

## Lecture 10

### Nucleosynthesis During Helium Burning and the s-Process

#### A. Thermodynamic Conditions (Massive Stars)

For the most part covered previously  
 $\tau_{\text{He}} \sim 10^6$  years (+- factor of three)

$$n=3 \quad M_\alpha = 18 \frac{\sqrt{1-\beta}}{\mu^2 \beta^4} \quad \mu = \frac{4}{3} \text{ (for pure helium)} \quad \text{solve for } \beta$$

$$T_c = 4.6 \times 10^6 \mu \beta \left( \frac{M_\alpha}{M_\odot} \right)^{2/3} \rho_c^{1/3} \text{ K}$$

E.g.,  $M_\alpha = 6$  (a  $20 M_\odot$  main sequence star)

$$\beta = 0.83$$

$$T_c = 1.7 \times 10^8 \left( \frac{\rho_c}{1000 \text{ g cm}^{-3}} \right)^{1/3} \text{ K}$$

need temperatures  $> 10^8$   
 to provide significant  
 energy generation by  $3\alpha$

So, typical temperatures are  $2 \times 10^8$  K (higher in shell burning later) when densities are over  $1000 \text{ gm cm}^{-3}$ . As the core evolves the temperature and density go up significantly. Note non-degenerate.

From Schaller et al (1992)  $Z = 0.02$  and central helium mass fraction of 50%.

Main sequence Mass	Current Mass	Approximate $M_\alpha$	$T_c/10^8$	$\rho_c/1000 \text{ g cm}^{-3}$
12	11.66	3	1.76	1.42
15	14.24	4	1.83	1.12
20	18.10	6	1.92	0.831
25	20.40	8	1.99	0.674
40	20.7	12	2.11	0.470

At  $1.9 \times 10^8$  K, the temperature sensitivity of the  $3\alpha$  rate is approximately  $T^{20}$ .

As discussed earlier during helium burning:

$$\frac{dY_\alpha}{dt} = -3\rho^2 Y_\alpha^3 \lambda_{3\alpha} - Y(^{12}\text{C}) Y_\alpha \rho \lambda_{\alpha\gamma}(^{12}\text{C})$$

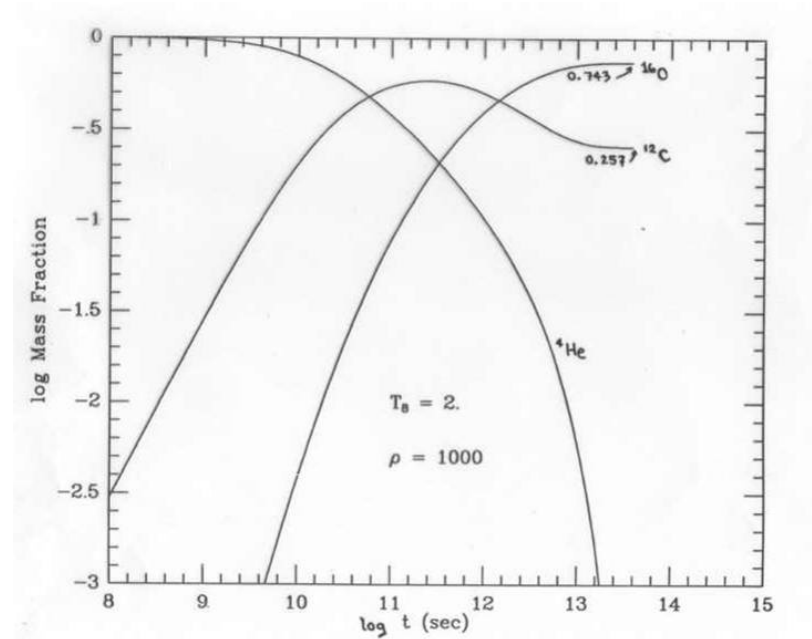
$$\frac{dY(^{12}\text{C})}{dt} = \rho^2 Y_\alpha^3 \lambda_{3\alpha} - Y(^{12}\text{C}) Y_\alpha \rho \lambda_{\alpha\gamma}(^{12}\text{C})$$

$$\frac{dY(^{16}\text{O})}{dt} = Y(^{12}\text{C}) Y_\alpha \rho \lambda_{\alpha\gamma}(^{12}\text{C})$$

Coulomb barrier and lack of favorable resonances inhibit alpha capture on  $^{16}\text{O}$ .

