Lecture 12: Overview of Stellar Evolution

Glatzmaier and Krumholtz 13
Prialnik 7
Pols 8

Summary of energy generation rates (Lecture 10)

$$\varepsilon_{pp} = 2.48 \times 10^{6} \ \rho Y_H^2 T_6^{-2/3} \, e^{-33.7/T_6^{1/3}} \ \text{erg g}^{-1} \text{ s}^{-1}$$

$$\varepsilon_{CNO} = 4.33 \times 10^{25} \, Y_p \, Z \, \rho \, T_9^{-2/3} e^{-15.19/T_9^{1/3}} \ \text{erg g}^{-1} \text{ s}^{-1}$$

(smaller S-factor than in texts)

$$\varepsilon_{3\alpha} = 5.11 \times 10^{8} \, \rho^2 X_\alpha^3 \ \mathrm{T}_9^{-3} \ \exp(-4.403 / \mathrm{T}_9) \ \text{erg g}^{-1} \text{ s}^{-1}$$

Consider stars supported mostly by ideal gas pressure contracting and obeying the Virial theorem.

When not powered by nuclear reactions, such stars radiate their gravitational binding energy on a Kelvin Helmholtz time scale. This is equivalent to radiating away the internal energy of the star. This requires a gravitational energy generation rate in erg g s^{-1} of

$$\varepsilon_{grav} = \frac{3P}{2\rho\tau_{KH}} = \frac{N_A k}{\mu}\frac{\bar{T}}{\tau_{KH}}$$

For the sun with $\bar{T} \sim 10^7$ K, $\tau_{KH} \sim 30\,My$, this gives

$\varepsilon_{grav} \sim 1$ erg g s^{-1} (*GK* say ~10)

A similar number can be obtained from simply dividing the present luminosity of the sun by its mass, $L_\odot / M_\odot \approx 1.9$ erg g s^{-1}. From previous discussions, e.g., of Eddington's standard model, note that this answer is independent of whether the source of the star's energy is nuclear or gravity.

Continuing to use Eddington's standard model, which also has $L \propto M^3$ suggests that, as a function of mass,

$$\varepsilon_{grav} \sim \frac{L}{M} \sim 2\left(\frac{M}{M_\odot}\right)^2 \; erg\ g\ s^{-1}$$

for stars near the main sequence.

"Ignition" on the main sequence will happen (can be defined by the point) when $\varepsilon_{nuc} = \varepsilon_{grav}$ for the whole star. In fact, because the nuclear reactions are very temperature sensitive, nuclear energy generation goes on in a much smaller fraction of the star's mass, which we shall call f ~ 0.1. Then

$$\left\langle \varepsilon_{nuc} \right\rangle_{whole\,star} \approx \left\langle \varepsilon_{grav} \right\rangle_{whole\,star} \Rightarrow$$

$$\varepsilon_{nuc,center} \approx \varepsilon_{grav} / f \approx 10\ \varepsilon_{grav}$$

$$\approx 20 \left(\frac{0.1}{f} \right) \left(\frac{M}{M_\odot} \right)^2 \text{ erg g}^{-1}\text{ s}^{-1}$$

(recovering GK for $1M_\odot$)

Putting ε_{pp} or (for T> 18MK) ε_{CNO} equal to the value required above gives the ignition line for main sequence (hydrogen burning) stars.

Helium burning

Continuing to consider non-degenerate stars (the helium flash must be considered separately), we can, as in your homework problem, also treat the helium core as a separate star with smaller mass.

Again using Eddington's model as a guide, the luminosity increases, for a given M, as μ^4, which implies an increase of $(1.34/0.61)^4 = 23$. In fact, since a) the central regions of helium burning stars are convective and b) the surface opacity is less, the luminosity increase is even greater. Typical horizontal branch stars have have helium core luminosities ~ 30 - 50 times solar and core masses about 0.5 solar masses. More massive stars that do not become HB stars have more massive helium cores and are even more luminous with L again going roughly as the cube of the helium core mass.

Helium burning

As a result helium ignition requires substantially higher energy generation rates at ignition. Even without this increase, the temperature required for helium ignition would be much hotter because of the larger Coulomb barriers involved and the smaller energy generation rate.

Near ignition for helium burning, the 0.5 $M_\odot$ helium core has

$$\varepsilon_{grav} \sim \frac{L}{M} \sim \frac{50\,L_\odot}{0.5\,M_\odot}\left(\frac{M}{0.5M_\odot}\right)^2 \text{ erg g}^{-1}\text{ s}^{-1}$$

and

$$\varepsilon_{nuc,cen} \sim 1000\left(\frac{L_\odot}{M_\odot}\right)\left(\frac{M}{0.5M_\odot}\right)^2\left(\frac{0.1}{f}\right)\text{erg g}^{-1}\text{ s}^{-1}$$

$$\sim 2000\left(\frac{M}{0.5M_\odot}\right)^2\left(\frac{0.1}{f}\right)\text{erg g}^{-1}\text{ s}^{-1} \qquad \text{since } \frac{L_\odot}{M_\odot} \approx 2\,\text{erg g}^{-1}\text{ s}^{-1}$$

