Introductory Nuclear Physics

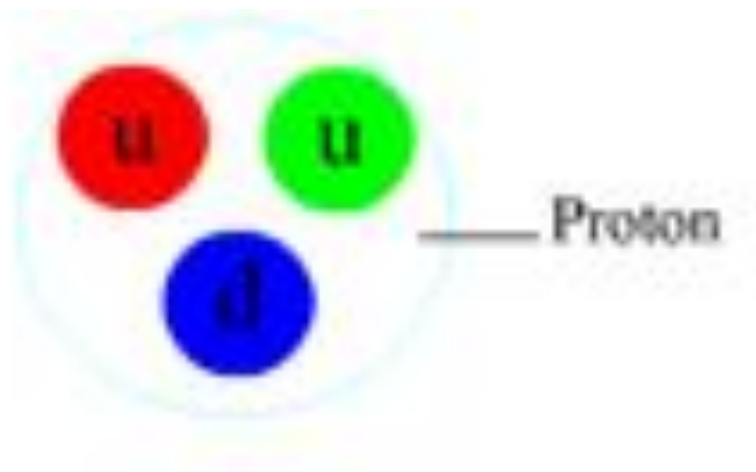
Glatzmaier and Krumholz 7
Prialnik 4
Pols 6
Clayton 4.1, 4.4

Each nucleus is a bound collection of N neutrons and Z protons. The *mass number* is A = N + Z, the *atomic number* is Z and the nucleus is written with the elemental symbol for Z

$$^{A}Z$$

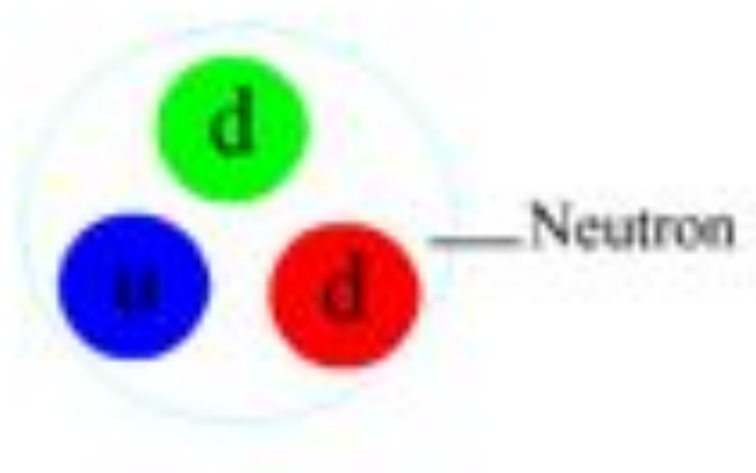
E.g. ^{12}C, ^{13}C, ^{14}C are isotopes of carbon all with Z= 6 and neutron numbers N = 6, 7, 8

The neutrons and protons are bound together by the *residual strong or color force*



proton = uud
neutron = udd

There are two ways of thinking of the strong force - as a residual color interaction (like a van der Waal's force) or as the exchange of mesons. Classically the latter has been used.



http://en.wikipedia.org/wiki/Nuclear_force

Mesons are quark-antiquark pairs and thus carry net spin of 0 or 1. They are Bosons while the baryons are Fermions. They can thus serve as coupling particles. Since they are made of quarks they experience both strong and weak interactions. The lightest three mesons consist only of combinations of u, d, $\bar{u}$, and $\bar{d}$ and are

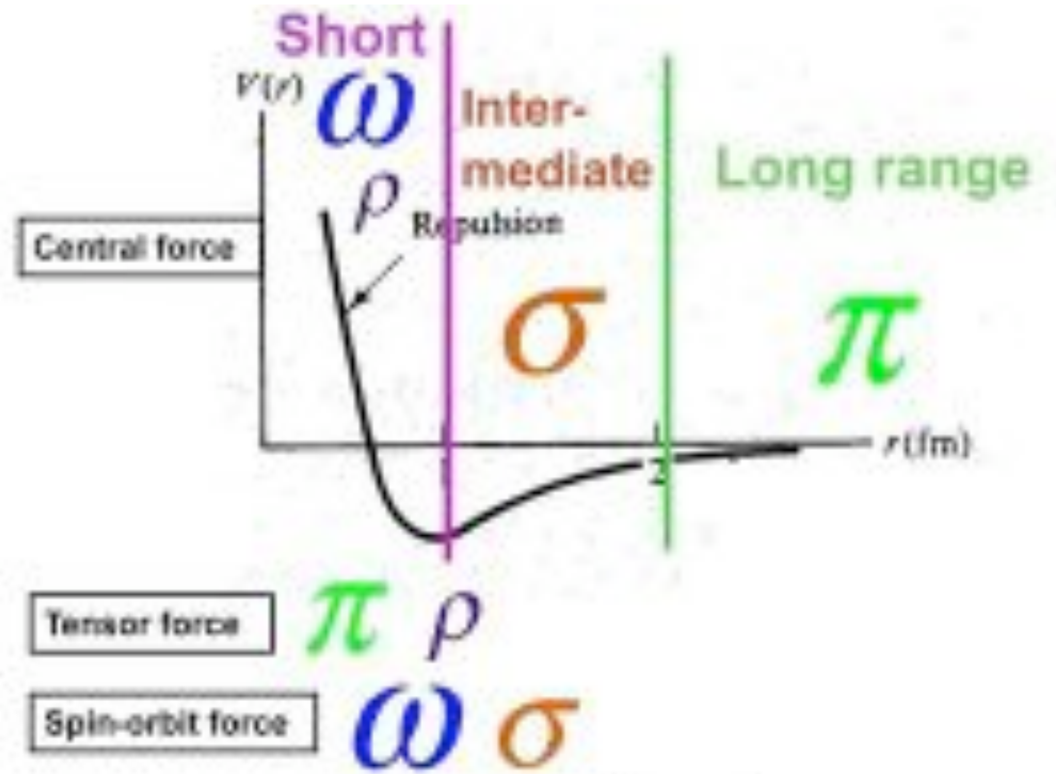
Name	Made of	Charge	Mass*c^2	τ (sec)
π^o	$\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$	0	135 MeV	8.4(-17)
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	± 1	139.6	2.6(-8)

$\pi^\pm \rightarrow \mu^\pm + \nu_\mu$ $\qquad$ $\pi^0 \rightarrow 2\gamma$, occasionally $e^+ + e^-$

There are many more mesons. Exchange of these lightest mesons give rise to a force that is complicated, but attractive. But at a shorter range, many other mesons come into play, notably the omega meson (782 MeV), and the nuclear force becomes repulsive.

http://en.wikipedia.org/wiki/List_of_mesons

There are many mesons with different masses. The heavier the mass, the shorter the lifetime by the uncertainty principle $\Delta E\ \Delta t \sim h$ and hence the shorter the range $r = c\ \Delta t = ch/mc^2 = h/mc$.



http://www.scholarpedia.org/article/Nuclear_Forces - Properties_of_the_nuclear_force

- The nuclear force is only felt among hadrons (2 or 3 bound quarks – e.g., nucleons, mesons).

1 Fermi = 1 fm
$=10^{-13}$ cm

- At typical nucleon separation (1.3 fm) it is a very strong attractive force.