**Several general features:**

- $^{12}\text{C}$  production favored by large density; oxygen by lower density
- $^{12}\text{C}$  produced early on,  $^{16}\text{O}$  later
- last few alpha particles burned most critical in setting ratio  $^{12}\text{C}/^{16}\text{O}$
- Energy generation larger for smaller  $^{12}\text{C}/^{16}\text{O}$ .



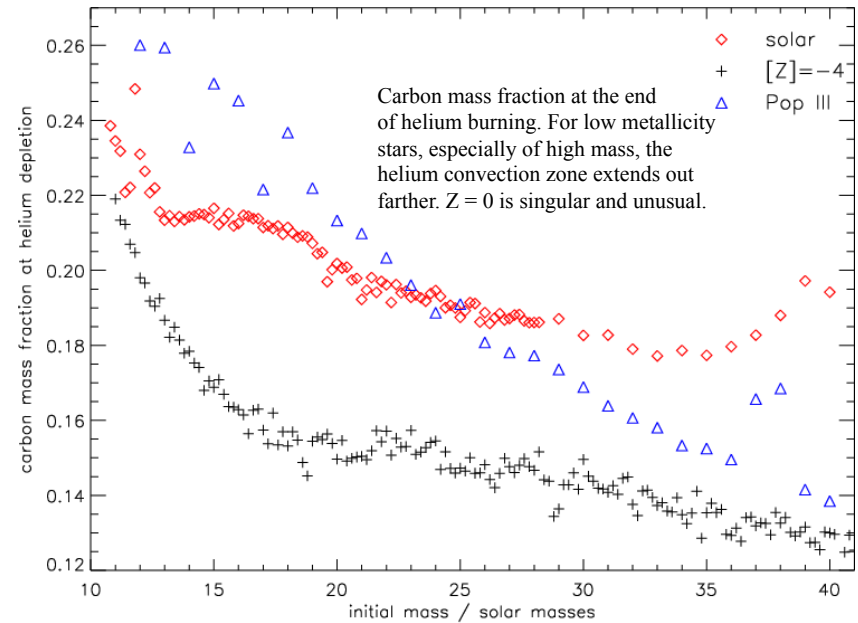
B1. Principal Nucleosynthesis

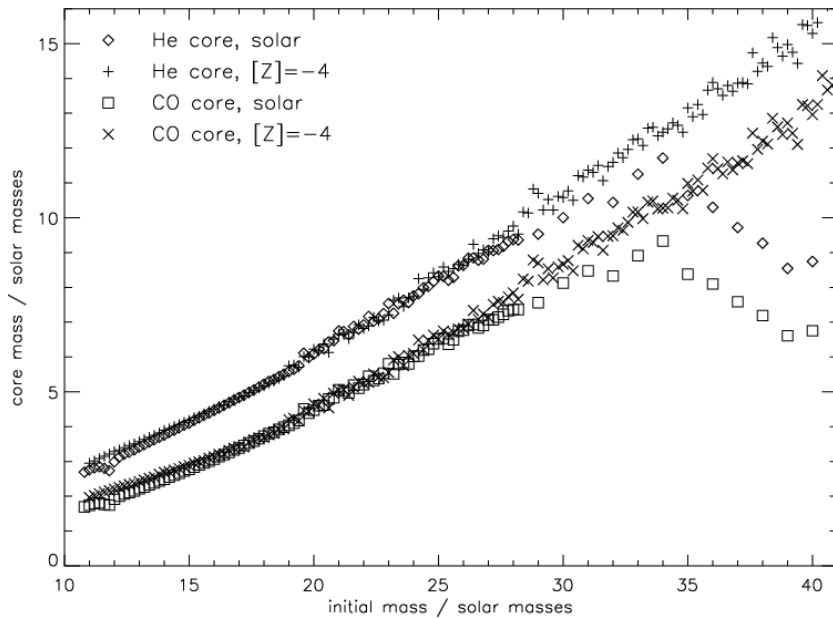
In massive stars after helium burning in the stars center (calculations included semiconvection).

