Non-Axisymmetric Rotating Potentials —

FIGURES of EQUILIBRIUM

- The Newton-Cassini Debate. →
  ~ 1680 s

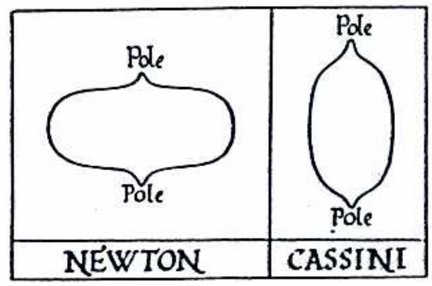


FIG. 2. An old-time caricature of the controversy between the opposing schools of Newton and Cassini with respect to the figure of the earth.

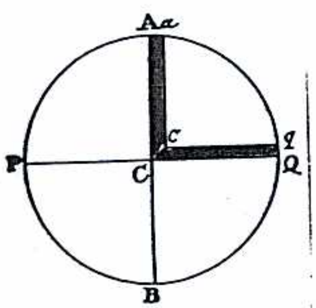


FIG. 1. Illustration from the Principia bearing on Newton's ments for the rotational flattening of the earth.

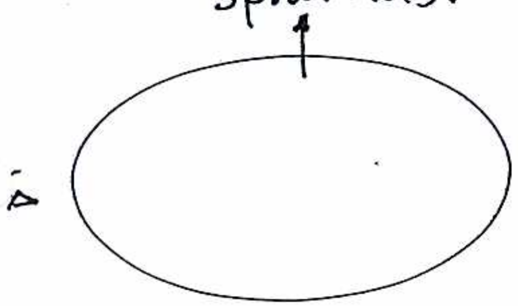
← Newton imagined two holes drilled to the center of the earth and filled with water. Centrepital acceleration impels the equatorial column to be longer if columns have same weight.

→ Maupertuis & Clairaut made Geodetic measurements in 1738 in Lapland and proved Newton correct.

↓

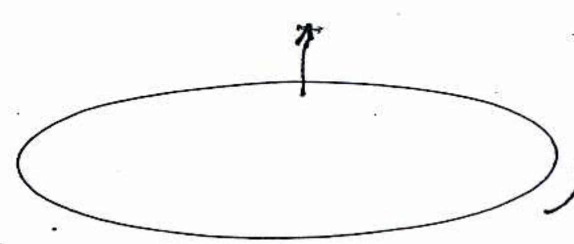
snide comments by Voltaire

- Maclaurin (1742) generalized Newton's results to uniformly rotating homogenous (incompressible) spheroids:



