Lecture 10

Nucleosynthesis During Helium Burning and the s-Process

A. <u>Thermodynamic Conditions</u> (Massive Stars)

For the most part covered previously $\tau_{\rm He} \sim 10^6$ years (+- factor of three)

n=3

$$M_{\alpha} = 18 \frac{\sqrt{1-\beta}}{\mu^2 \beta^4}$$
 $\mu = \frac{4}{3}$ (for pure helium) solve for β
 $T_{c} = 4.6 \times 10^{6} \mu \beta \left(\frac{M_{\alpha}}{M_{\odot}}\right)^{2/3} \rho_{c}^{-1/3}$ K

E.g., $M_{\alpha} = 6$ (a 20 M_{\odot} main sequence star) $\beta = 0.83$ $T_{c} = 1.7 \times 10^{8} \left(\frac{\rho_{c}}{1000 \,\mathrm{g \, cm^{-3}}}\right)^{1/3} \mathrm{K}$

need temperatures > 10^8 to provide significant energy generation by 3α

So, typical temperatures are 2×10^8 K (higher in shell burning later) when densities are over 1000 gm cm⁻³. As the core evolves the temperature and density go up significantly. Note non-degenerate.

From Schaller et al (1992) Z = 0.02 and central helium mass fraction of 50%.

Main sequene Mass	ce Current Mass	Approxin M_{α}	nate $T_c/10^8$	$p_{c}/1000 \text{ g cm}^{-3}$
12	11.66	3	1.76	1.42
15	14.24	4	1.83	1.12
20	18.10	6	1.92	0.831
25	20.40	8	1.99	0.674
40	20.7	12	2.11	0.470

At 1.9 x 10⁸ K, the temperature sensitivity of the 3α rate is approximately T^{20} .

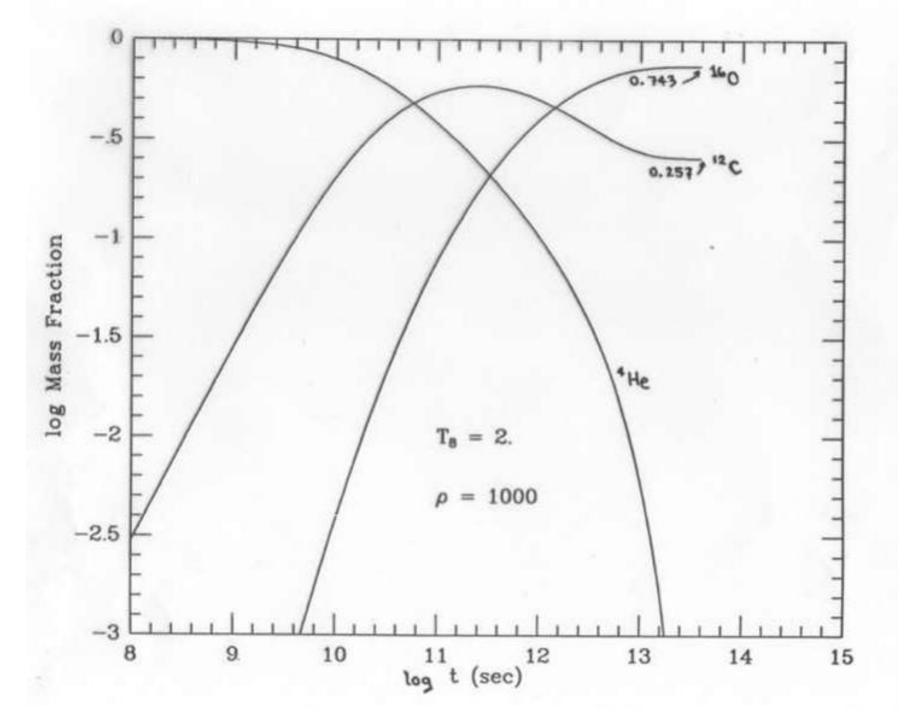
As discussed earlier during helium burning:

$$\frac{dY_{\alpha}}{dt} = -3\rho^2 Y_{\alpha}^3 \lambda_{3\alpha} - Y(^{12}C)Y_{\alpha}\rho\lambda_{\alpha\gamma}(^{12}C)$$
$$\frac{dY(^{12}C)}{dt} = \rho^2 Y_{\alpha}^3 \lambda_{3\alpha} - Y(^{12}C)Y_{\alpha}\rho\lambda_{\alpha\gamma}(^{12}C)$$
$$\frac{dY(^{16}O)}{dt} = Y(^{12}C)Y_{\alpha}\rho\lambda_{\alpha\gamma}(^{12}C)$$

Coulomb barrier and lack of favorable resonances inhibit alpha capture on ¹⁶O.

Several general features:

- ¹²C production favored by large density; oxygen by lower density
- ¹²C produced early on, ¹⁶O later
- last few alpha particles burned most critical in setting ratio ¹²C/¹⁶O
- Energy generation larger for smaller ${}^{12}C/{}^{16}O$.