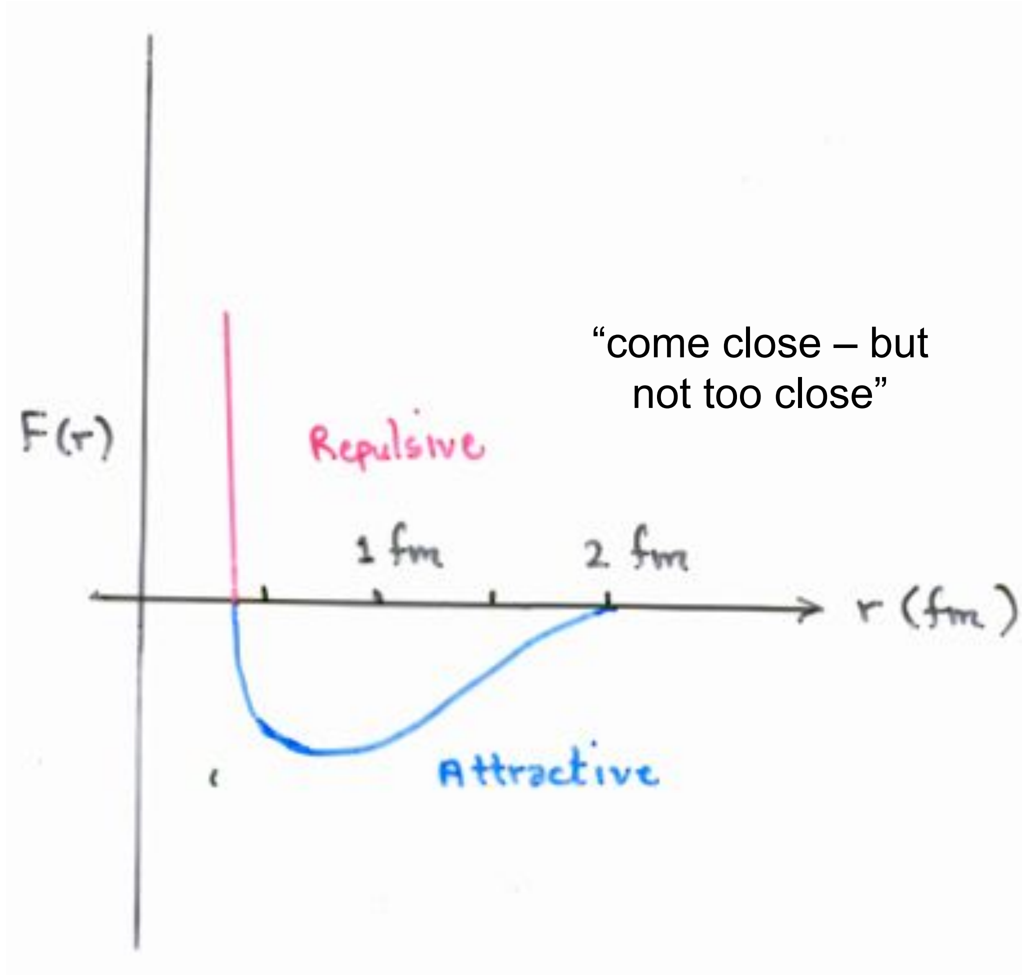
- At much smaller separations between nucleons the force is very powerfully repulsive, which keeps the nucleons at a certain average separation.

- Beyond about 1.3 fm separation, the force exponentially dies off to zero. It is less than the Coulomb force beyond about 2.5 fm (1 fm = 10^{-13} cm).

- The *NN* force is nearly independent of whether the nucleons are neutrons or protons. This property is called *charge independence or isospin independence*.

- The *NN* force depends on whether the spins of the nucleons are parallel or antiparallel.

- The *NN* force has a noncentral or tensor component.



“come close – but not too close”

Because of the nature of the force, putting nucleons in the nucleus is like putting (magnetic) marbles into a (spherical) fishbowl. The nucleus is virtually incompressible and its volume is proportional to A = N+Z.

Since the nucleons are Fermions they obey FD statistics and have a Fermi energy

$$p_0 = \left(\frac{3h^3}{8\pi} n \right)^{1/3}$$

1.4×10^{14} g cm^{-3} (x 2)

where n = $\frac{0.17}{2}$ fm^{-3} $= 8.5 \times 10^{37}$ cm^{-3} is the density of n or p.

Here h = 6.626 $\times 10^{-27}$ erg s . This implies a speed for the nucleons of about c / 4. and a peak Fermi energy, $\varepsilon_F = \frac{p_0^2}{2M} = 39$ MeV.

The average Fermi energy is 3/5 of this

$$\langle \varepsilon_F \rangle = 23 \text{ MeV per nucleon}$$

Coulomb energies are much smaller than this.
To zeroth order the nucleus is a degenerate gas of nucleons confined by the (residual) strong force. For heavy nuclei though the electrical energy does become important since it goes as Z^2.

Also, because of its short range, smaller than a typical nucleus, nucleons only interact with their nearest neighbors. Experimentally, we see the effect of "saturation". Nuclear binding energy goes very nearly linearly in A, at least for A $\gtrsim 4$. Recall that gravitational and electrical binding energy depend on M^2 and e^2 respectively. The minimum energy state is a sphere. The largest known deformation is ^{176}Lu, about 20%. The nuclear volume is also linear in A. Hence, for $A \gtrsim 12$, the radius is $\propto A^{1/3}$ and the density constant to 10%. Specifically

Removing each marble from the bowl takes the same energy

$$R \propto A^{1/3}$$

$$R \approx 1.12\,A^{1/3}\ \mathrm{fm}$$

The nuclear crossing time for a typical nucleon, $\tau \sim 10R/c \sim 10^{-22}$ s sets the time scale for the shortest nuclear reactions.

The electrical force between charged nucleons is

$$E_{\mathrm{Coul}} = \frac{e^2}{R} = \frac{1.44\ \mathrm{MeV}}{R(\mathrm{fm})} << \epsilon_F$$

for any one nucleon. However, since electrical energy rises as Z^2 and nuclear binding goes only as A, for large mass nuclei, the electrical force does become non-negligible.

The nuclear force is independent of charge and is the same between neutrons and protons. However it does depend on spin and orientation. The triplet state $(\uparrow\uparrow)$ [1] of two

nuclear force is
spin dependent

nucleons has different binding (stronger) that the singlet state (↑↓). An important astrophysical example of this is the deuteron, ^{2}H. The triplet state ($J = 1^+$) is bound. The di-neutron, n↑n↓ and di-proton, p↑p↓, are not (note that p↑p↑ and n↑n↑ are forbidden by the Pauli exclusion principle).

The binding energy of a nucleus:

- the energy available to hold nucleus together

- For a bunch of well-separated nucleons: the binding energy is zero
- Bring them together: strong force glues them together. However, energy has to come from somewhere: binding energy must come from a **reduction** in nuclear mass

Formally, it is the difference between mass of component protons and neutrons and that of actual nucleus, related through $E= mc^2$:

nb. $BE(p) = 0$
$BE(n) = 0$

$$BE(A,Z) = Z\, m_p c^2 + N\, m_n c^2 - M(A,Z)\, c^2$$

Binding energy is a **positive quantity**
(even though the strong potential in which the nucleons sit is **negative**)

Binding energy per nucleon

→ the average energy state of nucleon is a sum of high energy "surface" nucleons with low energy "bulk" nucleons
→ nucleus minimizes energy by minimizing surface area – a sphere

Unless weak interactions are involved (to be discussed later), the total energy released or absorbed in a given reaction

$$Q = \sum BE(species\,out) - \sum BE(species\,in)$$

Q is measured in MeV where
1 MeV = 1.6022 x 10^{-6} erg.

E.g. the binding energy of a proton is 0

The binding energy of a ^{4}He nucleus is 28.296 MeV.

4 p $\rightarrow ^4$ He thus releases 28.296 MeV per helium formed or 6.8 $\times 10^{18}$ erg g^{-1} (modulo some losses to neutrinos and p,n mass changes)

Semi-Empirical Mass Formulae

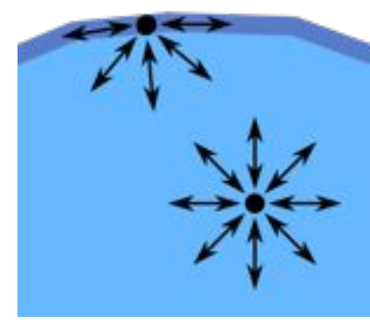
- A phenomenological understanding of nuclear binding energies as function of A, Z and N.
- Assumptions:
  - Nuclear density is constant.
  - We can model effect of short range attraction due to strong interaction by a liquid drop model.
  - Coulomb corrections can be computed using electro magnetism (even at these small scales)
  - Nucleons are fermions in separate wells (Fermi gas model ➔ asymmetry term)
  - Corrections for spin and shell closures.

Liquid Drop Model

- Phenomenological model to understand binding energies.
- Consider a liquid drop
  - Intermolecular force repulsive at short distances, attractive at intermediate distances and negligible at large distances ➔ constant density. Molecules on the “inside” are in a lower energy state than those at the surface, so surface area is minimized
  - n=number of molecules, T=surface tension, BE=binding energy E=total energy of the drop, a,b=free constants

$$E=-an + 4\pi R^2 T \quad \Rightarrow \quad BE=an-bn^{2/3}$$

surface area ~ $n^{2/3}$



- Analogy with nucleus
  - Nucleus has constant density
  - From nucleon-nucleon scattering experiments we know:
    - Nuclear force has short range repulsion and is attractive at intermediate distances.

