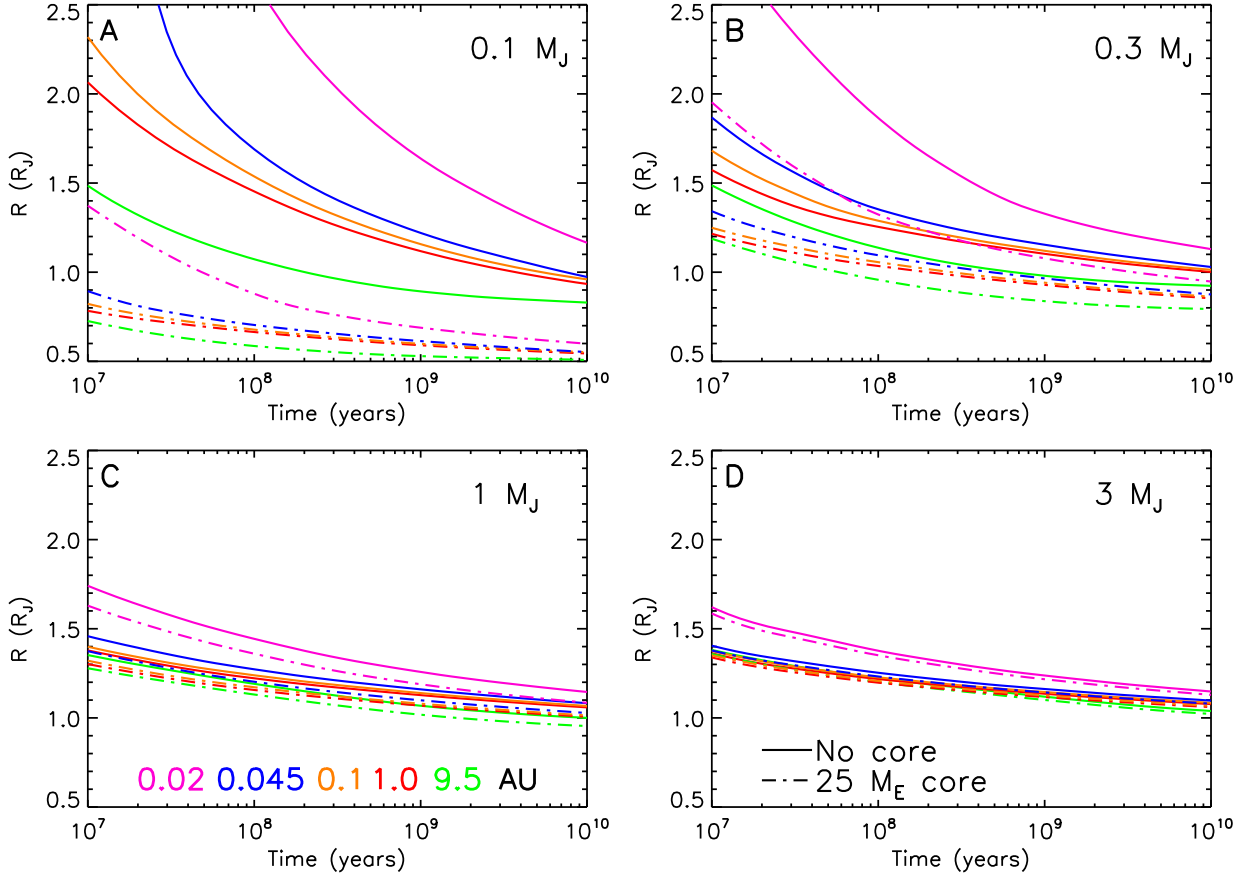


Fig. 5.— Planetary radii as a function of time for masses of $0.1 M_J$ ($32 M_{\oplus}$, A), $0.3 M_J$ (B), $1.0 M_J$ (C), and $3.0 M_J$ (D). The five curve colors code for the five different orbital separations from the Sun, shown in (C). Solid lines indicate models without cores and dash-dot lines indicate models with a core of $25 M_{\oplus}$.