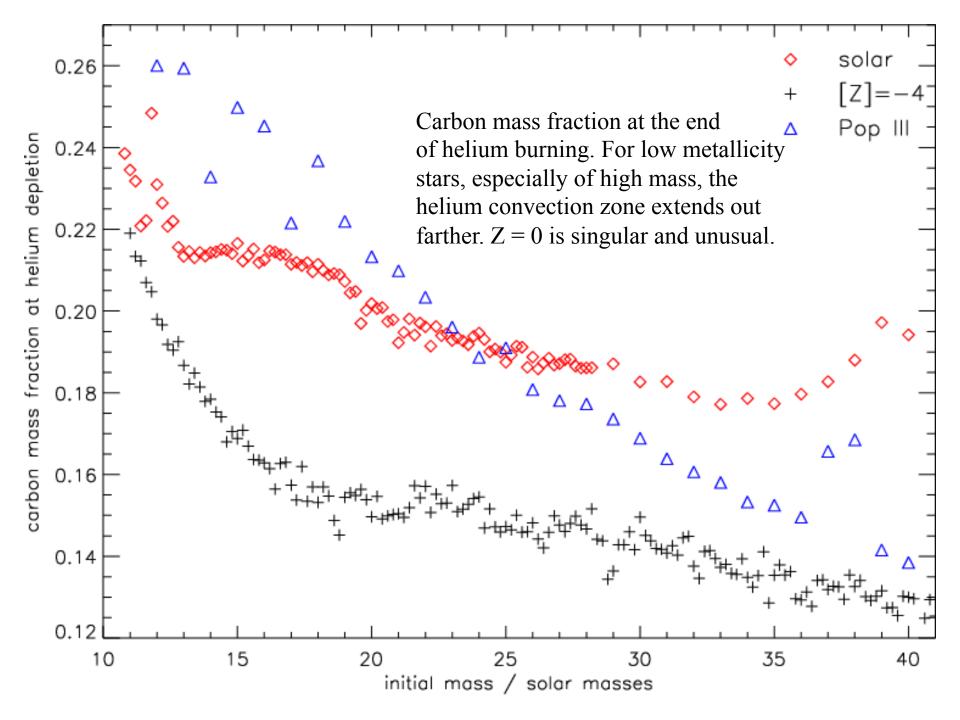
B1. Principal Nucleosynthesis

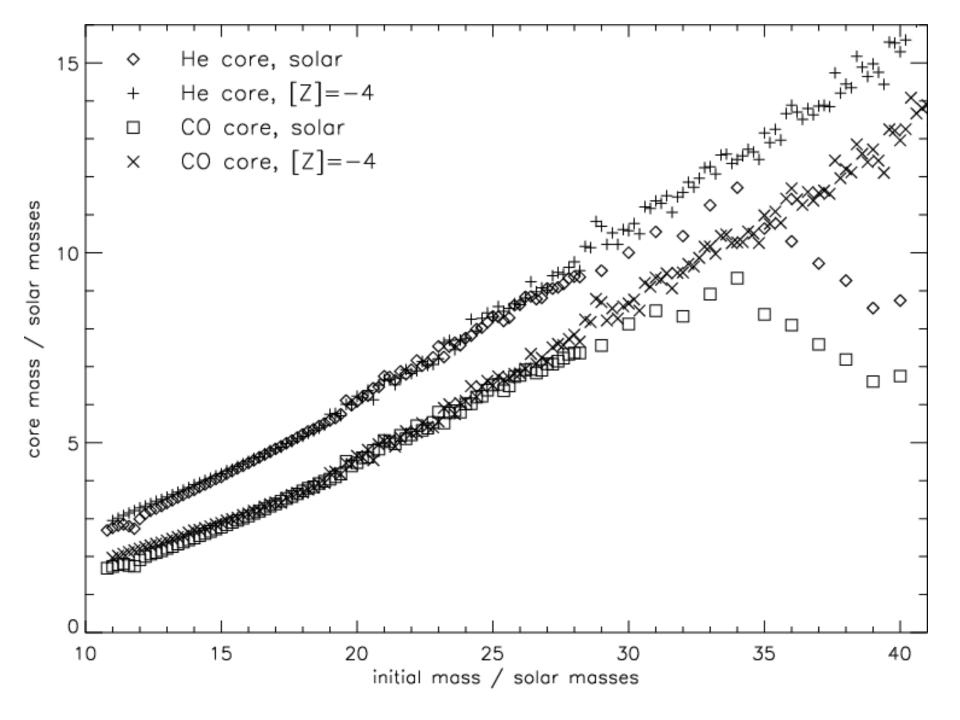
In massive stars after helium burning in the stars center (calculations included semiconvection).

	M/M_{\odot}	$\underline{X}(^{12}\underline{C})$	$\underline{X(^{16}O)}$	$\underline{X(^{20}Ne)}$
In the sun,	12	0.247	0.727	1.91(-3)
$^{12}C/^{16}O = 0.32$	15	0.188	0.785	3.02(-3)
	19	0.153	0.815	6.44(-3)
	25	0.149	0.813	1.16(-2)
	35	0.147	0.807	1.84(-2)

Buchman ${}^{12}C(\alpha,\gamma){}^{16}O$ multiplied by 1.2.

If the star contains appreciable metals there is, as we shall see also ²²Ne and ¹⁸O.⁴





Within uncertainties, helium burning in massive stars (over 8 solar masses) could be the origin in nature of ${}^{12}C$. It is definitely the origin of ${}^{16}O$

Complications:

- If the helium core grows just a little bit towards the end of helium burning, the extra helium convected in greatly decreases the ¹²C synthesis.
- Mass loss from very massive WR stars can greatly increase the synthesis of both ¹²C and ¹⁶O in stars over 35 solar masses
- The uncertain rate for ${}^{12}C(\alpha,\gamma){}^{16}O$
- ¹²C/¹⁶O ratio may be affected by post-helium burning evolution and by black hole formation above some critical main sequence mass. ¹⁶O is made in the more massive (massive) stars.

B. <u>Trace</u> Element <u>Nucleosynthesis</u> - <u>Charged</u> Particles During hydrogen burning CNO → ¹⁴N so that at helium ignition

$$\chi(^{14}N) = 14(Y_i(^{12}c) + Y_i(^{14}N) + Y_i(^{16}0))$$

= 0.013(Z/Z_0)

Early during helium burning $\begin{bmatrix}
14 \\ N(x, x) \\ F(e^{+}y) \\ 0
\end{bmatrix}$ NO. From this point onwards have a net neutron excess $\boxed{\eta = 1 - 2Ye} -1 \le \eta \le 1$

= $\sum (N_i - Z_i) Y_i$ $\eta = 0$ if Z = N

Before the above reaction the composition was almost entirely ⁴He and ¹⁴N, hence $\eta \approx 0$ (actually a small posive value exists because of ⁵⁶Fe and the like).

After this reaction

$$\eta = 0.0019 \, \frac{Z}{Z_{\odot}}$$

During helium core burning, ¹⁸O is later mostly destroyed by ¹⁸O(α, γ)²²Ne.

During helium shell burning which does not go to completion in massive stars, much of 180 remains undestroyed and this is the source of 180 in nature. Convection helps to preserve it.

	Lifetimes (yr)	$X_{d} = V_2$	q = 1000.
	T8 = 1.8	Ts = 2	
160 (x, Y)	3.5 (10)	2.3(9)	survives
180 (x, x)	2700	98	destroyed unless preserved by convection
14 N (x, x)	15	1.1	destroyed
22 Ne (a,n) + (a,r)		2.2(5)	partly destroyed

So one expects that, depending on mass, some but not all of the ²²Ne will burn towards the end of helium burning when the temperature goes up.

The following table gives the temperature at the center of the given model and the mass fractions of ²²Ne, ²⁵Mg, and ²⁶Mg each multiplied by 1000, when the helium mass fraction is 1% and zero

	Μ	T _C	²² Ne	²⁵ Mg	²⁶ Mg
7 1	12	2.42	13.4	0.51	0.61
/oosley, 1d Heger			12.3	1.17	1.05
(2007)	15	2.54	12.7	0.98	0.91
			11.1	1.99	1.80
	19	2.64	11.5	1.73	1.54
			9.4	2.90	2.87
	25	2.75	9.8	2.67	2.59
			6.96	4.05	4.54
	35	2.86	7.37	3.87	4.22
			4.41	5.18	6.39

The remainder of the ²²Ne will burn early during carbon burning, but then there will be more abundant "neutron poisons".

These numbers are quite sensitive to the uncertain reaction rate for $^{22}Ne(\alpha,n)^{25}Mg$ and may be lower limits to the ^{22}Ne consumption.

C. The s-Process in Massive Stars

Late during helium burning, when the temperature rises to about 3.0 x 10⁸ K, ²²Ne is burned chiefly by the reaction $^{22}Ne(\alpha,n)^{25}Mg$ (with some competition from $^{22}Ne(\alpha,\gamma)^{26}Mg$).

Where do the neutrons go?

Some go on ⁵⁶Fe but that fraction is only:

 $\sigma_{56} Y_{56}$

$$J = \frac{160}{\sum \sigma_i Y_i}$$
160 $Y_{16} \approx \frac{0.5}{16} = 3.1 \times 10^{-2}$ $Y_{16} \sigma_{16} \approx 1.2 \times 10^{-3}$ But, ¹⁷O(α ,n)²⁰Ne destroys the ¹⁷O and restores the neutron
22Ne $Y_{22} \approx \frac{0.005}{22} = 2.3 \times 10^{-4}$ $Y_{22} \sigma_{22} \approx 1.3 \times 10^{-5}$
25Mg $Y_{25} \approx \frac{0.005}{25} = 2 \times 10^{-4}$ $Y_{25} \sigma_{25} \approx 1.3 \times 10^{-3}$ For σ in mb
56Fe $Y_{56} \approx \frac{0.0013}{56} = 2.3 \times 10^{-5}$ $Y_{56} \sigma_{56} \approx 2.7 \times 10^{-4}$

30 keV neutron capture cross sections

(mostly from Bao et al, ADNDT, 2000)

Nucleus	<u></u>	Nucleus	<u> </u>	
	(mb)		(mb)	
12 0	0.0154	545	07.0	
^{12}C	0.0154	⁵⁴ Fe	27.6	
*16O	0.038	⁵⁶ Fe	11.7	
²⁰ Ne	0.119	⁵⁷ Fe	40.0	
²² Ne	0.059	⁵⁸ Fe	12.1	
²⁴ Mg	3.3	⁵⁸ Ni	41.0	
²⁵ Mg	6.4	⁶⁴ Zn	59	
²⁶ Mg	0.126	⁶⁵ Zn	162	
²⁸ Si	2.9	⁶⁶ Zn	35	
		⁸⁸ Sr	6.2	(closed shell)

The large cross section of ${}^{25}Mg$ is particularly significant since it is made by ${}^{22}Ne(\alpha,n)$ ${}^{25}Mg$.