	$M/M_{\odot}$	$X(^{12}\text{C})$	$X(^{16}\text{O})$	$X(^{20}\text{Ne})$
In the sun,	12	0.247	0.727	1.91(-3)
$^{12}\text{C}/^{16}\text{O} = 0.32$	15	0.188	0.785	3.02(-3)
	19	0.153	0.815	6.44(-3)
	25	0.149	0.813	1.16(-2)
	35	0.147	0.807	1.84(-2)

*Buchan  $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$  multiplied by 1.2.*

If the star contains appreciable metals there is, as we shall see also  $^{22}\text{Ne}$  and  $^{18}\text{O}$ .





*Within uncertainties, helium burning in massive stars (over 8 solar masses) could be the origin in nature of  $^{12}\text{C}$ . It is definitely the origin of  $^{16}\text{O}$*

#### Complications:

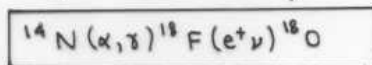
- If the helium core grows just a little bit towards the end of helium burning, the extra helium convected in greatly decreases the  $^{12}\text{C}$  synthesis.
- Mass loss from very massive WR stars can greatly increase the synthesis of both  $^{12}\text{C}$  and  $^{16}\text{O}$  in stars over 35 solar masses
- The uncertain rate for  $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$
- $^{12}\text{C}/^{16}\text{O}$  ratio may be affected by post-helium burning evolution and by black hole formation above some critical main sequence mass.  $^{16}\text{O}$  is made in the more massive (massive) stars.

#### B. Trace Element Nucleosynthesis - Charged Particles

During hydrogen burning  $\text{CNO} \rightarrow ^{14}\text{N}$  so that at helium ignition

$$X(^{14}\text{N}) = 14 (Y_i(^{12}\text{C}) + Y_i(^{14}\text{N}) + Y_i(^{16}\text{O})) = 0.013 (Z/Z_\odot)$$

Early during helium burning



NB. From this point onwards have a net neutron excess

$$\eta = 1 - 2Y_e = \sum (N_i - Z_i) Y_i$$

$-1 \leq \eta \leq 1$   
 $\eta = 0$  if  $Z = N$

Before the above reaction the composition was almost entirely  $^4\text{He}$  and  $^{14}\text{N}$ , hence  $\eta \approx 0$  (actually a small positive value exists because of  $^{56}\text{Fe}$  and the like).

After this reaction

$$\eta = 0.0019 \frac{Z}{Z_\odot}$$

During helium core burning,  $^{18}\text{O}$  is later mostly destroyed by  $^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}$ .

During helium shell burning which does not go to completion in massive stars, much of  $^{18}\text{O}$  remains undestroyed and this is the source of  $^{18}\text{O}$  in nature. Convection helps to preserve it.

	Lifetimes (yr)		$X_\alpha = 1/2$	$\rho = 1000$	
	$T_8 = 1.8$	$T_8 = 2$			
$^{16}\text{O}(\alpha, \gamma)$	3.5 (10)	2.3 (9)			survives
$^{18}\text{O}(\alpha, \gamma)$	2700	98			destroyed unless preserved by convection
$^{14}\text{N}(\alpha, \gamma)$	15	1.1			destroyed
$^{22}\text{Ne}(\alpha, n) + (\alpha, \gamma)$	4.1 (6)	2.2 (5)			partly destroyed

So one expects that, depending on mass, some but not all of the  $^{22}\text{Ne}$  will burn towards the end of helium burning when the temperature goes up.