Maximum Temperature Achieved in a Contracting Core

We found for all polytropes a relation

$$P_c = C_n G M^{2/3} \rho_c^{4/3}$$

with C_n a weak function of n, equal to C_n = 0.478 for n = 3/2. This polytropic index is appropriate for both non-relativistic degeneracy and fully convective stars supported by ideal gas. For ideal gas the central temperature thus depends on the central density as

$$\mathrm{T}_c = \frac{C_n G \mu M^{2/3}}{N_A k} \rho_c^{1/3}$$

Maximum Temperature Achieved in a Contracting Core

The temperature will continue to rise until the gas becomes degenerate (if it ever does). Then the pressure if it becomes degenerate will be given by

$$P_{NR\,\deg} = K_{NR}\left(\rho Y_e\right)^{5/3} \qquad K_{NR} = 1.004 \times 10^{13} \text{ dyne cm}^{-2}$$

which also = $C_{3/2} G M^{2/3} \rho_c^{4/3}$ so

$$\rho_c^{1/3} = \frac{C_{3/2} G M^{2/3}}{K_{NR} Y_e^{5/3}} \Rightarrow \rho_c = \left(\frac{C_{3/2} G}{K_{NR}}\right)^3 \frac{M^2}{Y_e^5}$$

Maximum Temperature Achieved in a Contracting Core

The maximum temperature will be reached, at the point when the gas first becomes degenerate and the two pressures are comparable, that is

$$P_{c,total} = C_{3/2} G M^{2/3} \rho_c^{4/3} = 2\left(\frac{\rho_c T_c N_A k}{\mu}\right) = 2\left(K_{NR}\left(\rho_c Y_e\right)^{5/3}\right)$$

from which it follows

$$T_c = \frac{C_{3/2} G \mu M^{2/3}}{2 N_A k} \rho_c^{1/3} = \frac{C_{3/2} G \mu M^{2/3}}{2 N_A k} \frac{C_{3/2} G M^{2/3}}{2 K_{NR} Y_e^{5/3}}$$

$$= \frac{C_{3/2}^2 G^2 \mu}{4 N_A k K_{NR} Y_e^{5/3}} M^{4/3} \qquad (\textit{Pols } 8.5)$$

$$= 5.7 \times 10^7 \left(\frac{M}{M_\odot}\right)^{4/3} K \qquad \text{for } \mu = 0.6,\ Y_e = 0.875$$

$$= 3.2 \times 10^8 \left(\frac{M}{M_\odot}\right)^{4/3} K \qquad \text{for } \mu = 1.33,\ Y_e = 0.50\ (\textit{helium})$$

Minimum mass star for H ignition

Set

$$\varepsilon_{pp} = 2.48 \times 10^6 \ \rho Y_H^2 T_6^{-2/3} e^{-33.7/T_6^{1/3}} \ \text{erg g}^{-1} \text{ s}^{-1}$$

evaluated at $T_{max} = 5.7 \times 10^7 \left(\frac{M}{M_\odot}\right)^{4/3} K$

equal to

$\varepsilon_{nuc,center} \approx 20\left(\frac{0.1}{f}\right)\left(\frac{M}{M_\odot}\right)^2$ erg g^{-1} s^{-1} and solve for M. Probably easiest to solve by iteration. Pick $M = 0.2 M_\odot$ and solve for T_{max} to get 6.7×10^6 K which gives (at $Y_H = 0.7, \rho = 100\, g\, cm^3$) $\varepsilon_{pp} = 0.6$. $\varepsilon_{nuc,center} \approx 20(.04) = 0.8$, a pretty close match.

Empirically better results are obtained with

$$= \frac{C_{3/2}^2 G^2 \mu}{2 N_A k K_{NR} Y_e^{5/3}} M^{4/3} = 11.4 \times 10^7 \left(\frac{\mu}{0.6}\right)\left(\frac{0.875}{Y_e}\right)^{5/3}\left(\frac{M}{M_\odot}\right)^{4/3}$$

Then 0.1 $M_\odot$ gives 5.3×10^6 K, $\varepsilon_{pp} = 0.16$. $\varepsilon_{nuc,center} \approx 20(.01) = 0.2$,

Minimum mass star for He ignition

$$\varepsilon_{3\alpha} = 5.11\times10^{8}\,\rho^{2}X_{\alpha}^{3}\;T_9^{-3}\;\exp\,(-4.403\,/\,T_9)\;\;\mathrm{erg\;\;g^{-1}\;s^{-1}}$$

evaluated at $T_{max} = 3.2\times10^{8}\left(\frac{M}{M_\odot}\right)^{4/3} K$

equal to

$$\varepsilon_{nuc,center} \approx 2000\left(\frac{M}{0.5M_\odot}\right)^{2}\left(\frac{0.1}{f}\right)\mathrm{erg\;g^{-1}\;s^{-1}}$$

Try ρ=$1.\times10^{4}$ $X_\alpha = 1.$, $M = 0.45M_\odot, 0.50M_\odot \Rightarrow T = 1.27, 1.10\times10^{8}$ K