* Igashira et al, ApJL, 441, L89, (1995); factor of 200 upwards revision

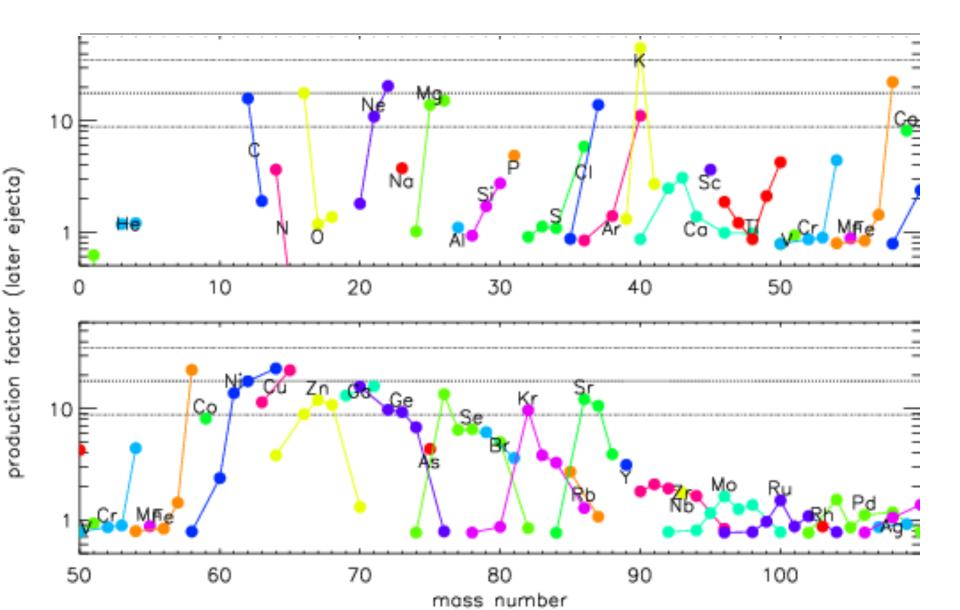
So a fraction $\frac{Y_{56}\sigma_{56}}{Y_{25}\sigma_{25}} \sim 10 - 20\%$ capture on iron. How many

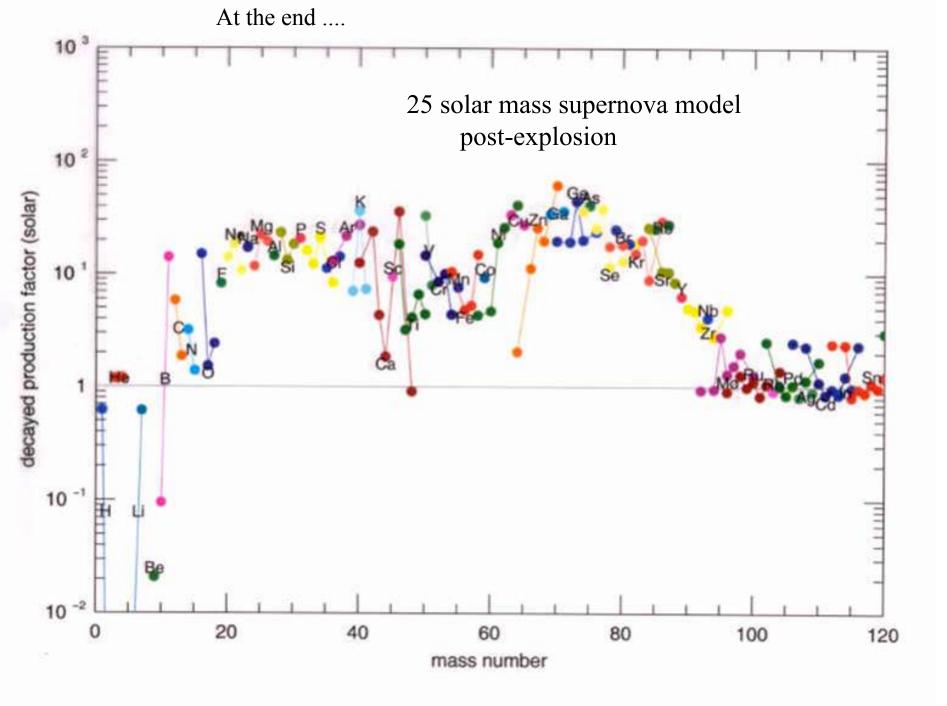
neutrons is this?

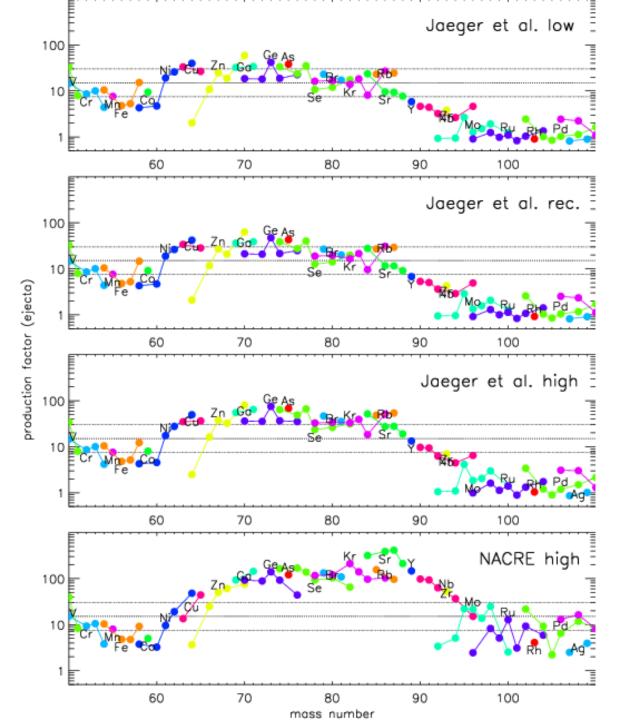
$$Y_n / Y_{Fe} \sim Y_{22} / Y_{56} = 42$$

where we have assumed a mass fraction of 0.02 for ²²Ne and 0.0013 for ⁵⁶Fe and that all ²²Ne burns by (α ,n).

This is about 4 - 8 *neutrons per iron* and obviously not nearly enough to change e.g., Fe into Pb, but the neutron capture cross sections of the isotopes generally increase above the iron group and the solar abundances decrease. A significant s-process occurs that produces significant quantities of the isotopes with A < 90. Composition of a 25 solar mass star at the end of helium burning compared with solar abundances (Rauscher et al 2001)

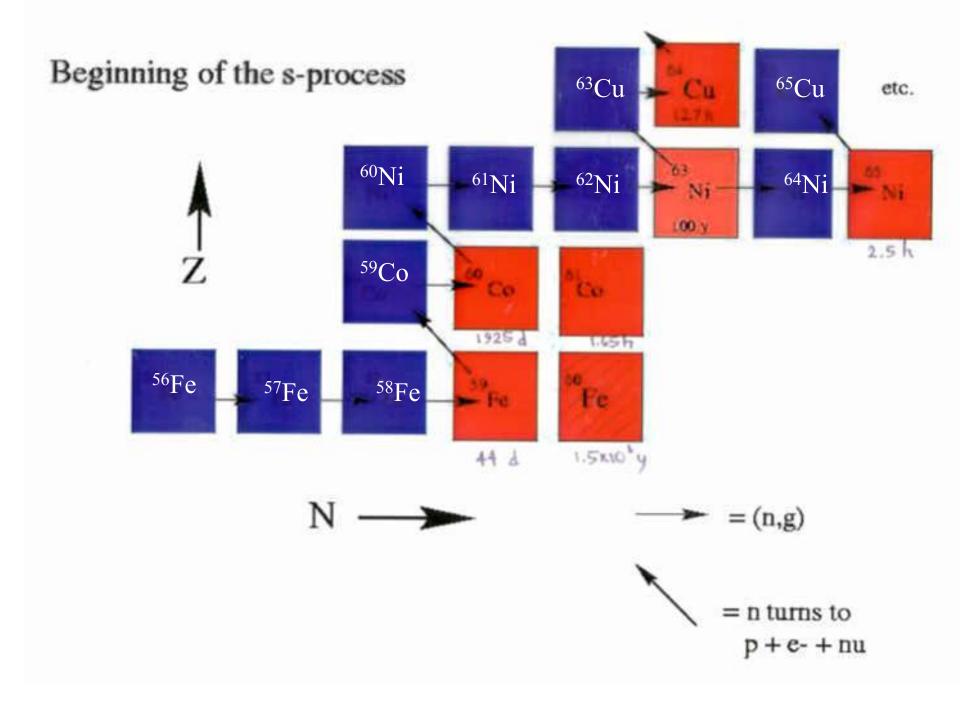






Results depend most sensitively upon the reaction rate for 22 Ne(α ,n) 25 Mg.