The following table gives the temperature at the center of the given model and the mass fractions of  $^{22}\text{Ne}$ ,  $^{25}\text{Mg}$ , and  $^{26}\text{Mg}$  each multiplied by 1000, when the helium mass fraction is 1% and zero

Woosley, and Heger (2007)

M	$T_C$	$^{22}\text{Ne}$	$^{25}\text{Mg}$	$^{26}\text{Mg}$
12	2.42	13.4	0.51	0.61
		12.3	1.17	1.05
15	2.54	12.7	0.98	0.91
		11.1	1.99	1.80
19	2.64	11.5	1.73	1.54
		9.4	2.90	2.87
25	2.75	9.8	2.67	2.59
		6.96	4.05	4.54
35	2.86	7.37	3.87	4.22
		4.41	5.18	6.39

The remainder of the  $^{22}\text{Ne}$  will burn early during carbon burning, but then there will be more abundant "neutron poisons".

These numbers are quite sensitive to the uncertain reaction rate for  $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$  and may be lower limits to the  $^{22}\text{Ne}$  consumption.

### C. The s-Process in Massive Stars

Late during helium burning, when the temperature rises to about  $3.0 \times 10^8$  K,  $^{22}\text{Ne}$  is burned chiefly by the reaction  $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$  (with some competition from  $^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$ ).

Where do the neutrons go?

Some go on  $^{56}\text{Fe}$  but that fraction is only:

$$f = \frac{\sigma_{56} Y_{56}}{\sum \sigma_i Y_i}$$

$^{16}\text{O}$	$Y_{16} \approx \frac{0.5}{16} = 3.1 \times 10^{-2}$	$Y_{16} \sigma_{16} \approx 1.2 \times 10^{-3}$	} But, $^{17}\text{O}(\alpha, n)^{20}\text{Ne}$ destroys the $^{17}\text{O}$ and restores the neutron
$^{22}\text{Ne}$	$Y_{22} \approx \frac{0.005}{22} = 2.3 \times 10^{-4}$	$Y_{22} \sigma_{22} \approx 1.3 \times 10^{-5}$	
$^{25}\text{Mg}$	$Y_{25} \approx \frac{0.005}{25} = 2 \times 10^{-4}$	$Y_{25} \sigma_{25} \approx 1.3 \times 10^{-3}$	For $\sigma$ in mb
$^{56}\text{Fe}$	$Y_{56} \approx \frac{0.0013}{56} = 2.3 \times 10^{-5}$	$Y_{56} \sigma_{56} \approx 2.7 \times 10^{-4}$	

### 30 keV neutron capture cross sections

(mostly from Bao et al, ADNDT, 2000)

Nucleus	$\sigma$ (mb)	Nucleus	$\sigma$ (mb)
$^{12}\text{C}$	0.0154	$^{54}\text{Fe}$	27.6
* $^{16}\text{O}$	0.038	$^{56}\text{Fe}$	11.7
$^{20}\text{Ne}$	0.119	$^{57}\text{Fe}$	40.0
$^{22}\text{Ne}$	0.059	$^{58}\text{Fe}$	12.1
$^{24}\text{Mg}$	3.3	$^{58}\text{Ni}$	41.0
$^{25}\text{Mg}$	6.4	$^{64}\text{Zn}$	59
$^{26}\text{Mg}$	0.126	$^{65}\text{Zn}$	162
$^{28}\text{Si}$	2.9	$^{66}\text{Zn}$	35
		$^{88}\text{Sr}$	6.2 (closed shell)

The large cross section of  $^{25}\text{Mg}$  is particularly significant since it is made by  $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ .