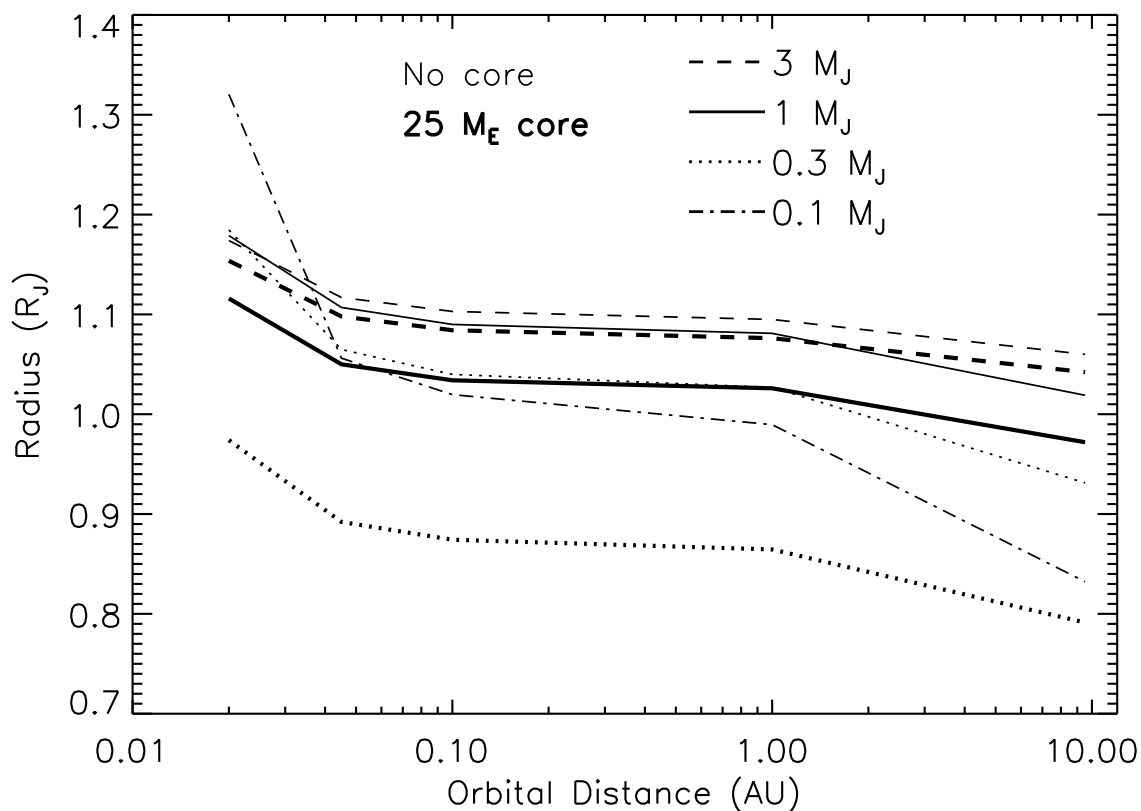


Fig. 6.— Planetary radii at 4.5 Gyr as a function of orbital distance from the Sun. Models are calculated at 0.02, 0.045, 0.1, 1.0, and 9.5 AU. Masses are 0.1, 0.3, 1.0, and $3.0 M_J$. Coreless planets (thin lines) and planets with a core of $25 M_\oplus$ of heavy elements (thick lines) are shown. Note the shape of these radius curves and the flattening between 0.1 and 1.0 AU. The $0.1 M_J$ planet with a $25 M_\oplus$ core is off the plot at $\sim 0.5 R_J$.

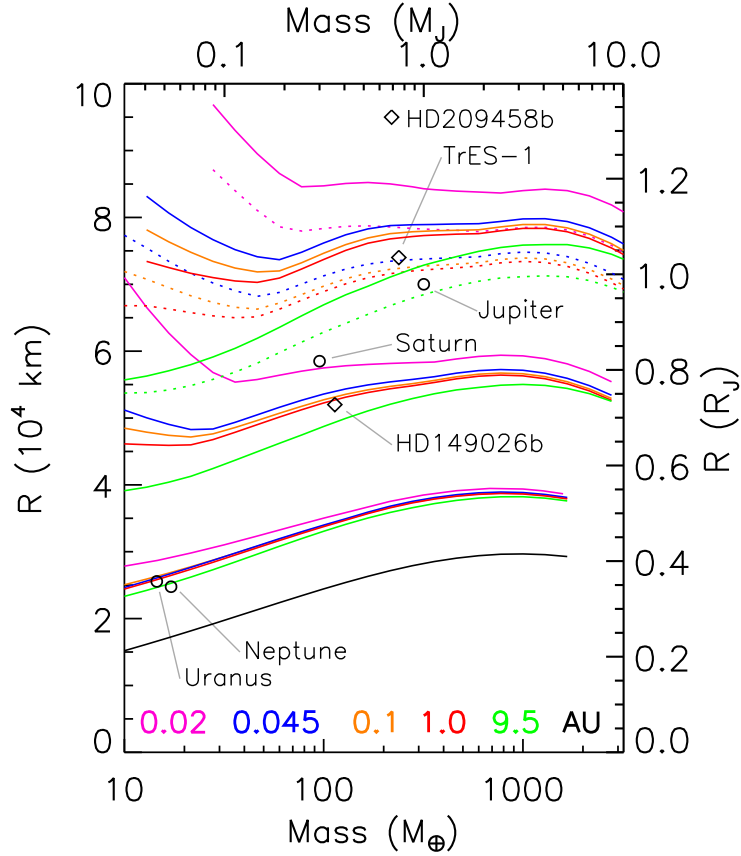


Fig. 7.— Planetary radii at 4.5 Gyr as a function of mass. Models are calculated at 0.02, 0.045, 0.1, 1.0, and 9.5 AU and are color coded at the bottom of the plot. The black curve is for a heavy element planet of 50% ice and 50% rock. The group of 5 colored curves above the black curve is for planets that are 90% heavy elements. The next higher set of 5 colored curves are for planets that are 50% heavy elements. The next higher set, shown in dotted lines, are 10% heavy elements. The highest set are for core-free planets of pure H/He. The open circles are solar system planets and the diamonds are extrasolar planets.