If ²²Ne does not burn until later (i.e., carbon burning there are much more abundant neutron poisons



Here "s" stands for "slow" neutron capture

$$\tau_{\beta} \ll \tau_{n\gamma} \qquad \tau_{\beta} = \frac{1}{\lambda_{\beta}} = \frac{\tau_{1/2}}{\ln 2} \qquad \frac{dY_{i}}{dt} = -Y_{i}Y_{n}\rho\lambda_{n\gamma}(i) + \dots$$
$$\tau_{n\gamma} = \left(\rho Y_{n}\lambda_{n\gamma}\right)^{-1} \qquad \tau_{n\gamma} = \left(\frac{1}{Y_{i}}\frac{dY_{i}}{dt}\right)^{-1}$$

This means that the neutron densities are relatively small

E.g. for a ²²Ne neutron source

$$\frac{dY_n}{dt} \approx 0 \approx \rho Y_{\alpha} Y(^{22}Ne)\lambda_{\alpha n}(^{22}Ne)$$

$$- \rho Y_n Y(^{25}Mg)\lambda_{n\gamma}(^{25}Mg)$$

 $\rho \approx 1000, X_{\alpha} \approx 0.5, X(^{22}Ne) \approx 0.005, X(^{25}Mg) \approx 0.005$

$$Y_{n} \approx \frac{Y_{\alpha} Y(^{22} Ne) \lambda_{\alpha n} (^{22} Ne)}{Y(^{25} Mg) \lambda_{n \gamma} (^{25} Mg)} \sim \frac{Y_{\alpha} \lambda_{\alpha n} (^{22} Ne)}{\lambda_{n \gamma} (^{25} Mg)}$$

$$Y_{\alpha} \sim 0.02/4; Y(^{22} Ne) \sim 2 Y(^{25} Mg); \rho = 1000$$

$$T_{8} \qquad \lambda_{\alpha n} (^{22} Ne) \qquad \lambda_{n \gamma} (^{25} Mg) \qquad n_{n} = \rho N_{A} Y_{n} \qquad \lambda_{n \gamma} (^{56} Fe)$$

$$2.0 \qquad 9.1(-17) \qquad 1.2(6) \qquad negligible$$

$$2.5 \qquad 1.5(-13) \qquad 1.1(6) \qquad \sim 10^{6} \qquad 1.9(6)$$

$$3.0 \qquad 2.6(-11) \qquad 1.0(6) \qquad \sim 10^{8} \qquad 1.9(6)$$

Most of the s-process takes place around $T_8 = 2.5 - 3$, so the neutron density is about $10^6 - 10^8$ cm⁻³ (depends on uncertain rate for (α ,n) on ²²Ne and on how much ²²Ne has burned).

At these neutron densities the time between capture, even for heavy elements with bigger cross sections than iron, is days. For ⁵⁶Fe itself it is a few years

Eg. at $T_8 = 2.5 (n_n \sim 10^6)$, the lifetime of ⁵⁶Fe is about

$$\tau({}^{56}Fe) = \left(\frac{1}{Y({}^{56}Fe)} \frac{dY({}^{56}Fe)}{dt}\right)^{-1} = \left(\rho Y_n \lambda_{n\gamma}({}^{56}Fe)\right)^{-1}$$

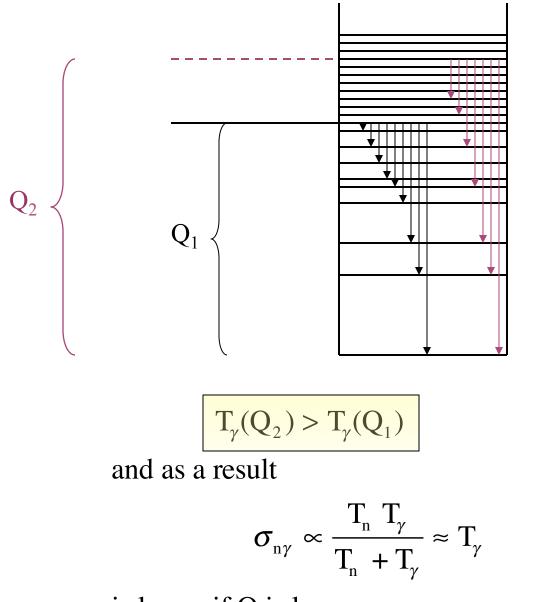
 $\approx 10^4$ year (less at higher temperatures)

The s-process in massive stars only goes on during a brief period at the end of helium burning. The time scale is lengthened by convection. Reaction Rates (n, y):

Either measured (Bao et al, ADNDT, 76, 70, 2000) or calculated using Hauser-Feshbach theory (Woosley et al., ADNDT, 22, 371, (1976) Holmes et al., ADNDT, 18, 305, (1976); Rauscher et al. ADNDT, 75, 1, (2000))

The calculations are usually good to a factor of two. For heavy nuclei within $kT \sim 30$ keV of Q_{ng} there are very many resonances.

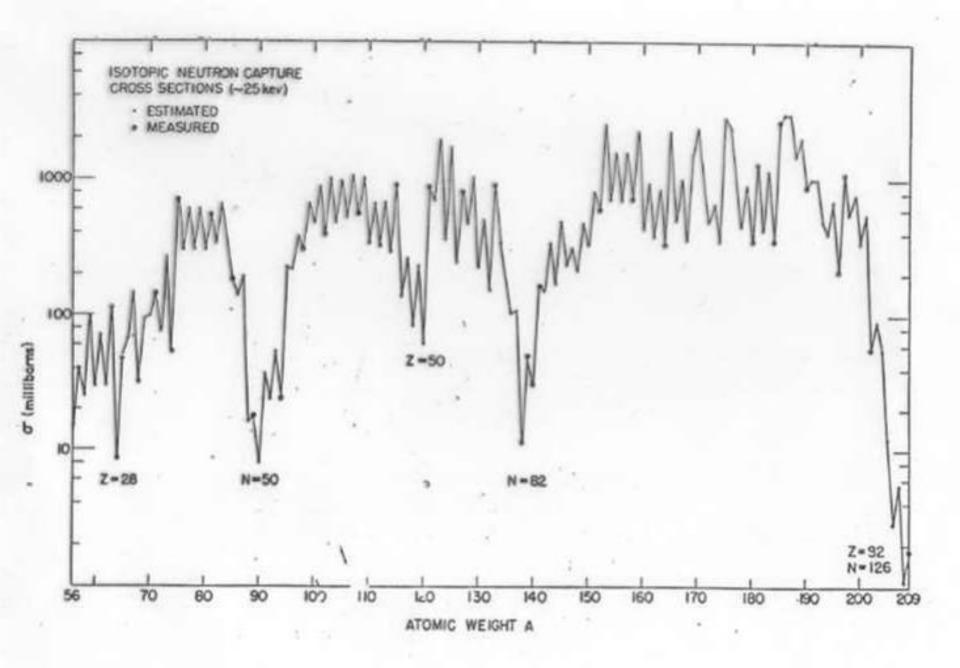
Occasionally, for light nuclei or near closed shells, direct capture is important: e.g., ¹²C, ^{20,22}Ne, ¹⁶O, ⁴⁸Ca



is larger if Q is larger

More levels to make transitions to at higher Q and also, more phase space for the outgoing photon.