$\varepsilon_{3\alpha} = 22{,}000\;\;,\;\;160.$

So in between 0.45 and 0.50 $M_\odot$ even allowing for uncertainty in the density.The answer is close to right because of the high power of temperature which 3α depends on for such low T (T^{41} at 1.0 x 10^{8} K)

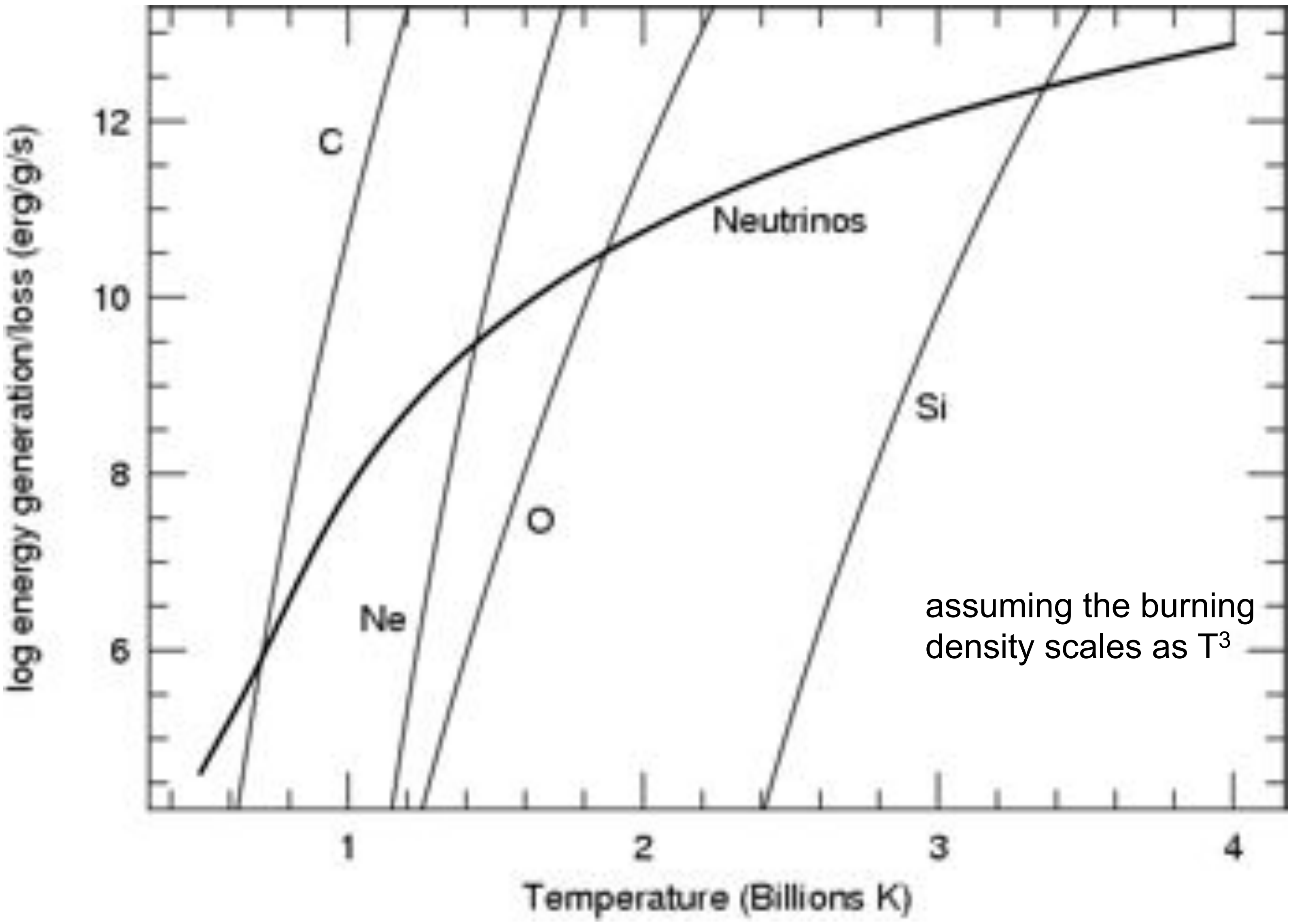
Beyond Helium Burning

Following the same approach it is not difficult to write down approximate energy generation rate formulae for carbon and oxygen burning. Silicon burning is more complicated. We shall discuss these later, but defer them for now since neutrino losses by the pair process play a dominant role there (Pols 6.5)