Volume and Surface Term

- If we can apply the liquid drop model to a nucleus
  - constant density
  - same binding energy for all constituents
- Volume term: $B_{Volume}(A) = +aA$
- Surface term: $B_{Surface}(A) = -bA^{2/3}$

a ~ 15 MeV
b ~ 18 MeV

- Since we are building a phenomenological model in which the coefficients a and b will be determined by a fit to measured nuclear binding energies we must include any further terms we may find with the same A dependence together with the above

Coulomb Energy

- The nucleus is electrically charged with total charge *Ze*
- Assume that the charge distribution is spherical and homogeneous and compute the reduction in binding energy due to the Coulomb interaction

$$E_{Coulomb} = \int_0^{Ze} \frac{Q(r)}{r} dQ \qquad Q(r) = Ze(r/R)^3 \quad dQ = 3Zer^2/R^3 dr$$

to change the integral to *dr* ; *R*=outer radius of nucleus

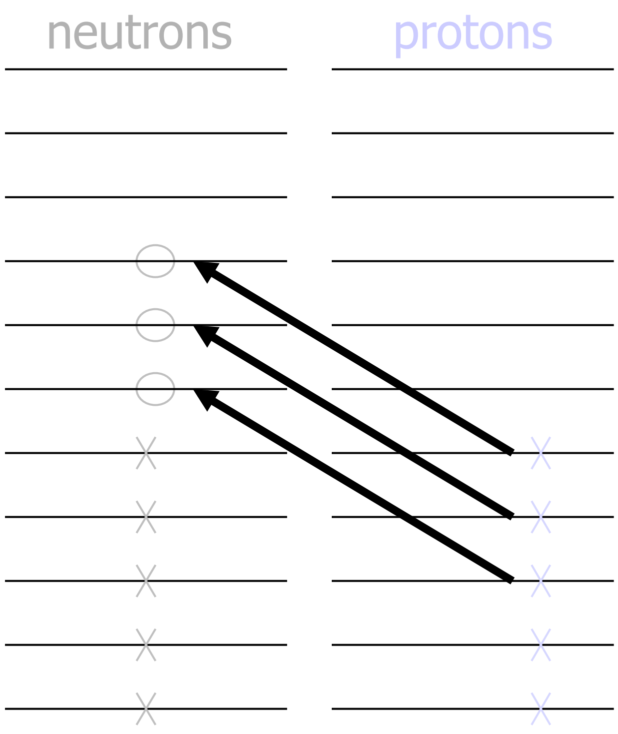
$$E_{Coulomb} = \int_0^{R} \frac{3(Ze)^2}{r} \frac{r^5}{R^6} dr = (3/5)\frac{(Ze)^2}{R}$$

… and remember $R=R_0A^{1/3}$

$$B_{Coulomb}(Z,A) = -c\,\frac{Z^2}{A^{1/3}}$$

Asymmetry Term

- Neutrons and protons are spin ½ fermions ➔ obey Pauli exclusion principle.
- If all other factors were equal nuclear ground state would have equal numbers of n & p.



Illustration

- n and p states with same spacing Δ.
- Crosses represent initially occupied states in ground state.
- If three protons were turned into neutrons
- the extra energy required would be $3\times3\ \Delta$

  but there would now be 6 more neutrons than protons.
- In general if there are Z-N excess neutrons over protrons the extra energy is ~ $((Z-N)/2)^2\ \Delta$. relative to Z=N.

Taking the Fermi energy, $(p_F^2 / 2m)$, of two separate gases (n and p) and perturbing (N-Z) one finds that the change in total Fermi energy for the two gases relative to Z= N, is proportional to $\frac{(N-Z)^2}{A}$ so long as (N-Z) << A

So far we have

$$BE = a\,A - b\,A^{2/3} - c\frac{(N-Z)^2}{A} - d\frac{Z^2}{A^{1/3}}$$

Spin pairing in the liquid drop model:

Spin pairing favours pairs of fermionic nucleons (similar to electrons in atoms)
i.e. a pair with opposite spin have lower energy than pair with same spin

Best case: even numbers of both protons and neutrons
Worst case: odd numbers of both protons and neutrons
Intermediate cases: odd number of protons, even number of neutrons or vice versa

→ Subtract small energy d required to decouple nucleons from binding energy:

$$\begin{aligned} \delta &= +a_p A^{-1/2} && \text{for both N \& Z odd} \\ &= 0 && \text{for N even, Z odd / Z even, N odd} \\ &= -a_p A^{-1/2} && \text{for both N \& Z even} \end{aligned}$$

→ a_p collects constants, $A^{-1/2}$ dependence provides best empirical fit to data

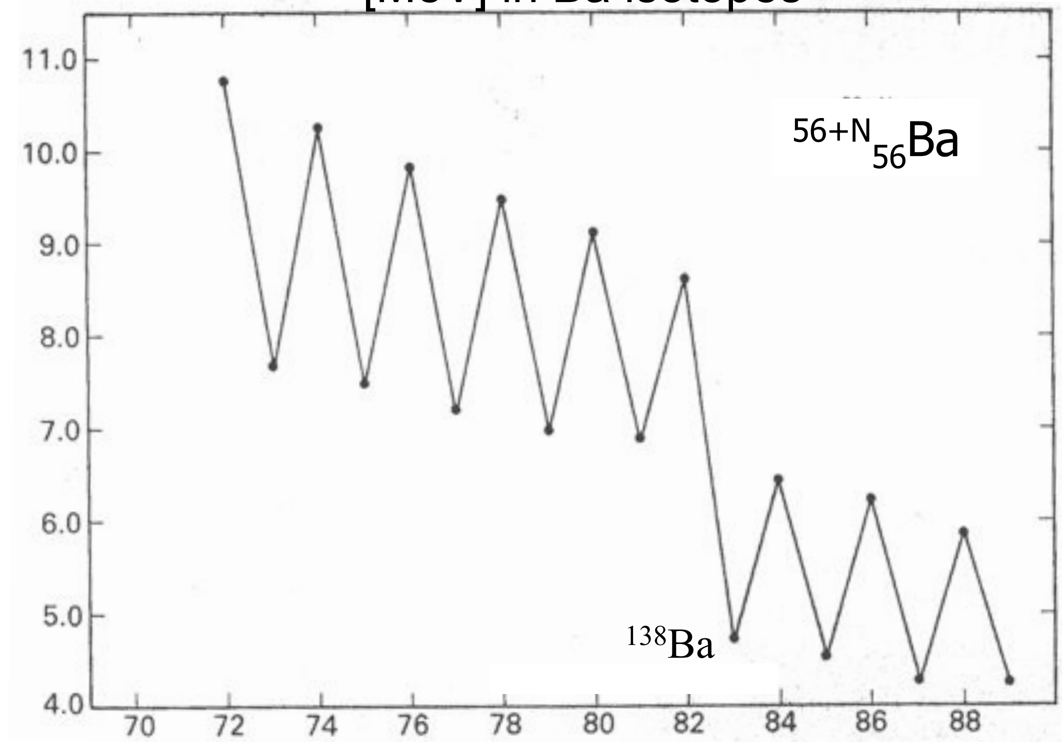
→ subtracting δ **reduces** BE for N and Z both odd

→ subtracting δ **adds** small amount to BE for N and Z both even

Pairing Term

- Nuclei with even number of n or even number of p more tightly bound then with odd numbers.
- Only 4 stable o-o nuclei (^{2}H, ^{6}Li, ^{10}B, ^{14}N) but 168 stable e-e nuclei.
- These differences in binding energy end up being reflected in cross sections and eventually, abundances, e.g., in the s-process.
  Elements with odd Z are less abundant

Neutron separation energy [MeV] in Ba isotopes



Neutron number

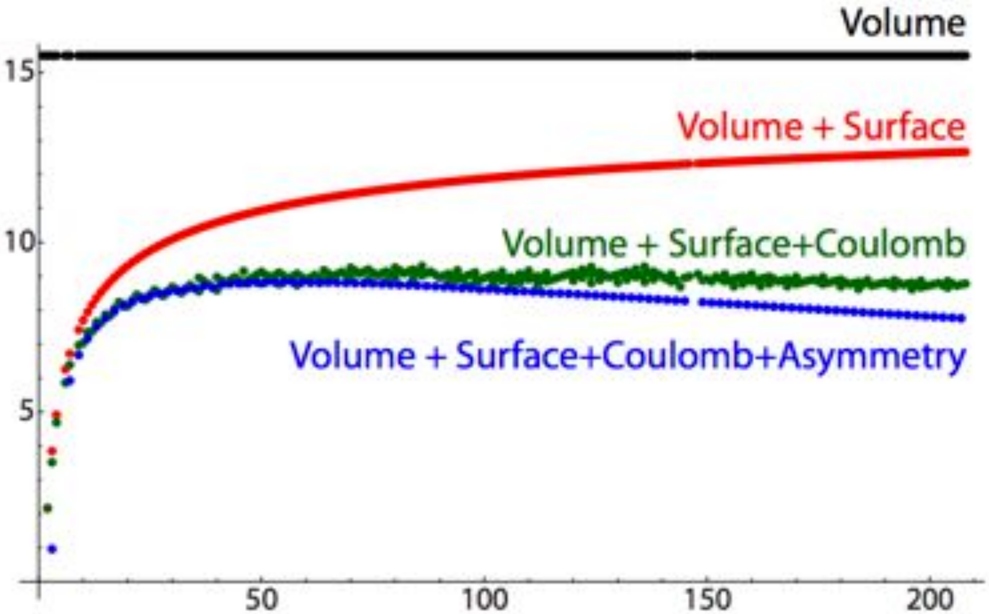
Similar to atoms, we shall find that nuclei have closed shells that are particularly sable, the nuclear equivalent of the noble gases. One such stable nucleus is ^{4}He; another is ^{16}O. Removing a nucleon from one of these "magic nuclei" with a closed shell takes more energy than a "valence" nucleon. E.g., it is easier to remove a neutron from ^{17}O than from ^{16}O (even accounting for the odd-even effect).