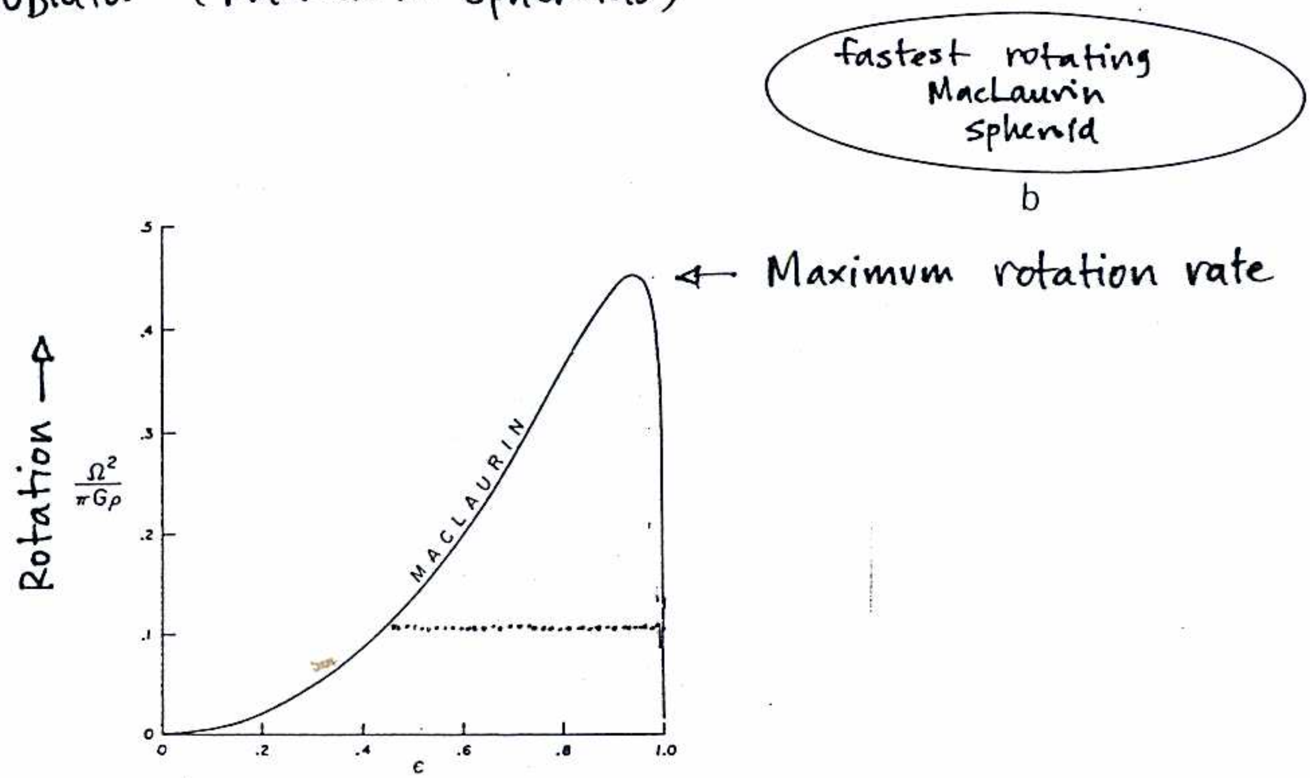
a

→ faster rotation →



b

- Simpson (1743) [as in "Simpson's Rule"] noticed that for any angular velocity less than a certain maximum value, there are two and only two possible "oblata" (MacLaurin spheroids):



→ oblateness →

0 = Sphere

1 = Flat Disk

$$\epsilon = \sqrt{1 - \frac{a_3}{a_1}}$$

a_3 = short axis

a_1 = long axis

- Jacobi (1834) was intrigued by the fact that 2 solutions exist at low rotation, and that the disk-like one cannot be found by considering small departures from sphericity...

  "One would make a grave mistake if one supposed that the spheroids of revolution are the only admissible figures of equilibrium"

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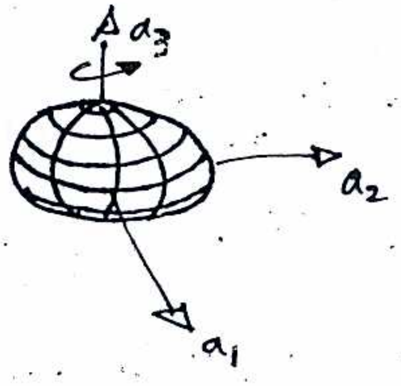
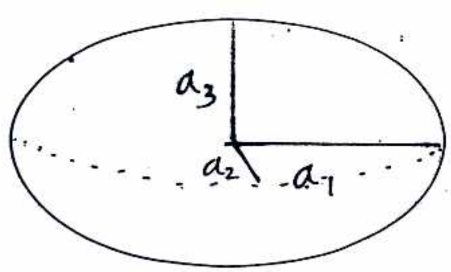
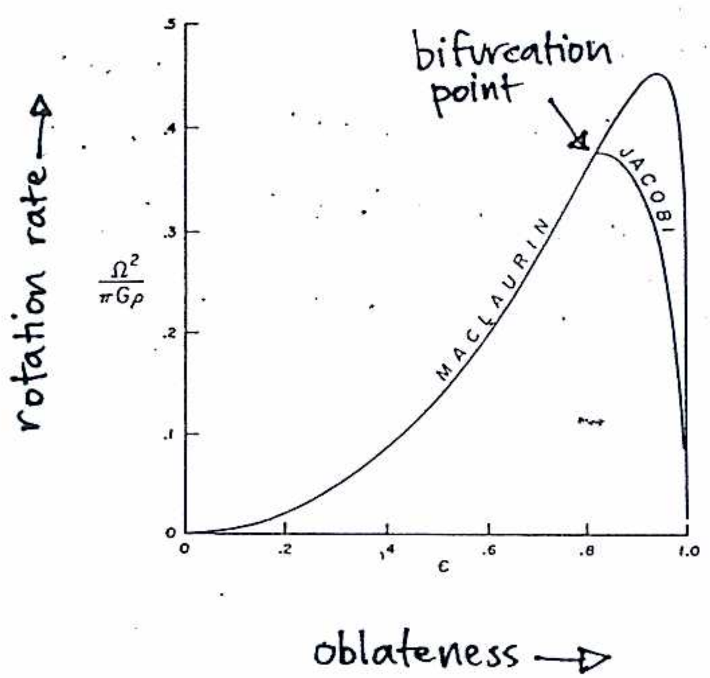
- Jacobi found that triaxial ellipsoidal figures of equilibrium are possible...