 E_{γ}^{3} for electric dipole



Rate Equations: Their Solutions and Implications

Assume constant density, temperature, cross section, and neutron density and ignore branching (would never assume any of these in a modern calculation). Then

$$\frac{dY(^{A}Z)}{dt} \equiv \frac{dY_{A}}{dt} = -Y_{A}Y_{n}\rho\lambda_{n\gamma}(A) + Y_{A-1}Y_{n}\rho\lambda_{n\gamma}(A-1)$$

and since
$$n_n = \rho N_A Y_n$$
 and $\lambda_{n\gamma} = N_A \langle \sigma_{n\gamma} v \rangle \approx N_A \sigma_A v_{thermal}$
defining $\tau \equiv \int \rho N_A Y_n v_{thermal} dt = \int n_n v_{thermal} dt$, one has
 $\frac{dn_A}{d\tau} = -n_A \sigma_A + n_{A-1} \sigma_{A-1}$

Note that τ has units of inverse cross section (inverse area).

If there were locations where steady state is achieved then

$$\frac{dn_A}{d\tau} \approx 0 = n_A \sigma_A - n_{A-1} \sigma_{A-1}$$

i.e., σn is locally a constant, and $n \propto \frac{1}{\sigma}$

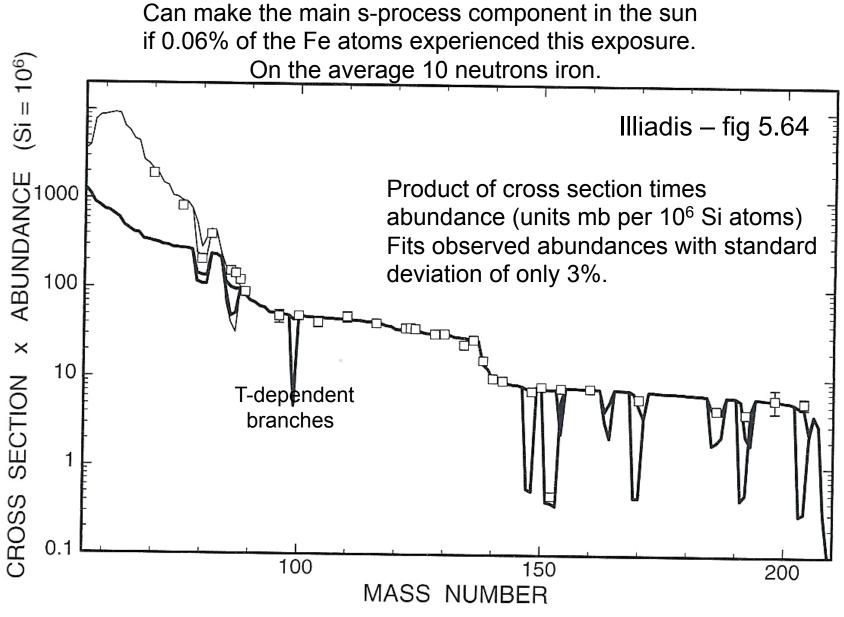
Attaining steady state requires a time scale longer than a few times the destruction lifetime of the species in the steady state group. One has "local" steady state because any flux that would produce, e.g., lead in steady state would totally destroy all the lighter s-process species.

The flow stagnates at various "waiting points" along the s-process path, particularly at the closed shell nuclei.

Eg., $n_n \sim 10^8 \text{ cm}^{-3} \Rightarrow \rho Y_n \sim n_n / N_A \sim 5 \times 10^{-17}$ $\lambda_{n\gamma}$ experimentally at helium burning temperatures is $10^5 - 10^8$ $\tau_{n\gamma} = \left(\rho Y_n \lambda_{n\gamma}\right)^{-1} = \left(\frac{d \ln Y_A}{dt}\right)^{-1} \sim 10 - 10^4 \text{ years}$

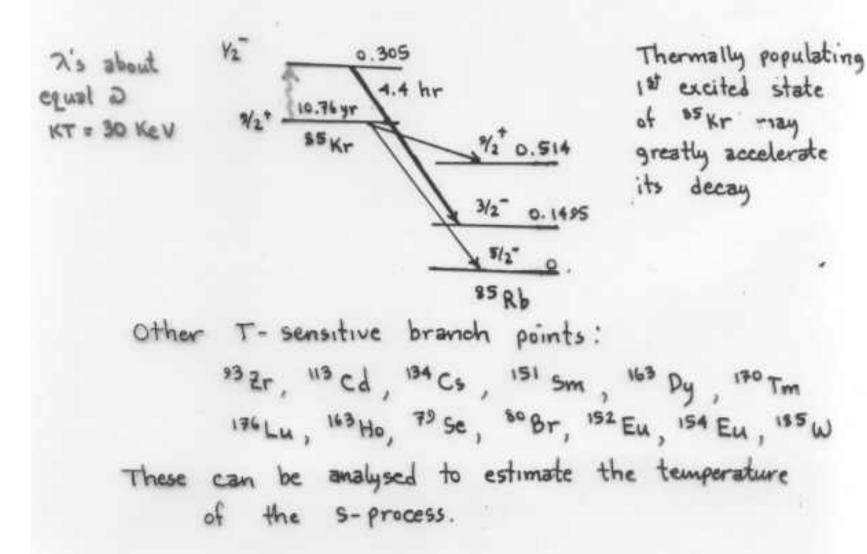
This can be greatly lengthened in a massive star by convection.

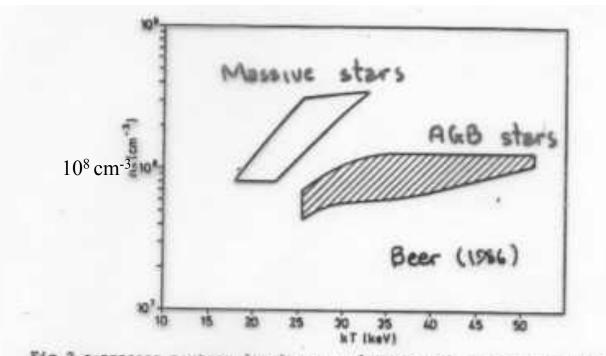
As a result nuclei with large cross sections will be in steady state while those with small ones are not. This is especially so in He shell flashes in AGB stars where the time scale for a flash may be only a few decades.

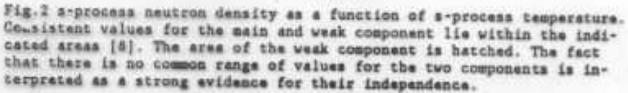


The symbols are s-only nuclei. The solid lines are the model results for a standard (exponential) set of exposure strengths. Below A = 90 there is evidence for a separate additional s-process component.