The pairing and shell corrections are purely empirical quantum mechanical corrections (for now) to the liquid drop model

Putting it all together:

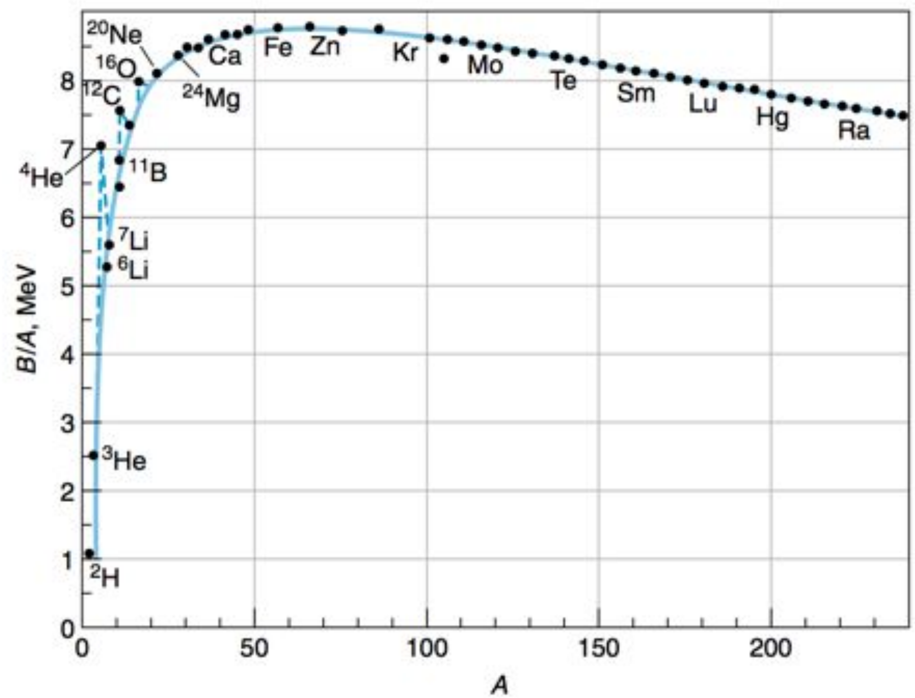
$$BE \approx a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - \delta(A) - S(A)$$

where terms 1 through 4 are the volume, surface, Coulomb, and symmetry energies respectively, $\delta(A)$ is the pairing correction, and $S(A)$ is the shell correction. Without the last two terms which are strictly quantum mechanical, this is known as the "liquid drop model" or the Bethe-Weizsäcker semi-empirical mass formula. Empirically, and crudely from fitting to known binding energies (deShalit and Feshbach, *Theoretical Nuclear Physics*, p. 126), we have $a_1 =$ 15.68 MeV, $a_2 =$ 18.56 MeV, $a_3 =$ 0.717 MeV, and $a_4 =$ 28.1 MeV. Also $\delta(A) =$ $+34/A^{3/4}$, 0, $-34/A^{3/4}$ MeV for odd-Z, odd-N; odd-A; and even-Z, even-N nuclei respectively. S(A) is complicated.



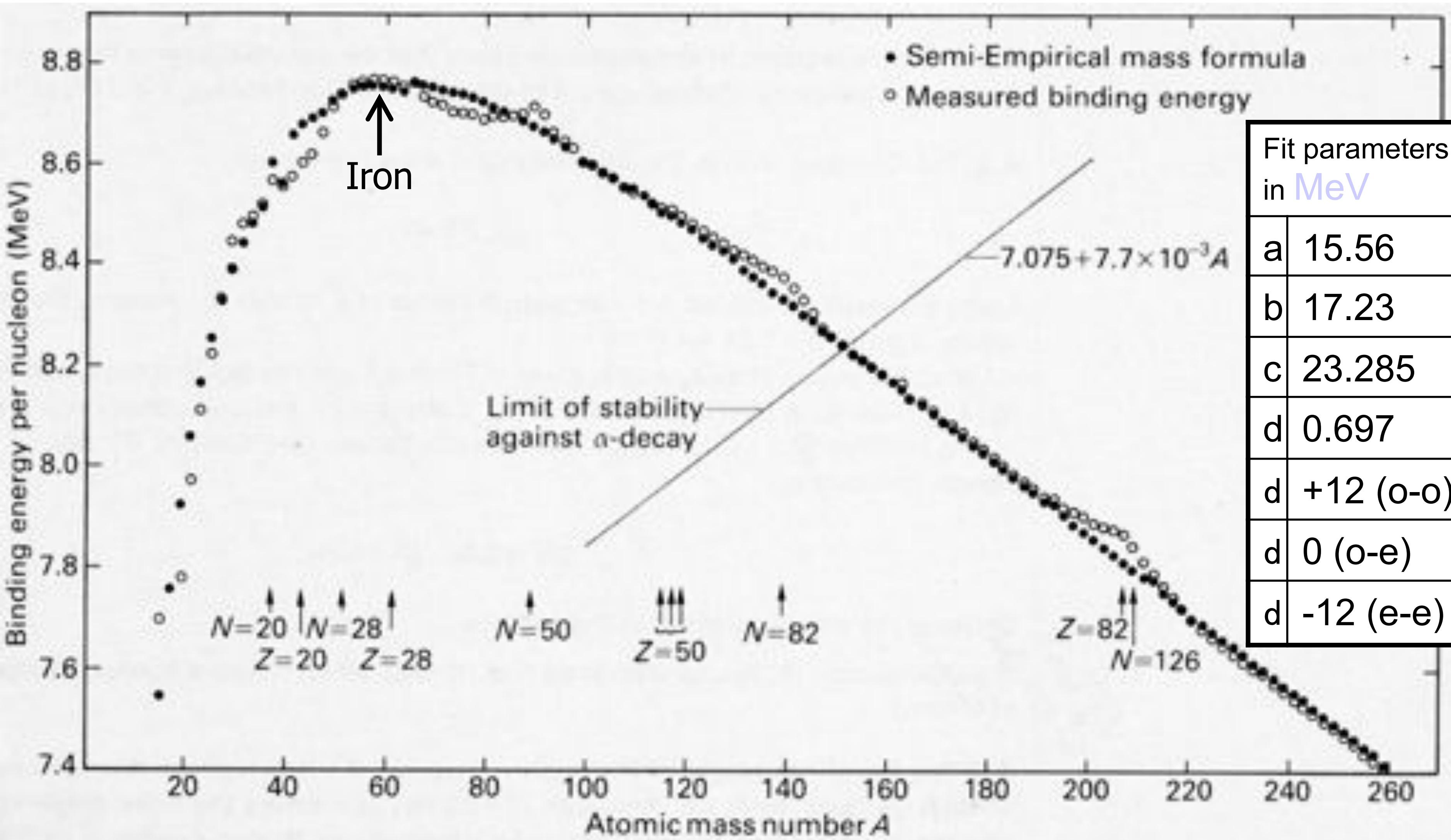
Plot of BE/A. At higher masses, an increasing penalty is paid to symmetry energy to save on Coulomb energy

Below iron energy can be released by fusion, above iron by fission.



Semi Empirical Mass Formula

Binding Energy vs. A for beta-stable odd-A nuclei



Fit parameters in MeV	
a	15.56
b	17.23
c	23.285
d	0.697
d	+12 (o-o)
d	0 (o-e)
d	-12 (e-e)

Utility

- Only makes sense for A greater than about 20
- Good fit for large A (<1% in most instances)
- Deviations are interesting - shell effects
- Explains the gross properties of the observed nuclei

Abundant light nuclei (up to Ca (Z = 20) have
Z = N (e.g., ^{4}He, ^{12}C, ^{16}O, ^{28}Si, etc.