Meyer (1842) showed that the Jacobian sequence of ellipsoids "bifurcates" from the MacLaurin sequence at the point where $\epsilon = 0.81267$. That is, the figure axis $a_1 \to a_2$ and the Jacobi spheroid turns into a Maclaurin spheroid:

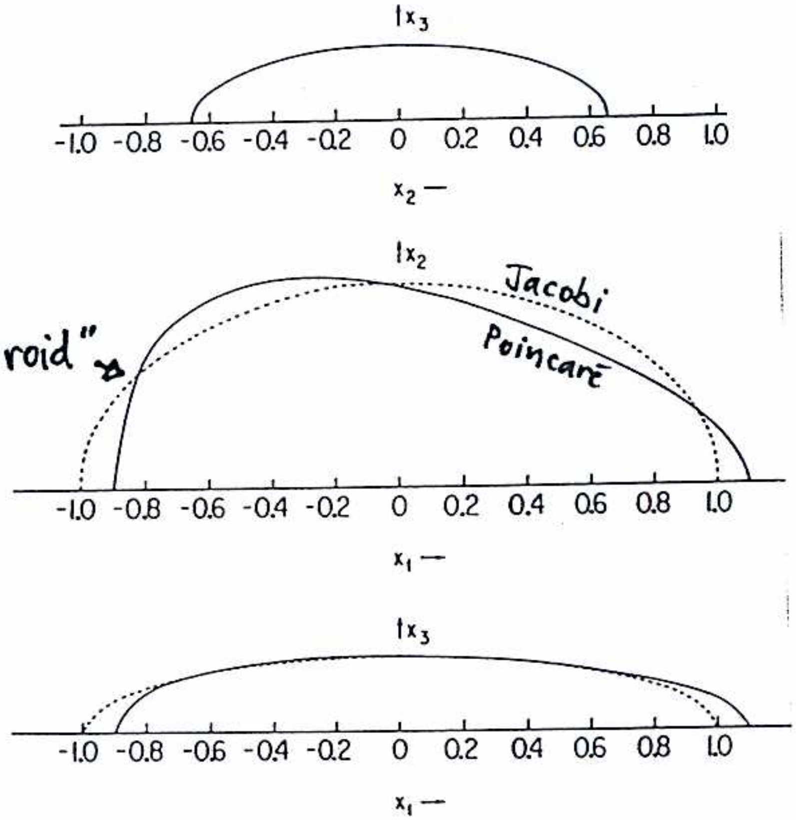


bifurcation spheroid



- Poincaré discovered in 1885 that along the sequence of Jacobian ellipsoids, another point of bifurcation occurs, and the Jacobian ellipsoids bifurcate into a sequence of pear-shaped (or egg-shaped) figures

Dotted lines show Jacobi Ellipsoid, solid lines show the Poincare "Pearoid"



Poincaré then imagined the slow evolution of a contracting spheroid in which $\tau_{viscous} \ll \tau_{K.H}$