The beta decay rates too are <u>usually</u> insensitive to T. However, occasionally







Implicit solution:

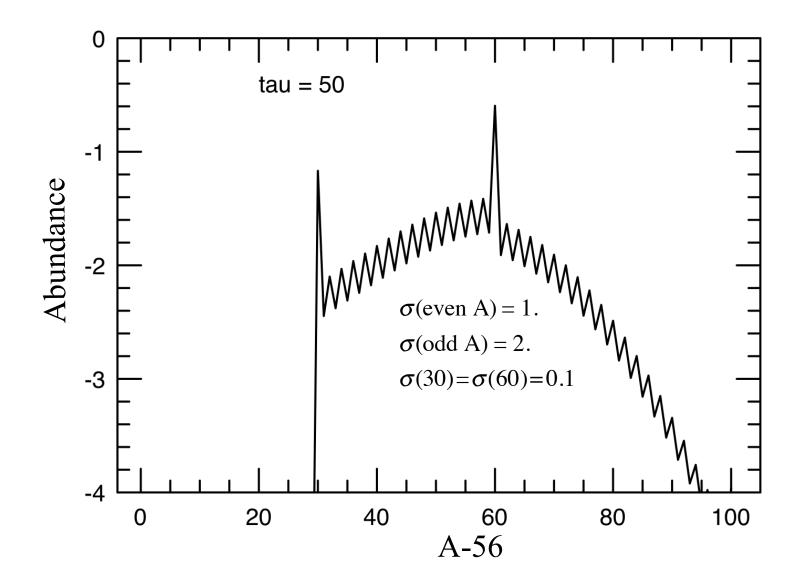
Assuming no flow downwards from A+1 and greater to A and below.

$$n_{new}(A) = \frac{n_{old}(A)/d\tau + n_{new}(A-1)\sigma_{n\gamma}(A-1)}{1/d\tau + \sigma_{n\gamma}(A)}$$

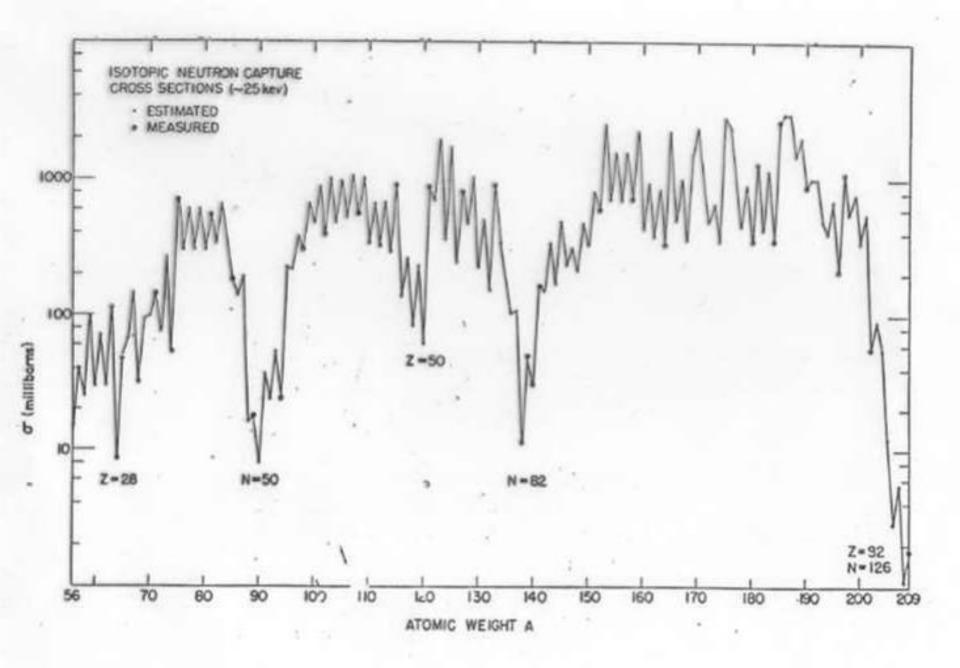
This works because in the do loop, $n_{new}(A-1)$ is updated to its new value before evaluating $n_{new}(A)$. Matrix inversion reduces to a recursion relation.

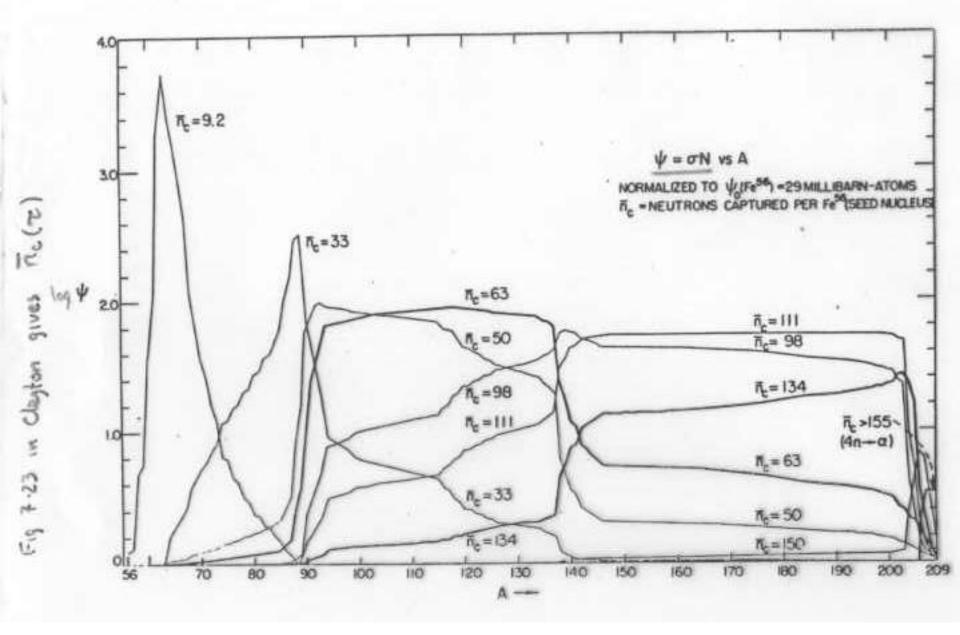
$$\frac{n_{new}(A) - n_{old}(A)}{d\tau} = -n_{new}(A)\sigma_A + n_{new}(A-1)\sigma_{A-1}$$

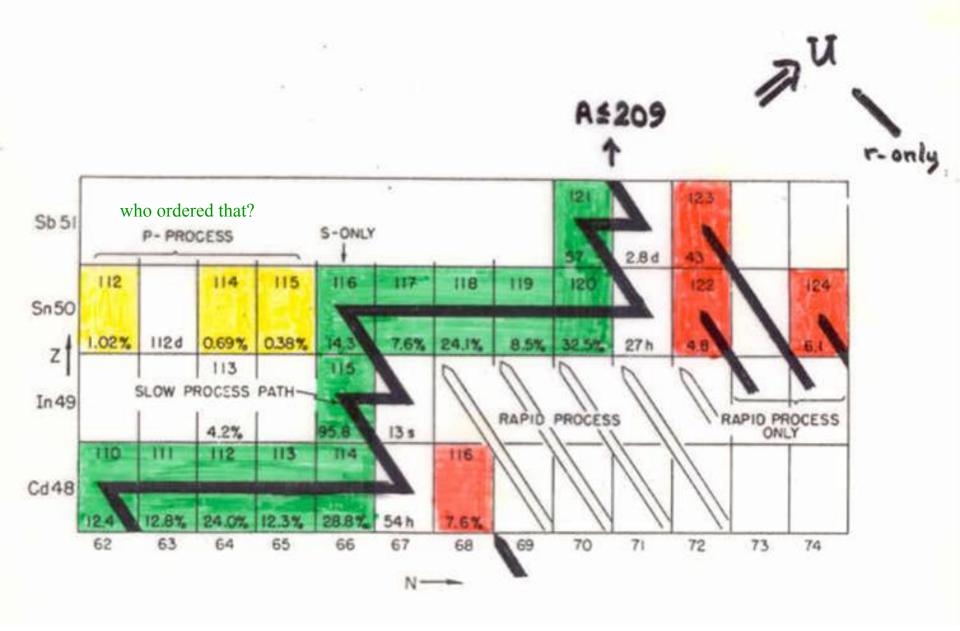
 $n_{new}(A-1)$ known from previous step in recursion relation



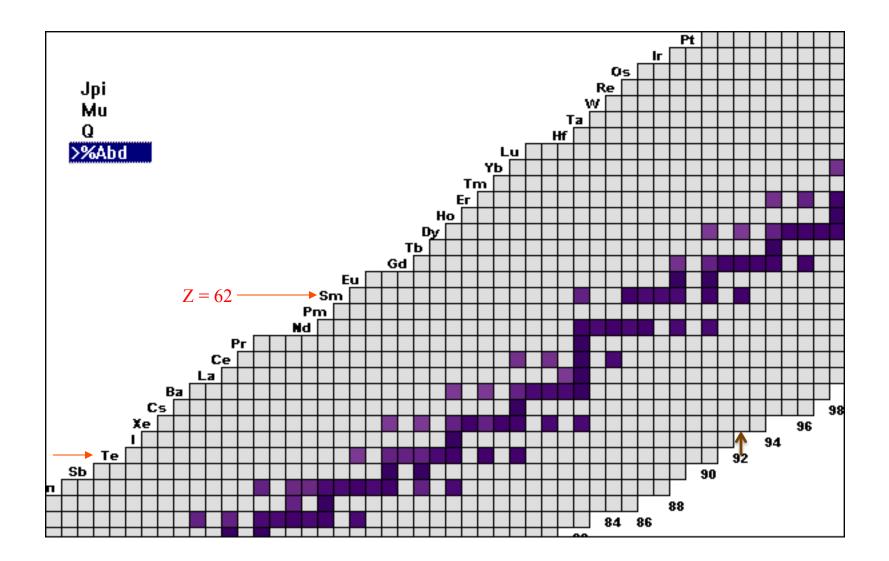
Sample output from toy model code micros2.f







e.g., 117 Sn, 118 Sn, 119 Sm, and 120 Sn are s,r isotopes. Sn is not a good place to look for s n = const though because it is a closed shell.