Above Ca N > Z

Most tightly bound nuclei are in the iron group

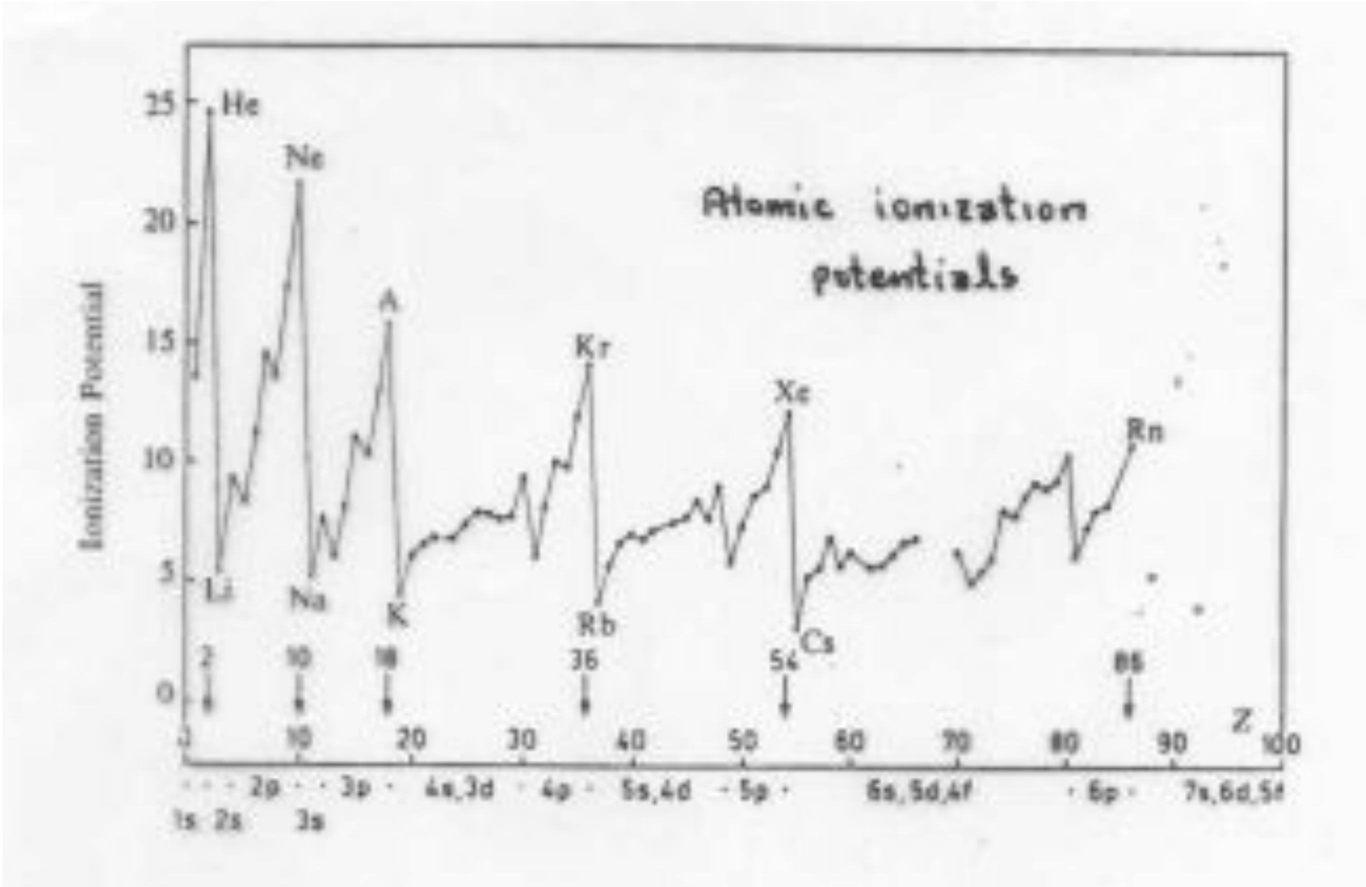
Very heavy nuclei are unstable to fission

What is the most tightly bound nucleus?

because of shell effects ($Z = 28$ is magic).

Nucleus	BE/A	Y_e
^{56}Ni	8.643	0.500
^{56}Fe	8.790	0.464
^{62}Ni	8.794	0.452

Nuclear Stability, The Shell Model



What are the equivalent closed shells for nuclei and how many neutrons or protons fit in each shell?

Empirically the closed shells are known.

Illiadis

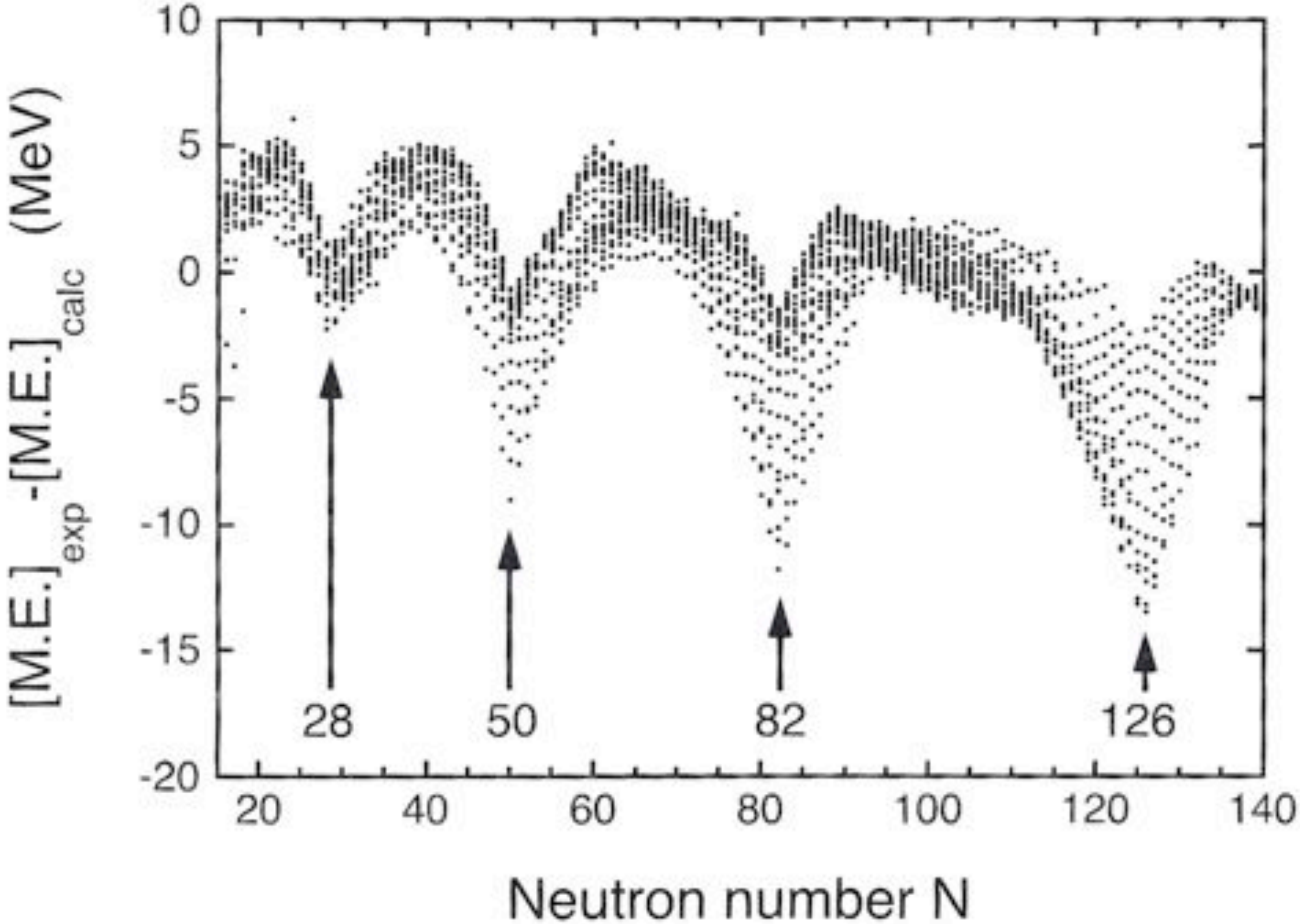
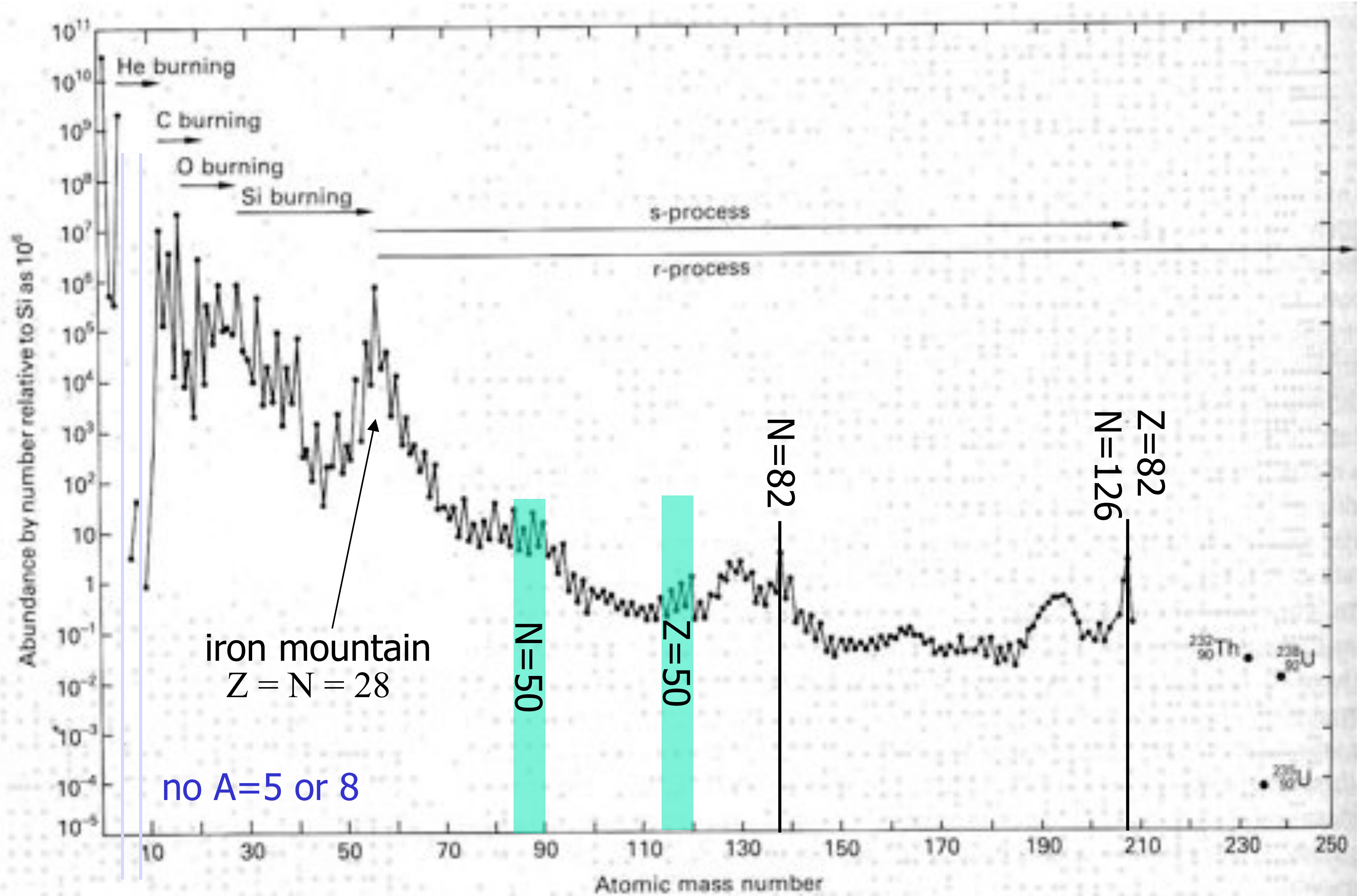