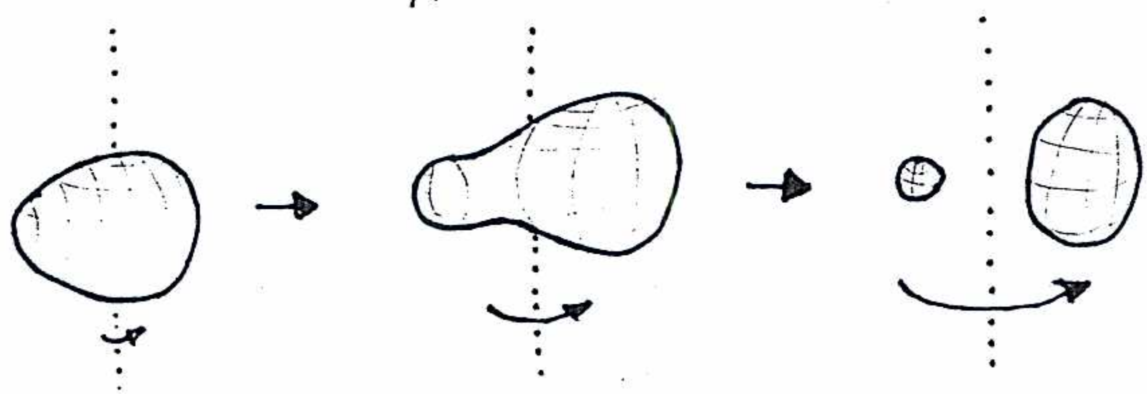
Maclaurin Spheroids

become secularly unstable to Jacobi ellipsoids

Which become secularly unstable to Pearoids

which ...

... might further spin up and eventually fission into two bodies – "the fission hypothesis"



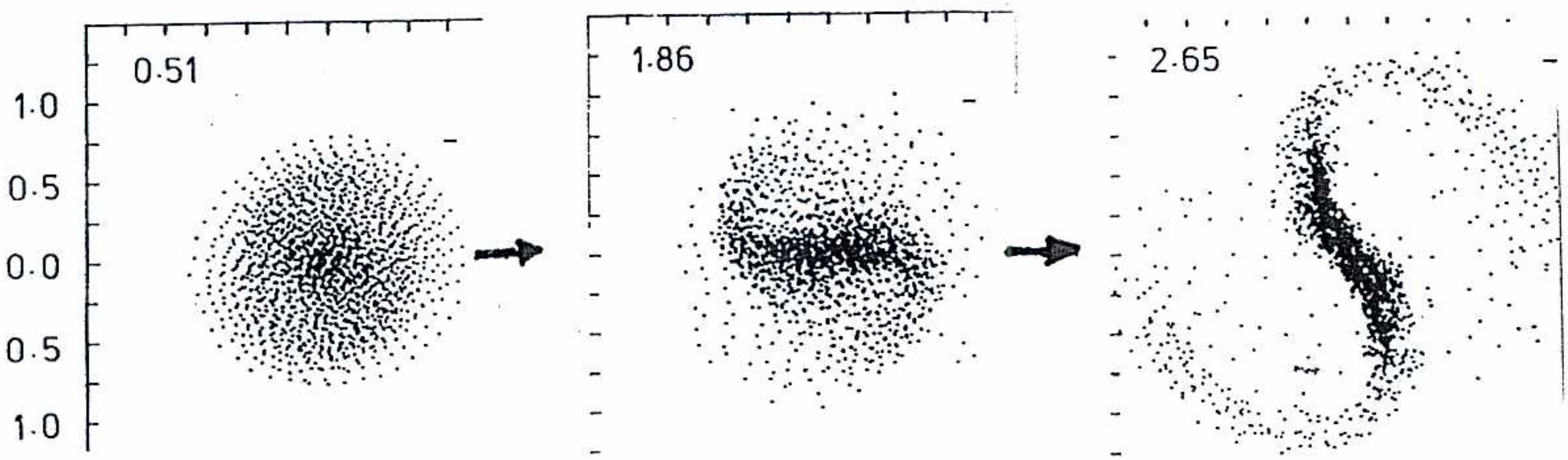
Darwin (1906) suggested that this might be the mechanism by which binary stars are formed.

Cartan (1924) showed that this fission process (if it occurs) would occur via a dynamical rather than secular instability (i.e. no dissipation would be required.)

Difficulties with the Newton-MacLaurin-Jacobi-Poincare-Darwin-Cartan Fission Hypothesis

[1.] Gaseous configurations with realistic degrees of central condensation (for example, polytropes), which are uniformly rotating, reach equatorial breakup prior to bifurcation into Jacobi-like triaxial ellipsoids. (James 1964, ApJ 140, 552)

[2.] Differentially rotating polytropes can become bar-unstable before reaching equatorial break-up. (Ostriker & Mark 1968, Bodenheimer & Ostriker 1973). The bar instability drives spiral waves which transport angular momentum outwards and stabilize the system against fission.



∘ figure from Durisen, Gingold, Tohline & Boss (1986)