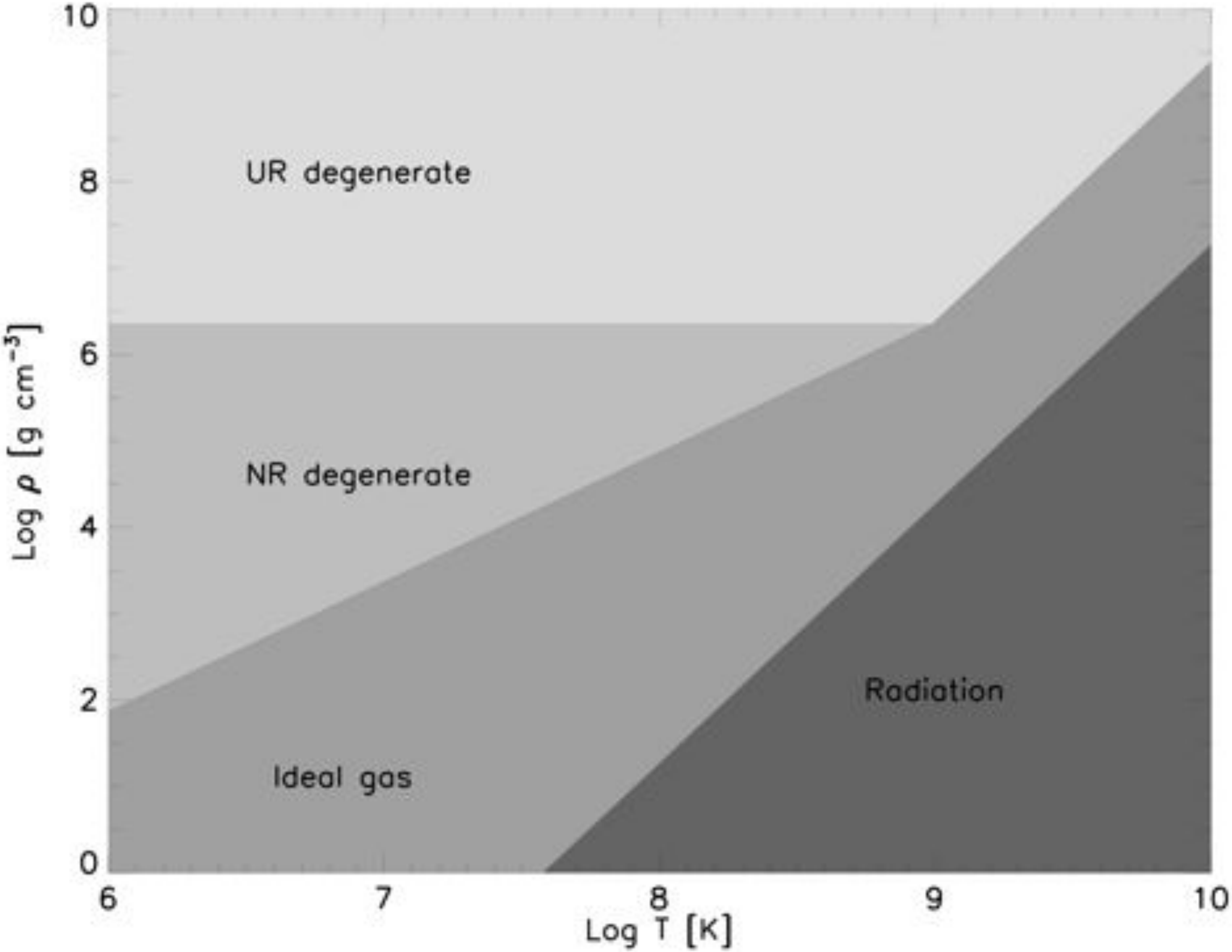
Setting neutrino losses equal to nuclear energy generation we will later determine that the heavy fuels burn at a nearly constant temperature (because of the huge temperature sensitivity. The densities vary.

Fuel	**Temperature (10^9 K)**	**Density (g cm^{-3})**
Carbon	0.8	2×10^5
Oxygen	1.8	2×10^6
Silicon	3.2	2×10^7