Fig. 1.11 Difference between experimental ground-state atomic mass excess (Audi et al. 2003) and the mass excess predicted by the spherical macroscopic part of the finite-range droplet (FRDM) mass formula (Möller et al. 1995) versus neutron number.

Abundance patterns reflect magic numbers



Ingredients in the Shell Model
– Mayer and Jensen 1963 Nobel Prize

Clayton 311 – 319

- Nuclear states are characterized by a
  wave function of given spin, angular momentum and parity

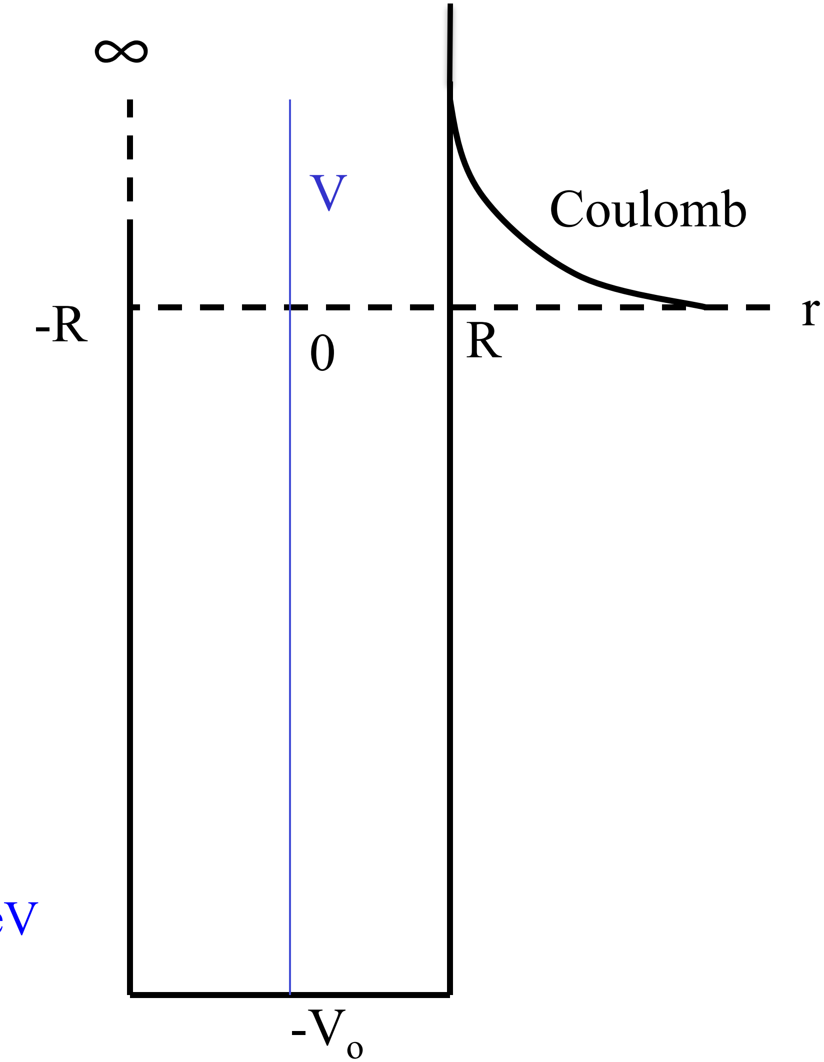
- Unlike the atom we do not solve for a central potential
  but instead for a square well (or variations thereon)

- spin-spin and spin-orbit coupling are more important for
  nuclei than for atoms

- There are two kinds of particles, n and p, not just e

- More than one particle can be excited at the same time.
  In atoms such states would be autoionizing.

A highly idealized nuclear potential looks something like this "infinite square well".