\* Igashira et al, *ApJL*, 441, L89, (1995); factor of 200 upwards revision

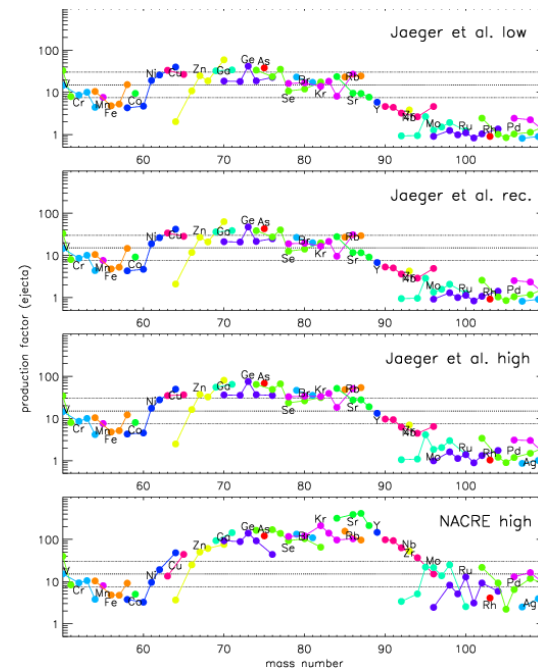
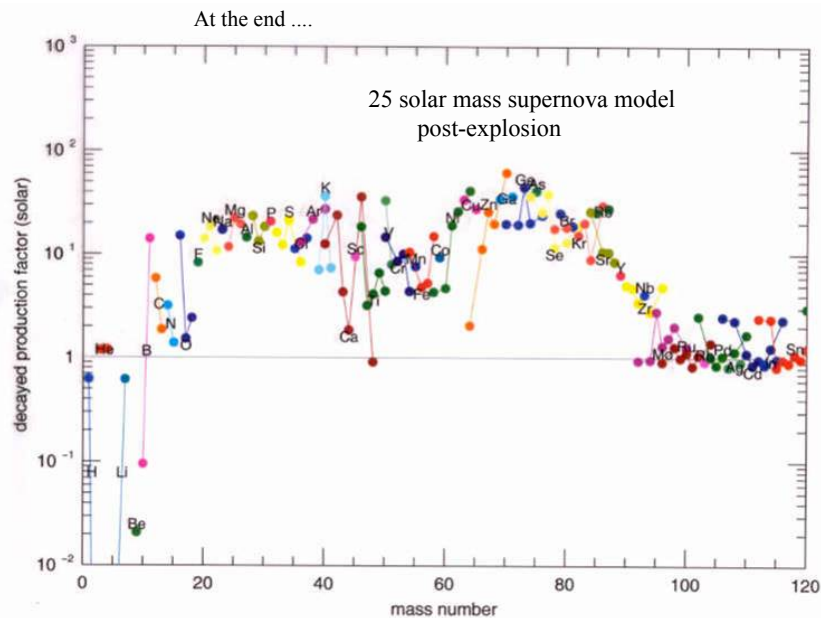
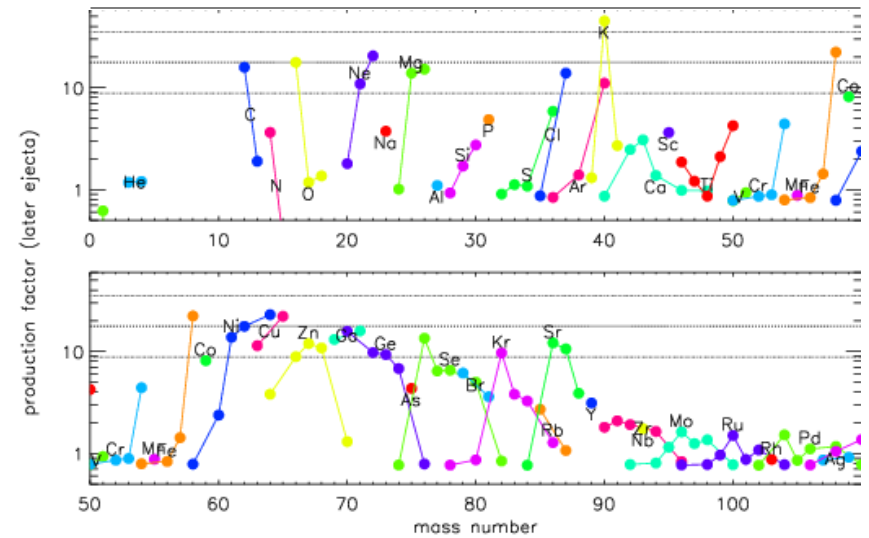
So a fraction  $\frac{Y_{56}\sigma_{56}}{Y_{25}\sigma_{25}} \sim 10 - 20\%$  capture on iron. How many neutrons is this?