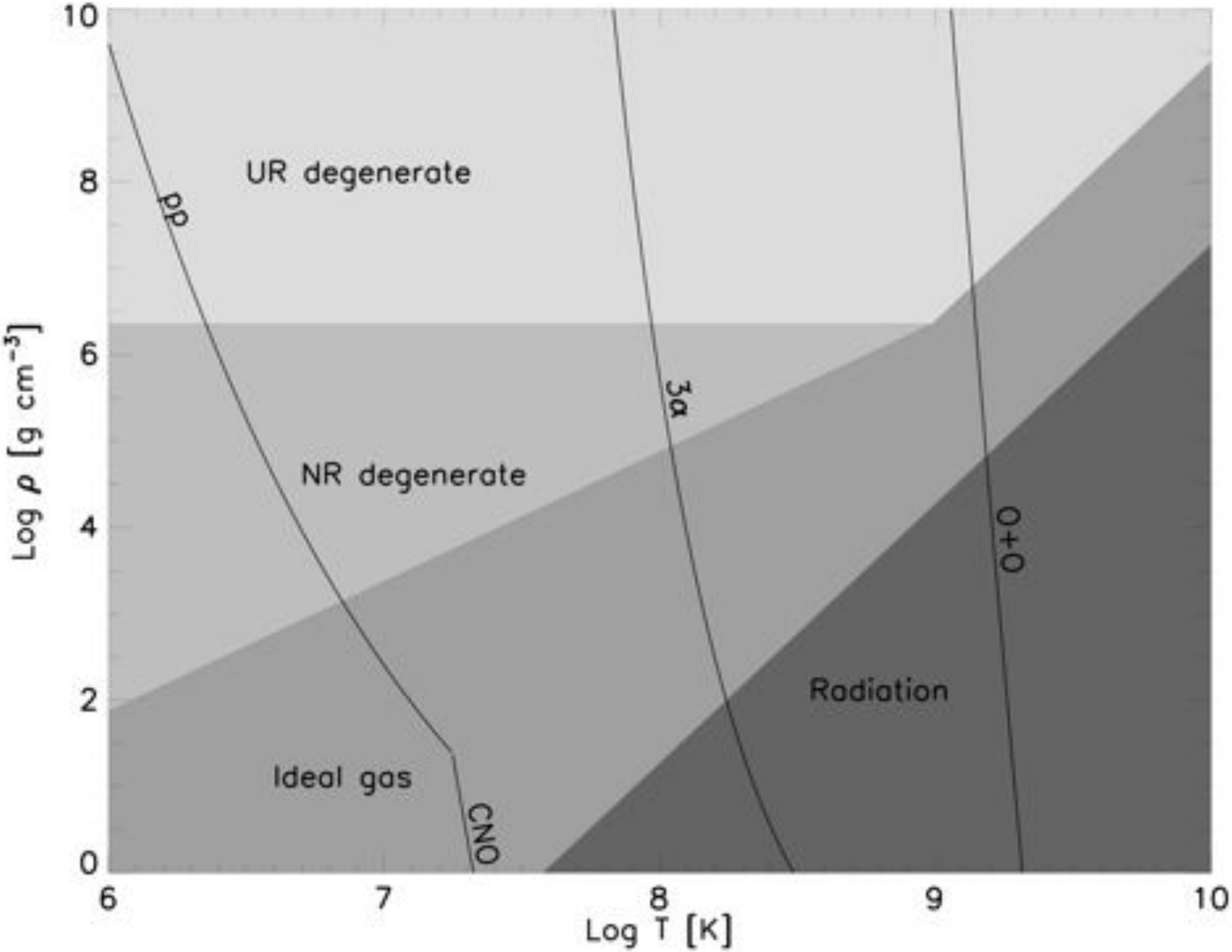
Woosley, Heger, and Weaver (RMP, 2002)



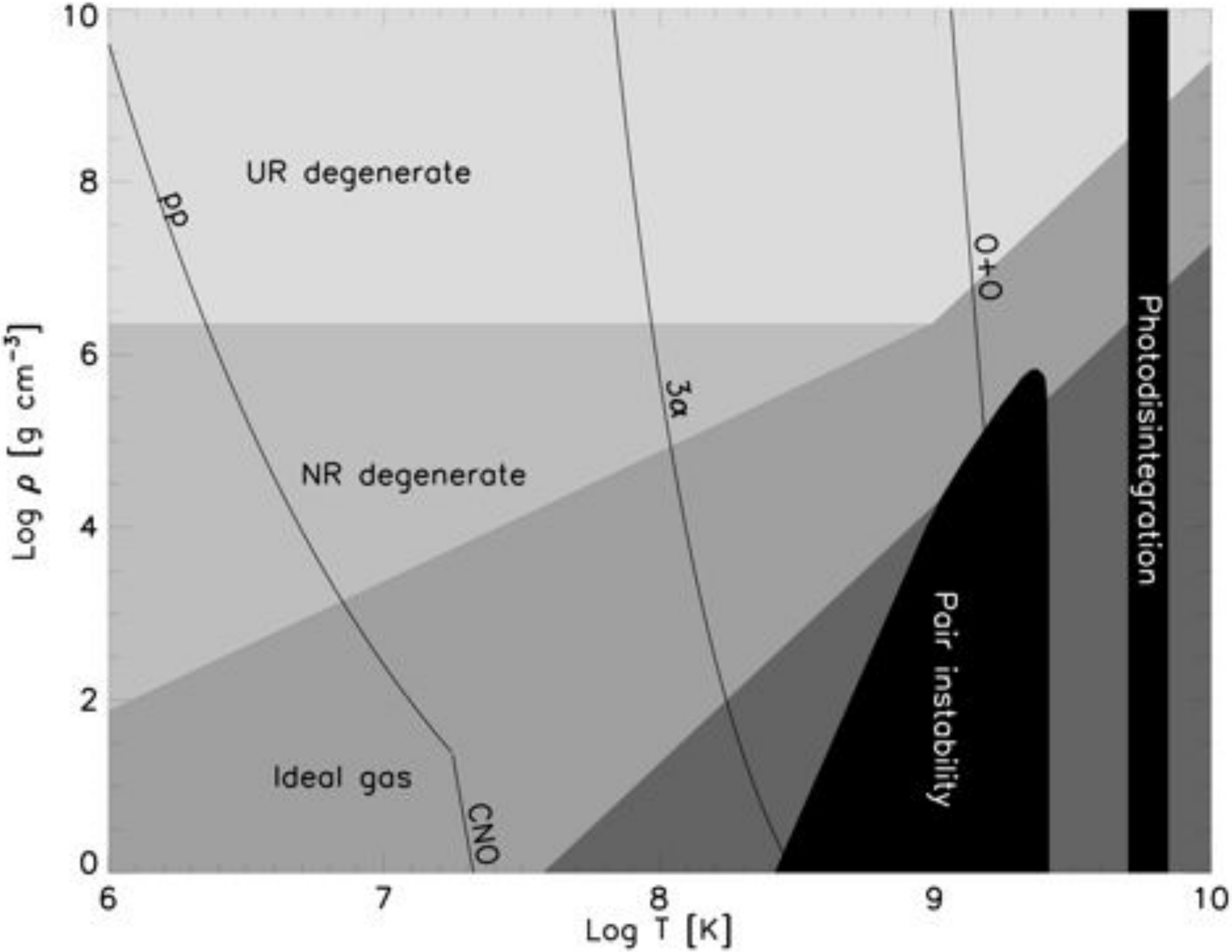
assuming the burning density scales as T^3