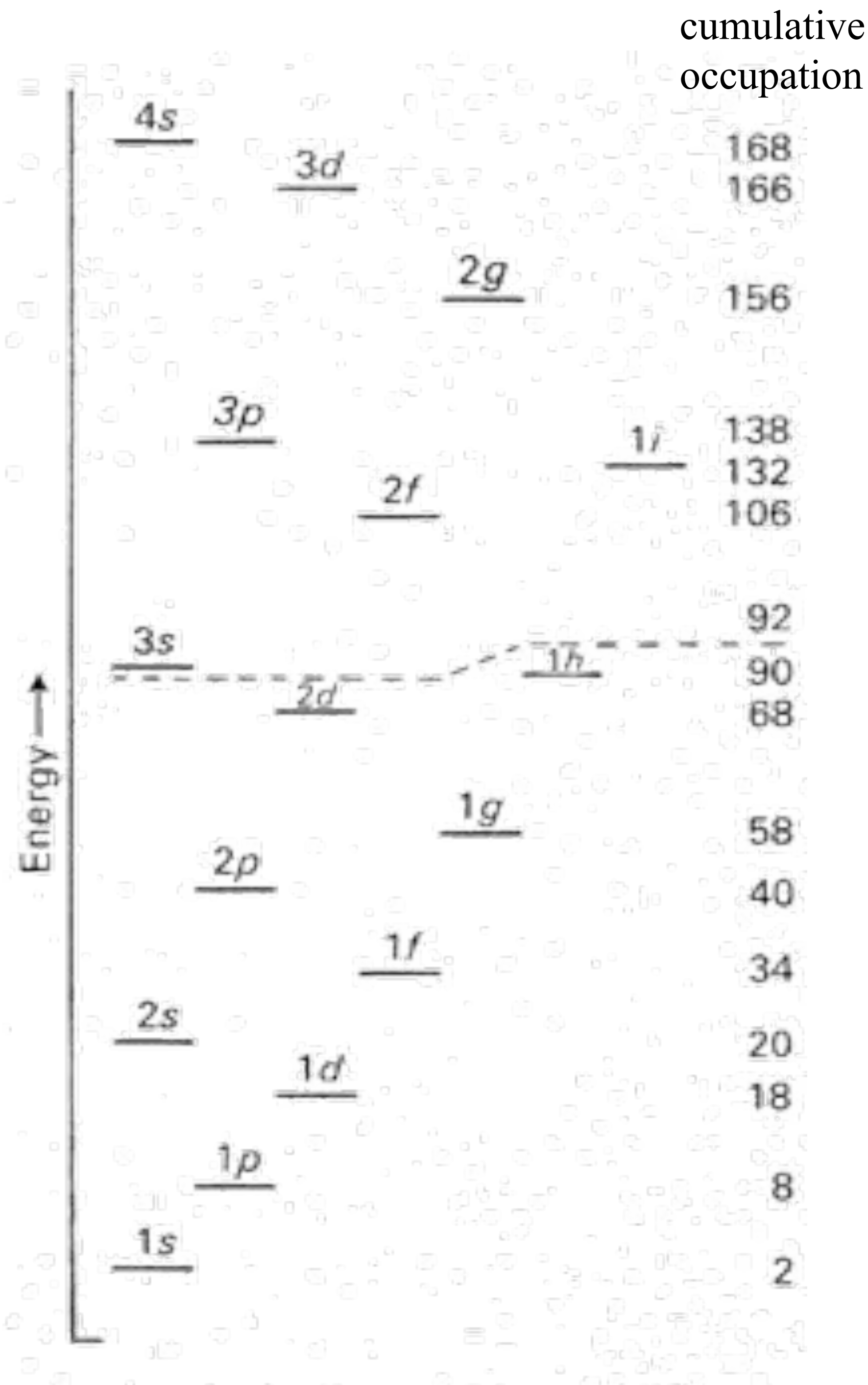
As is common in such problems one applies boundry conditions to Schroedinger's equation.

$$V = -V_o \qquad r < R$$
$$\quad = \infty \qquad r \geq R$$

$$\Psi(R) = 0$$
$$\Psi'(R) = 0 \qquad V_o \approx 50 - 60 \text{ MeV}$$

(In the case you have probably seen before of electronic energy levels in an atom, one would follow the same procedure, but the potential would be the usual [attractive] 1/r potential.)

Infinite Square Well Solutions



desired magic numbers

126

82

50

28

20

8

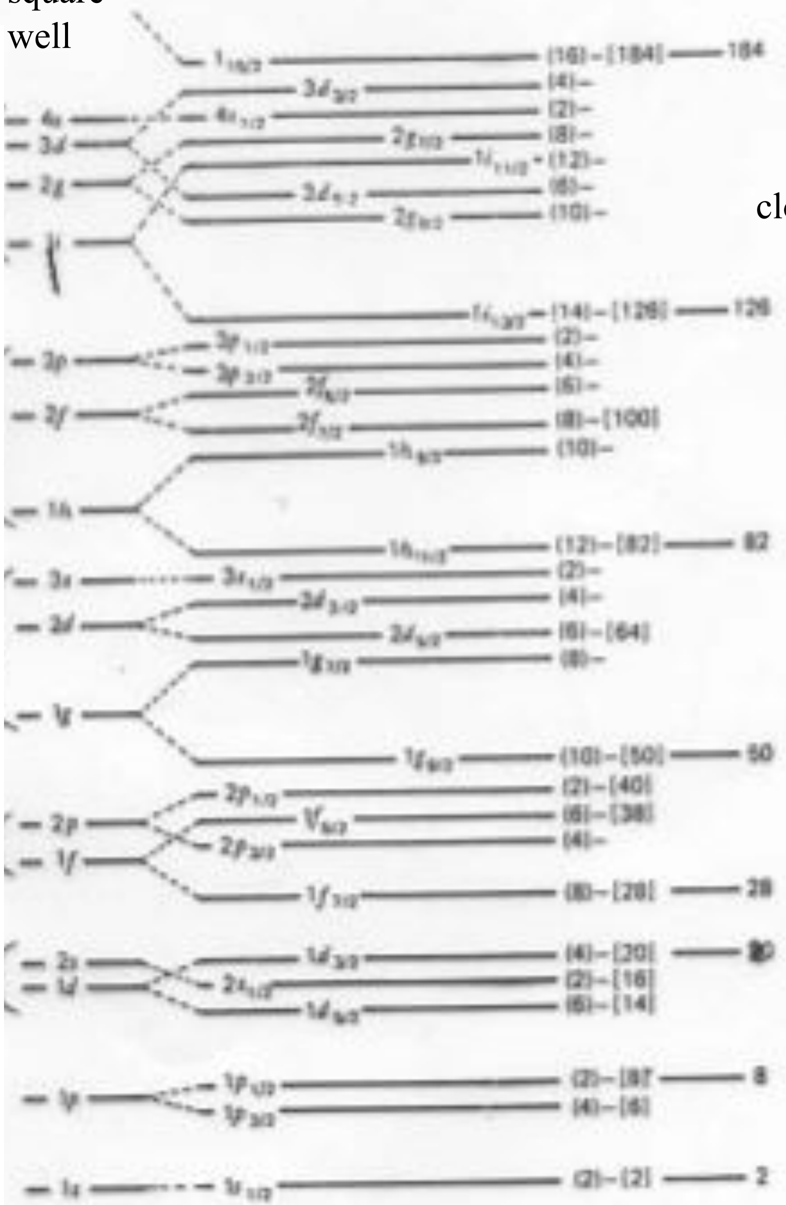
2

dotted line is to distinguish 3s, 2d, and 1h.

infinite square well

fine structure splitting

closed shells



Protons:

For neutrons
see Clayton p. 315
The closed shells are the same but the ordering of states differs from $1g_{7/2}$ on up. For neutrons $2d_{5/2}$ is more tightly bound. The $3s_{1/2}$ and $2d_{3/2}$ are also reversed.

For neutrons the level scheme is the same as for protons up to N = 50. Above that the Coulomb repulsion of the protons has an effect and favors orbits (for protons) with higher angular momentum. Thus for example the 51^{st} neutron is in the d level of j = 5/2 while for protons it is in the g level of j = 7/2. The effect is never enough to change the overall shell closures and magic numbers.

Maria Goeppert Mayer – Nobel talk - 1963

The correct energy level ordering then becomes :

Neutrons: $1s_{1/2}^{2}$ $1p_{3/2}^{4}$ $1p_{1/2}^{2}$ $1d_{5/2}^{6}$ $2s_{1/2}^{2}$ $1d_{3/2}^{4}$

$1f_{7/2}^{8}$ $2p_{3/2}^{4}$ $1f_{5/2}^{6}$ $2p_{1/2}^{2}$ $1g_{9/2}^{10}$ etc

Protons same through $1g_{9/2}$ but differs at next level $2d_{5/2}$ for n

$1g_{7/2}$ for p

Each state can hold (2j+1) nucleons.

The numbers where each of these shells close are

2, (6), 8, (14, 16), 20, 28, (32 38, 40), 50

where the calculated shell gaps are relatively small for the numbers in parenthesis

Remember | 2, 8, 20, 28, 50, 82, 126 |

Examples: ^{4}He, ^{16}O, ^{40}Ca, ^{56}Ni, ^{90}Zr

Nuclear Stability

A necessary condition for nuclear stability is that, for a collection of "A" nucleons, there exists no more tightly bound aggregate.

- E. g., a single ^{8}Be nucleus has less binding energy than two ^{4}He nuclei, hence ^{8}Be quickly splits into two heliums.

- An equivalent statement is that the nucleus ^{A}Z is stable if there is no collection of A nucleons that weighs less.

- However, one must take care in applying this criterion, because while unstable, some nuclei live a very long time. An operational definition of "unstable" is that the isotope has a measurable abundance and no decay has ever been observed (ultimately all nuclei heavier than the iron group are unstable, but it takes almost forever for most of them to decay). Exception – nuclei heavier than lead, e.g., U, Pu, etc.

In addition to just splitting up into lighter nuclei, which involves only the strong and electromagnetic forces, a nucleus may be unstable by the weak interaction.