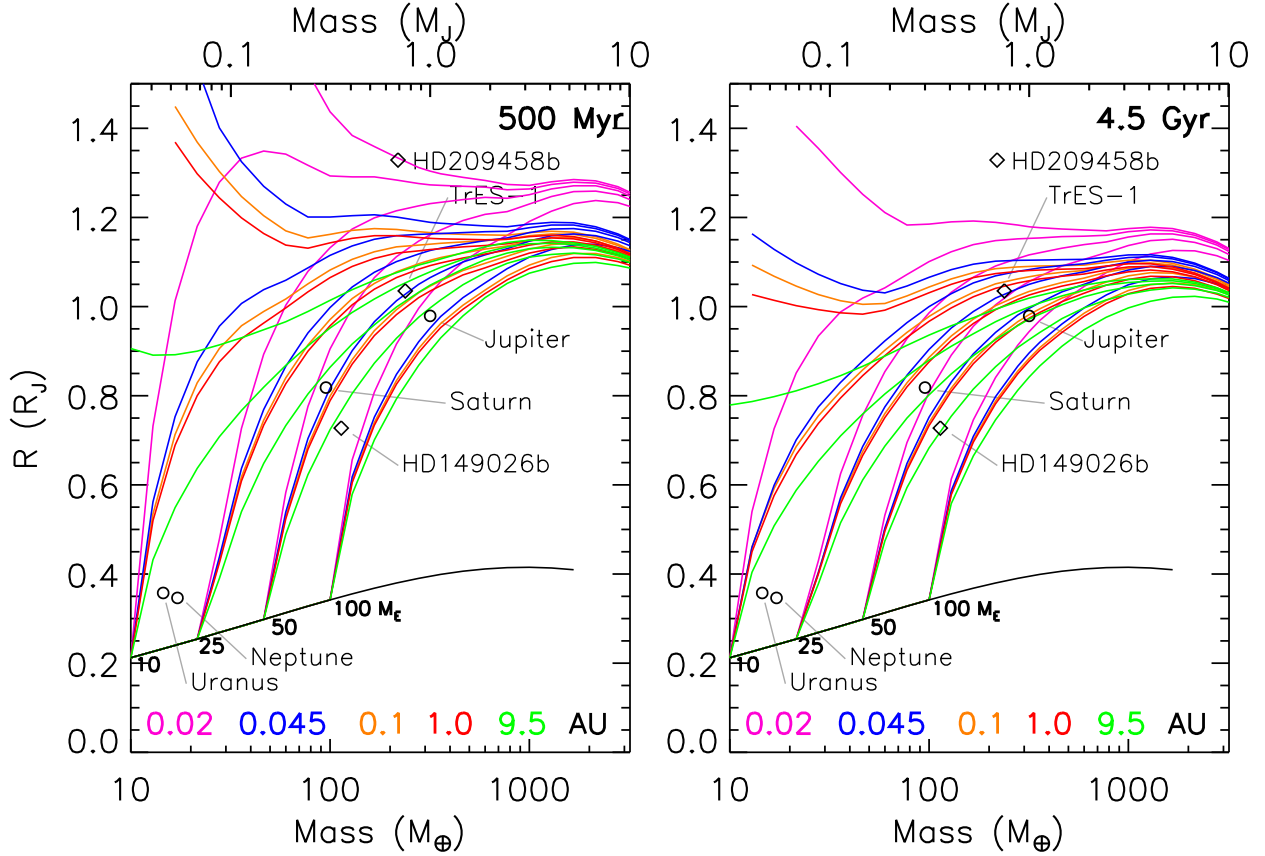


Fig. 8.— Planetary radii with various core masses at 500 Myr (A) and 4.5 Gyr (B). Models are calculated at 0.02, 0.045, 0.1, 1.0, and 9.5 AU and are color coded at the bottom of the plot. The black curve is for a heavy element planet of 50% ice and 50% rock. Models with no core, and core masses of 10, 25, 50, and 100 M_\oplus are computed. Labels in thick black text show where curves with a given constant core mass fall upon the mass/radius curve for heavy elements. Core-free models are the curves that terminate at low mass at the upper left. The open circles are solar system planets and the diamonds are extrasolar planets.