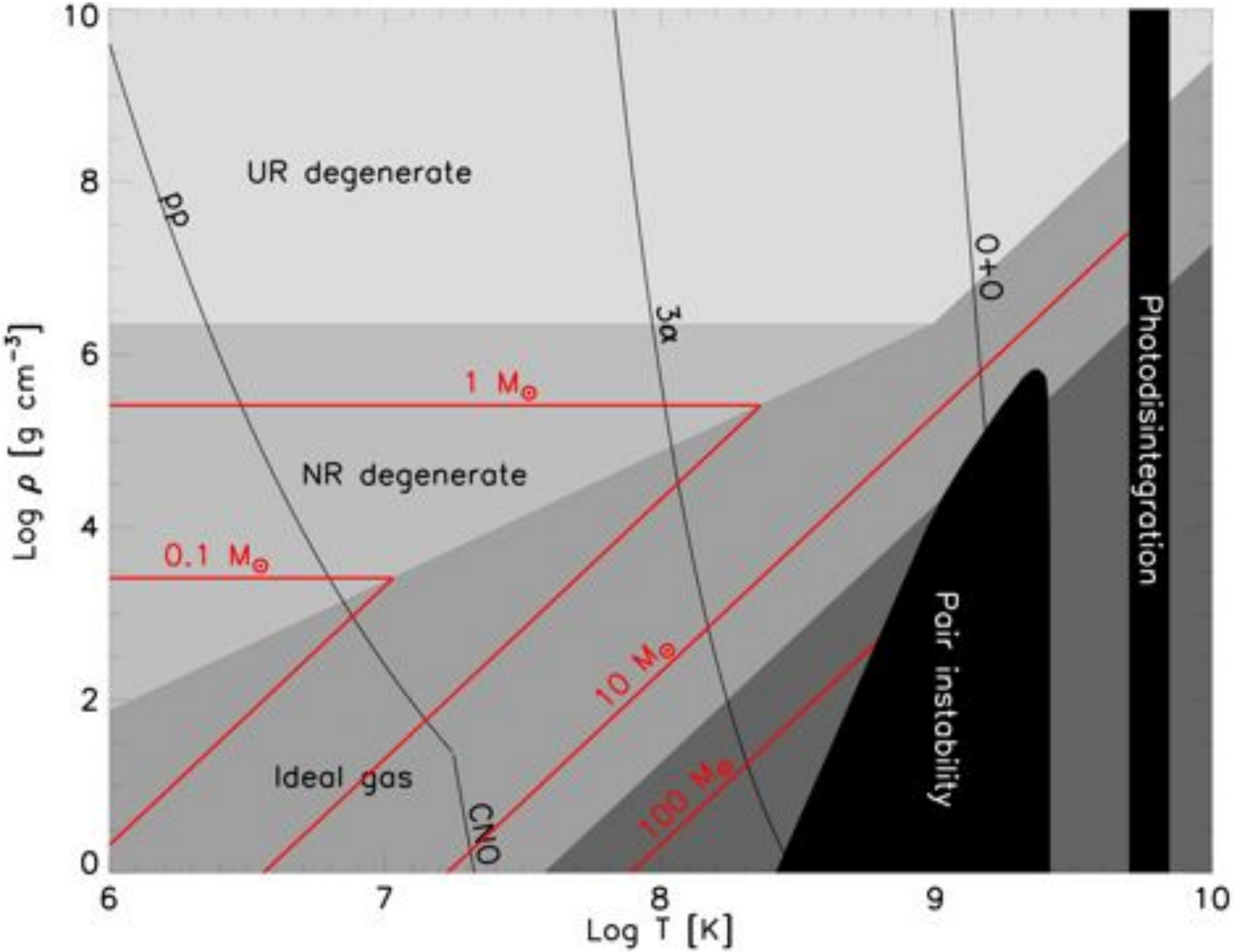
UR degenerate
NR degenerate
Ideal gas
Radiation
Log ρ [g cm^-3]
Log T [K]
0
2
4
6
8
10
6
7
8
9
10



UR degenerate
NR degenerate
Ideal gas
Radiation
pp
CNO
3α
O+O
Log ρ [g cm⁻³]
Log T [K]
10
8
6
4
2
0
6
7
8
9
10



UR degenerate
NR degenerate
Ideal gas
pp
CNO
3α
O+O
Pair instability
Photodisintegration
Log ρ [g cm^-3]
Log T [K]
0
2
4
6
8
10
6
7
8
9
10