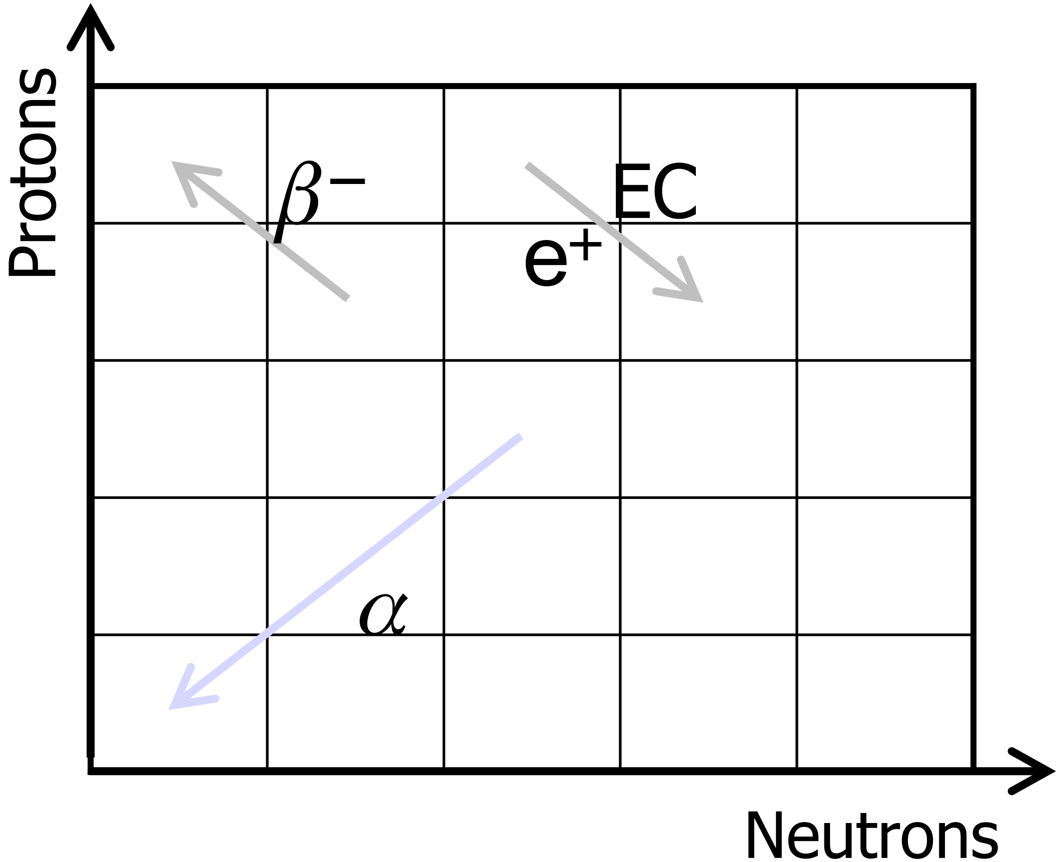
Weak decays:

beta − decay	$n \rightarrow p + e^{-} + \bar{\nu}_e$	must add energy
e^{+} − decay	$p \rightarrow n + e^{+} + \nu_e$	
e^{-} − capture	$p + e^{-} \rightarrow n + \nu_e$	

While the free neutron can decay, the other two reactions happen only inside of nuclei where the binding energy change between the parent nucleus and products can more than make up for the n-p mass difference.

or, in the last case where there exists a supply of energetic electrons.

Classification of Decays



α-decay:
- emission of Helium nucleus
- Z → Z-2
- N → N-2
- A → A-4

e^--decay (or β-decay)
- emission of e^- and $\bar{\nu}$
- Z → Z+1
- N → N-1
- A = const

e^+-decay
- emission of e^+ and ν
- Z → Z-1
- N → N+1
- A = const

Electron Capture (EC)
- absorbtion of e^- and emiss $\bar{\nu}$
- Z → Z-1
- N → N+1
- A = const

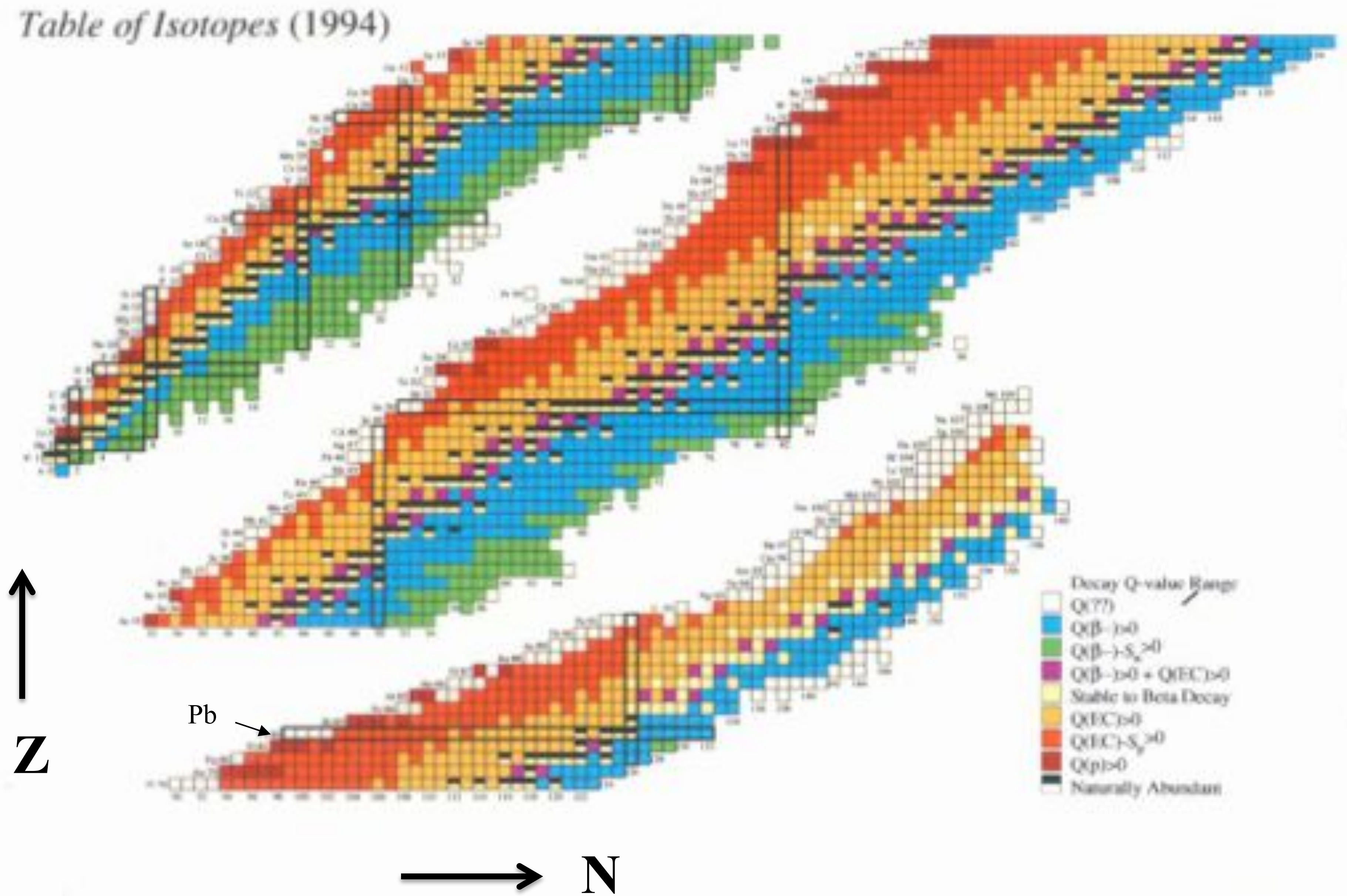


Table of Isotopes (1994)
Decay Q-value Range
Q(??)
Q(β-)>0
Q(β-)-S$_n$>0
Q(β-)>0 + Q(EC)>0
Stable to Beta Decay
Q(EC)>0
Q(EC)-S$_p$>0
Q(p)>0
Naturally Abundant
Pb
Z
N

In terms of binding energy

$$Q_{\beta} = BE(^{A}Z+1) - BE(^{A}Z) + 0.782\,\text{MeV}$$

$$Q_{e^{+}} = BE(^{A}Z-1) - BE(^{A}Z) - 1.804\ \text{MeV}$$

$$Q_{ec} = BE(^{A}Z-1) - BE(^{A}Z) - 0.782\ \text{MeV}$$

Tables of binding energies (given as BE/A) and reaction Q-values are given at

http://www.nndc.bnl.gov/masses/mass.mas03

Decay properties and lists of isotopes can be found at

http://www.nndc.bnl.gov/wallet/wc8.html