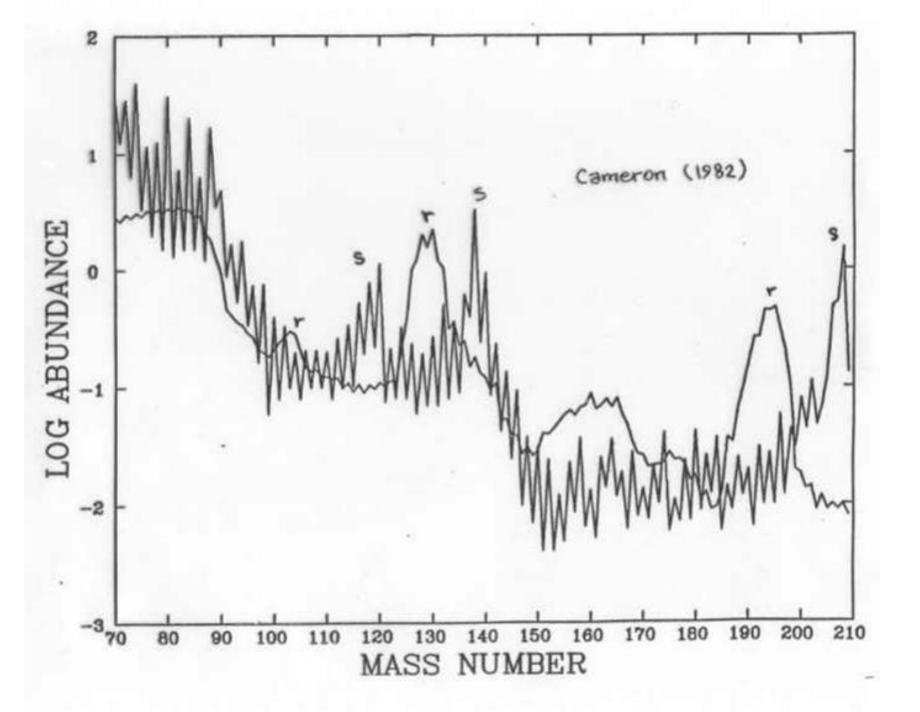
http://www.nndc.bnl.gov/chart/

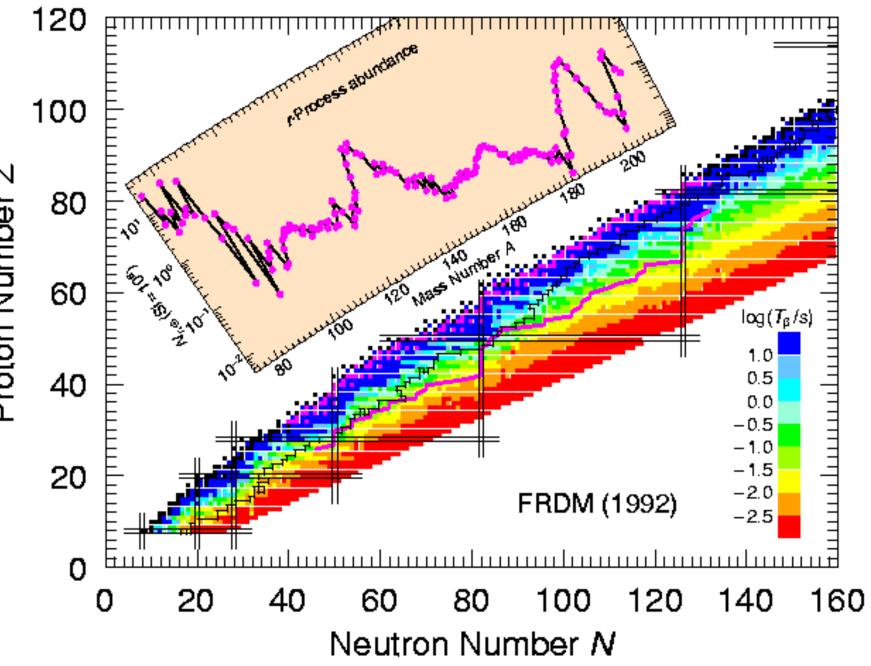
See

	similarly	for Sm	~ (mb)	n	σп
2=62	144 Sm	9	92 2 9	7.42(-3)	0.68
	147 Sm	r, 5	1000\$100	3.71 (-2)	37 ± 4
	148 Sm	5	267 ±12	2.70(-2)	7.2 ± 0.3 ←
	149 Sm	r,s	14541 66	3.52(-2)	48.3 ± 2.2
	iso Sm	5	447 ± 26	1,79 (-2)	\$.0. ± 0.5 +
	152 Sm	5, 7	375 123	6.41(-2)	24.2 ± 1.5
	194 Sm	r	293±19	5.45(-2)	16.0 \$ 1.0

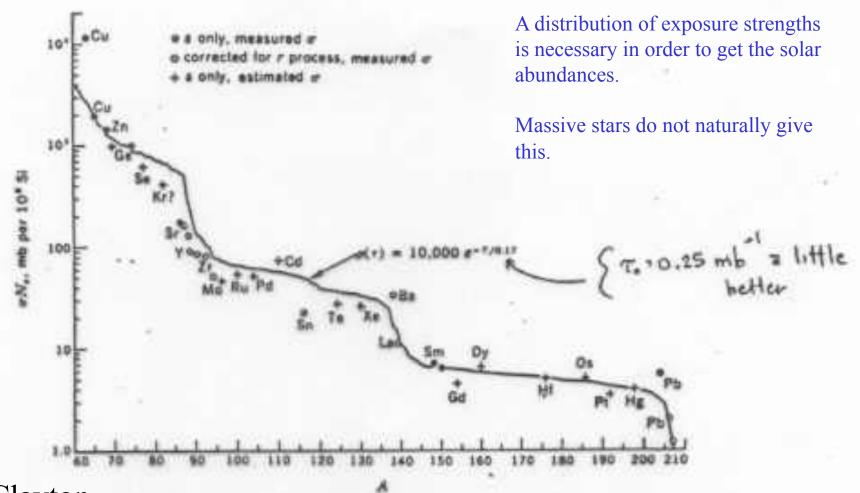
 $\tau_{1/2}(^{151}\text{Sm}) = 90 \text{ years}; \ \tau_{1/2}(^{153}\text{Sm}) = 46 \text{ hours}$

Based upon the abundances of s-only isotopes and the known neutron capture cross sections one can subtract the s-portion of s,r isotopes to obtain the r- and s-process yields separately.