UR degenerate
NR degenerate
Ideal gas
pp
CNO
3α
O+O
1 M⊙
0.1 M⊙
10 M⊙
100 M⊙
Pair instability
Photodisintegration
Log ρ [g cm⁻³]
Log T [K]
10
8
6
4
2
0
6
7
8
9
10

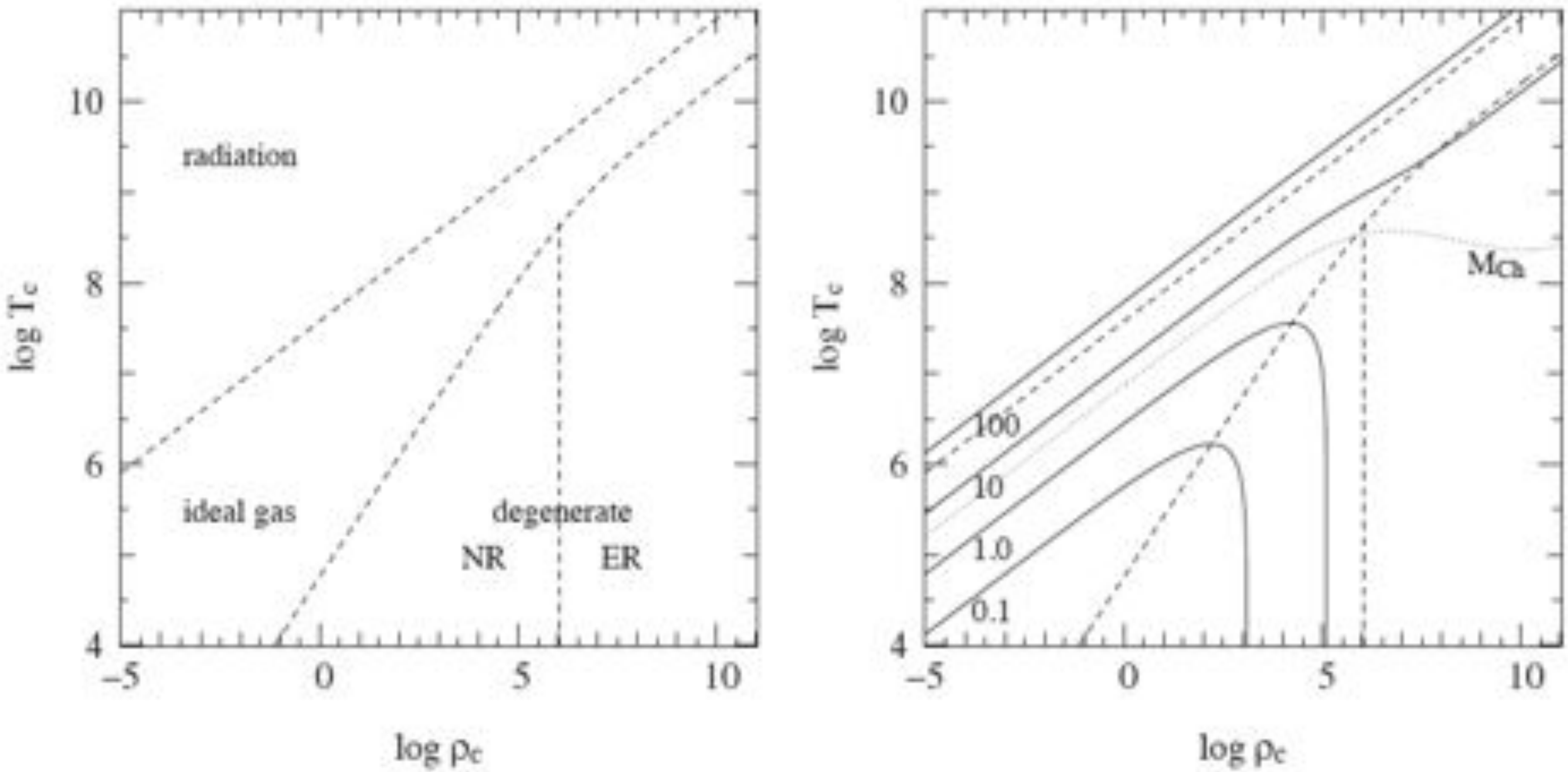
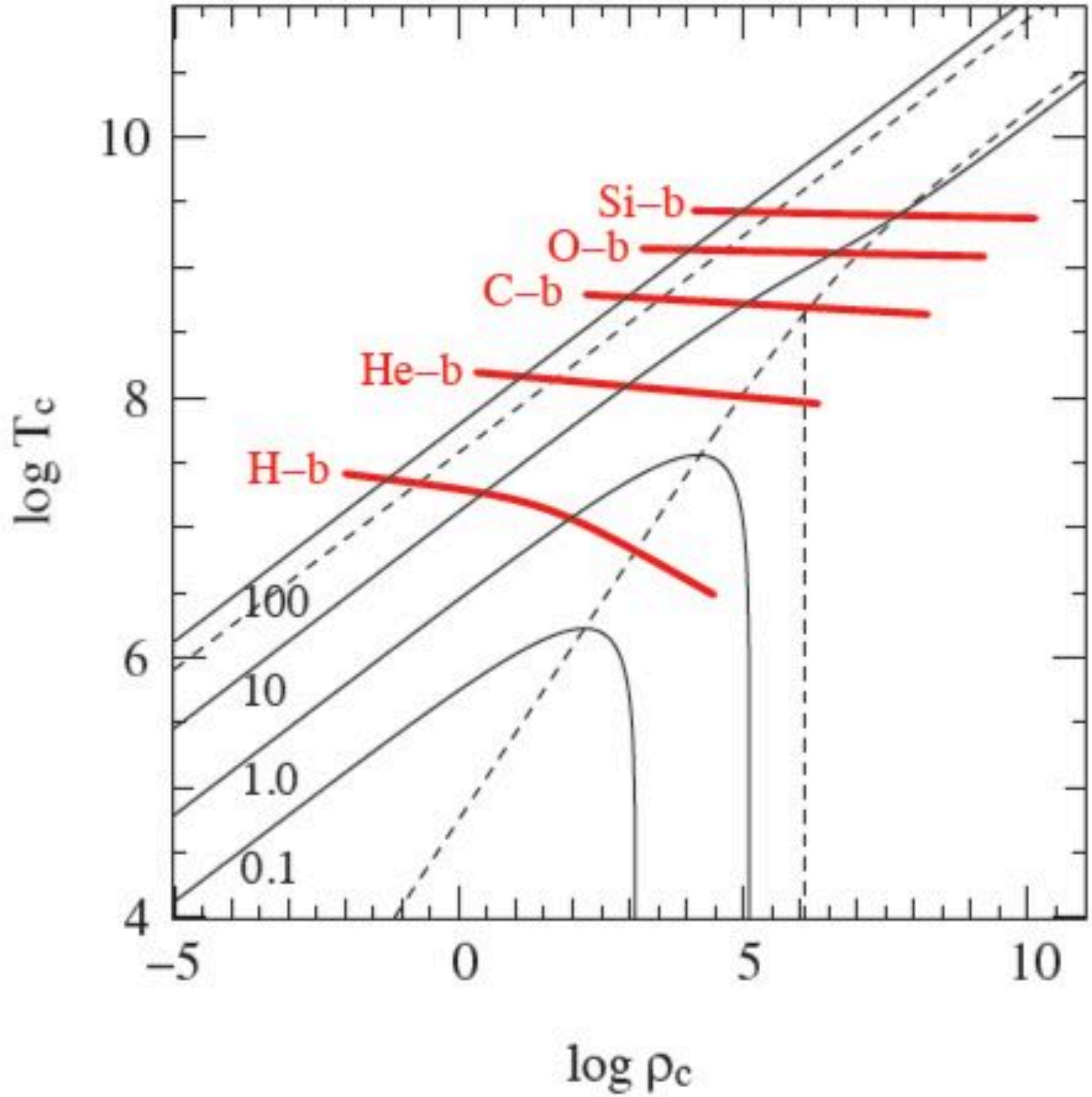