$$Y_n / Y_{Fe} \sim Y_{22} / Y_{56} = 42$$

where we have assumed a mass fraction of 0.02 for  $^{22}\text{Ne}$  and 0.0013 for  $^{56}\text{Fe}$  and that all  $^{22}\text{Ne}$  burns by  $(\alpha, n)$ .

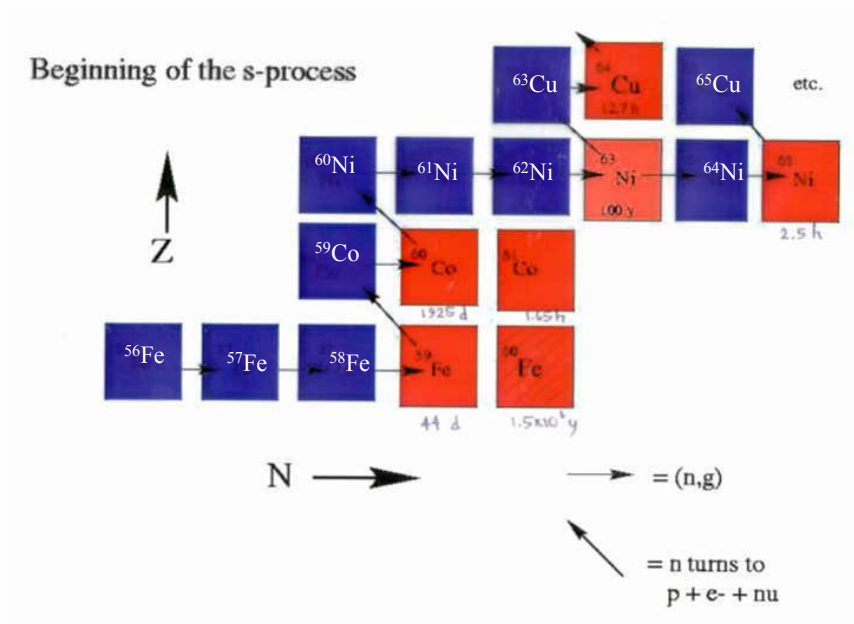
This is about **4 - 8 neutrons per iron** and obviously not nearly enough to change e.g., Fe into Pb, but the neutron capture cross sections of the isotopes generally increase above the iron group and the solar abundances decrease. A significant s-process occurs that produces significant quantities of the isotopes with  $A < 90$ .

Composition of a 25 solar mass star at the end of helium burning compared with solar abundances (Rauscher et al 2001)



Results depend most sensitively upon the reaction rate for  $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ .

If  $^{22}\text{Ne}$  does not burn until later (i.e., carbon burning) there are much more abundant neutron poisons



Here "s" stands for "slow" neutron capture

$$\tau_\beta \ll \tau_{n\gamma} \quad \tau_\beta = \frac{1}{\lambda_\beta} = \frac{\tau_{1/2}}{\ln 2} \quad \frac{dY_i}{dt} = -Y_i Y_n \rho \lambda_{n\gamma}(i) + \dots$$

$$\tau_{n\gamma} = (\rho Y_n \lambda_{n\gamma})^{-1} \quad \tau_{n\gamma} = \left( \frac{1}{Y_i} \frac{dY_i}{dt} \right)^{-1}$$