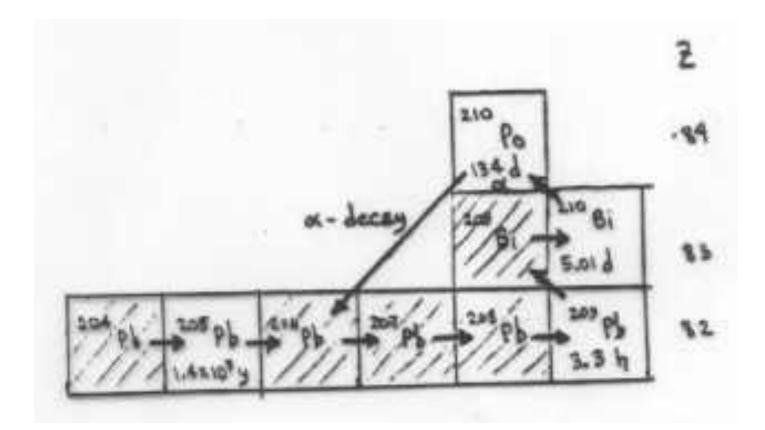
Proton Number Z

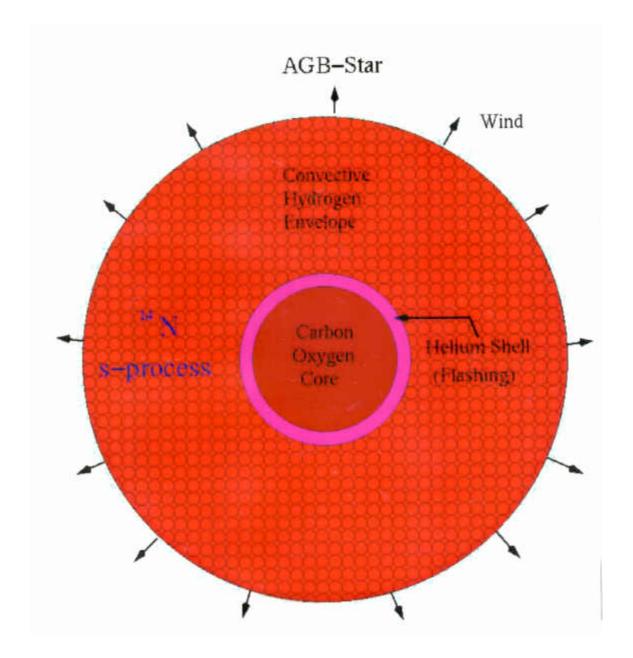


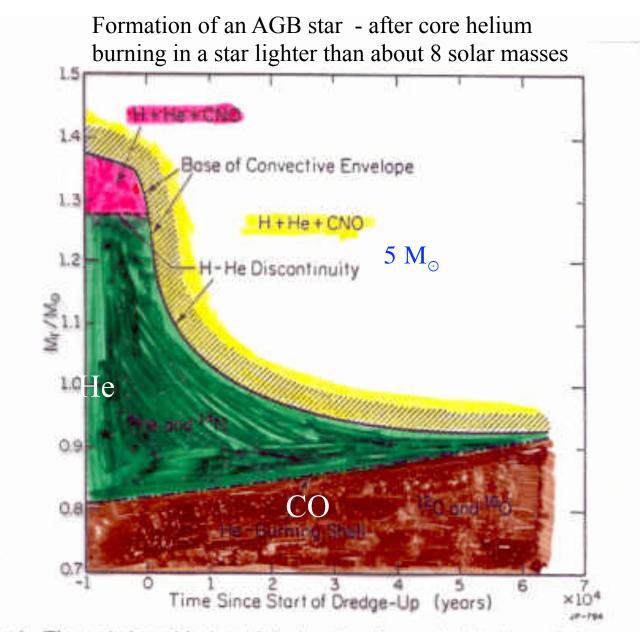
Clayton

Fig. 7-20 The solar-system v.N. curve. The product of the neutron-capture cross sections for kT = 30 kev times the nuclide abundance per 10^s silicon stores is plotted versus the atomic mass number A. The solid curve is the calculated result of an exponential distribution of neutron exposures. [P. A. Seeger, W. A. Fowler, and D. D. Cloyton, Astrophys. J. Suppl., 11:121 (1965). By permission of The University of Chicogo Press. Copyright 1965 by The University of Chicogo.]

Termination of the s-Process







12

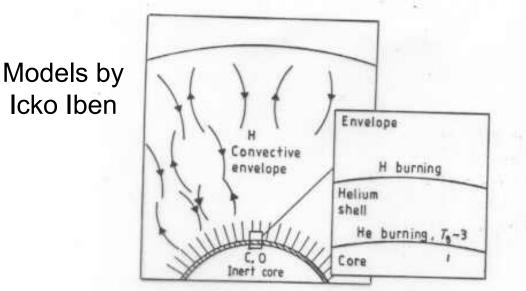
Figure 1 The variation with time of the location (in mass) of the base of the convective envelope and of the center of the helium-burning shell during the second dredge-up phase in a model of mass $S M_{\odot}$ and initial composition (Y, Z) = (0.28, 0.001).

 $\frac{dP}{dm} = \frac{GM(R)}{4\pi R^4}$

Thin shell instability:

If a shell is sufficiently thin, its pressure is set by the gravitational potential in which it rests. During a flash, this does not change. Burning raises the temperature but does not change the pressure so the density in the shell goes down. But the energy generation is very sensitive to the temperature and continues to go up. Finally sufficient burning occurs to cause enough overall expansion to reduce the pressure. This instability tends to happen at the edges of compact objects where the burning shells are quite thin compared with the radius of the core and the gravitational potential is thus constant.

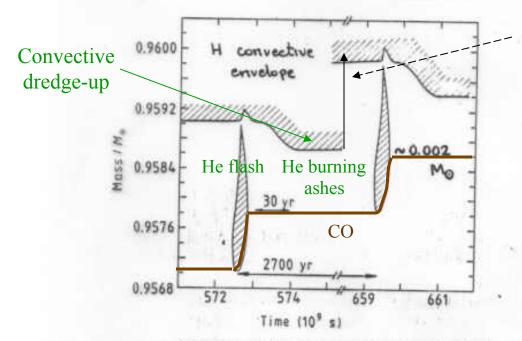
Schwarzshild & Harm (1965) and Weigert (1966), Yoon, Langer and van der Sluys, *A&A*, **425**, 207 (2004)



Part of the ashes of each helium shell flash are incorporated into the fuel for the next flash.

This naturally gives a power law distribution of exposures

Schematic structure of a $7M_{\odot}$ mass star during chell burning. For illustration, the helium shell is greatly exaggerated. Helium shell burning occurs in collect, which are triggered when the CNO cycle at the bottom of the envelope has produced a critical amount of helium.



H-shell burning extends helium layer in preparation for the next flash

During each flash have a mixture of He, C, ¹³C or ²²Ne, new seed nuclei and s-process from prior flashes Important nuclear physics modification:

s-process giants derived from AGB stars in the solar neighborhood do not show the large ²⁶Mg excesses one would expect if the neutron source were ²²Ne(a,n)²⁵Mg [as it surely is in massive stars]. Moreover these stars are too low in mass for ²²Ne(a,n)²⁵Mg to function efficiently. A different way of making neutrons is required. Probably

 ${}^{4}He(2\alpha,\gamma){}^{12}C(p,\gamma){}^{13}N(e^{+}\nu){}^{13}C$ ${}^{13}C(\alpha,n){}^{16}O$

with the protons coming from mixing between the helium burning shell and the hydrogen envelope. Each p mixed in becomes an n.

McWilliam and Lambert, *MNRAS*, **230**, 573 (1988) and Malaney and Boothroyd, *ApJ*, **320**, 866 (1987) Hollowell and Iben, *ApJL*, **333**, L25 (1988); *ApJ*, **340**, 966, (1989) many more since then The metallicity history of the s-process can be quite complicated.

In the simplest case in massive stars with a ²²Ne neutron source it is independent of metallicity until quite low values of Z.

At very low Z things can become complicated because of the effect of neutron poisons, ¹²C and ¹⁶O. In AGB stars the mixing between H and He shells is Z dependent. Some metal poor stars are actually very s-process rich. Cristallo et al (ApJ, 696, 797 (2009)).