Figure 8.2. The equation of state in the $\log T_c - \log \rho_c$ plane (left panel), with approximate boundaries between regions where radiation pressure, ideal gas pressure, non-relativistic electron degeneracy and extremely relativistic electron degeneracy dominate, for a composition of $X = 0.7$ and $Z = 0.02$. In the right panel, schematic evolution tracks for contracting stars of $0.1 - 100\,M_{\odot}$ have been added.



log T_c
log ρ_c
Si–b
O–b
C–b
He–b
H–b
100
10
1.0
0.1
10
8
6
4
–5
0
5
10

Table 8.1. Characteristics of subsequent gravitational contraction and nuclear burning stages. Column (3) gives the total gravitational energy emitted per nucleon since the beginning, and column (5) the total nuclear energy emitted per nucleon since the beginning. Column (6) gives the minimum mass required to ignite a certain burning stage (column 4). The last two columns give the fraction of energy emitted as photons and neutrinos, respectively.

phase	T (10^6 K)	total E_{gr}/n	main reactions	total E_{nuc}/n	M_{min}	γ (%)	ν (%)
grav.	$0 \rightarrow 10$	~ 1 keV/n				100	
nucl.	$10 \rightarrow 30$		$^1H \rightarrow {}^4He$	6.7 MeV/n	$0.08\,M_\odot$	~95	~5
grav.	$30 \rightarrow 100$	~ 10 keV/n				100	
nucl.	$100 \rightarrow 300$		$^4He \rightarrow {}^{12}C, {}^{16}O$	≈ 7.4 MeV/n	$0.3\,M_\odot$	~100	~0
grav.	$300 \rightarrow 700$	~ 100 keV/n				~50	~50
nucl.	$700 \rightarrow 1000$		$^{12}C \rightarrow$ Mg, Ne	≈ 7.7 MeV/n	$1.1\,M_\odot$	~0	~100
grav.	$1000 \rightarrow 1500$	~ 150 keV/n					~100
nucl.	$1500 \rightarrow 2000$		$^{16}O \rightarrow$ S, Si	≈ 8.0 MeV/n	$1.4\,M_\odot$		~100
grav.	$2000 \rightarrow 5000$	~ 400 keV/n	Si $\rightarrow \ldots \rightarrow$ Fe	≈ 8.4 MeV/n			~100