This means that the neutron densities are relatively small

E.g. for a  $^{22}\text{Ne}$  neutron source

$$\frac{dY_n}{dt} \approx 0 \approx \rho Y_\alpha Y(^{22}\text{Ne}) \lambda_{\alpha n}(^{22}\text{Ne}) - \rho Y_n Y(^{25}\text{Mg}) \lambda_{n\gamma}(^{25}\text{Mg})$$

$$\rho \approx 1000, X_\alpha \approx 0.5, X(^{22}\text{Ne}) \approx 0.005, X(^{25}\text{Mg}) \approx 0.005$$

$$Y_n \approx \frac{Y_\alpha Y(^{22}\text{Ne}) \lambda_{\alpha n}(^{22}\text{Ne})}{Y(^{25}\text{Mg}) \lambda_{n\gamma}(^{25}\text{Mg})} \sim \frac{Y_\alpha \lambda_{\alpha n}(^{22}\text{Ne})}{\lambda_{n\gamma}(^{25}\text{Mg})}$$

$$Y_\alpha \sim 0.02 / 4; Y(^{22}\text{Ne}) \sim 2 Y(^{25}\text{Mg}); \rho = 1000$$

$T_8$	$\lambda_{\alpha n}(^{22}\text{Ne})$	$\lambda_{n\gamma}(^{25}\text{Mg})$	$n_n = \rho N_A Y_n$	$\lambda_{n\gamma}(^{56}\text{Fe})$
2.0	9.1(-17)	1.2(6)	<i>negligible</i>	
2.5	1.5(-13)	1.1(6)	$\sim 10^6$	1.9(6)
3.0	2.6(-11)	1.0(6)	$\sim 10^8$	1.9(6)

Most of the s-process takes place around  $T_8 = 2.5 - 3$ , so the neutron density is about  $10^6 - 10^8 \text{ cm}^{-3}$  (depends on uncertain rate for  $(\alpha, n)$  on  $^{22}\text{Ne}$  and on how much  $^{22}\text{Ne}$  has burned).

At these neutron densities the time between capture, even for heavy elements with bigger cross sections than iron, is days. For  $^{56}\text{Fe}$  itself it is a few years

Eg. at  $T_8 = 2.5$  ( $n_n \sim 10^6$ ), the lifetime of  $^{56}\text{Fe}$  is about

$$\tau(^{56}\text{Fe}) = \left( \frac{1}{Y(^{56}\text{Fe})} \frac{dY(^{56}\text{Fe})}{dt} \right)^{-1} = (\rho Y_n \lambda_{n\gamma}(^{56}\text{Fe}))^{-1}$$

$$\approx 10^4 \text{ year (less at higher temperatures)}$$

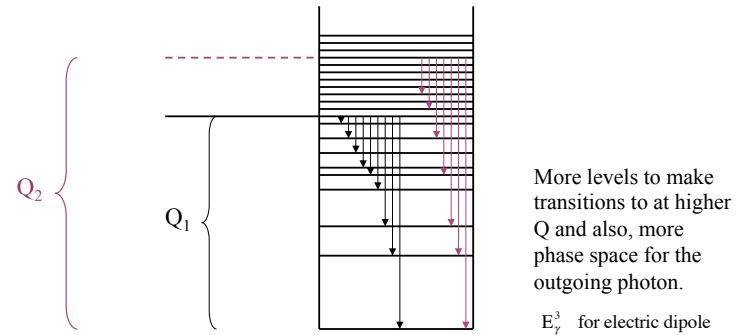
The s-process in massive stars only goes on during a brief period at the end of helium burning. The time scale is lengthened by convection.

Reaction Rates (n,γ):

Either measured (Bao et al, ADNDT, 76, 70, 2000)  
 or calculated using Hauser-Feshbach theory  
 (Woosley et al., ADNDT, 22, 371, (1976)  
 Holmes et al., ADNDT, 18, 305, (1976);  
 Rauscher et al. ADNDT, 75, 1, (2000))

The calculations are usually good to a factor of two. For heavy nuclei within  $kT \sim 30$  keV of  $Q_{ng}$  there are very many resonances.

Occasionally, for light nuclei or near closed shells, direct capture is important: e.g.,  $^{12}\text{C}$ ,  $^{20,22}\text{Ne}$ ,  $^{16}\text{O}$ ,  $^{48}\text{Ca}$

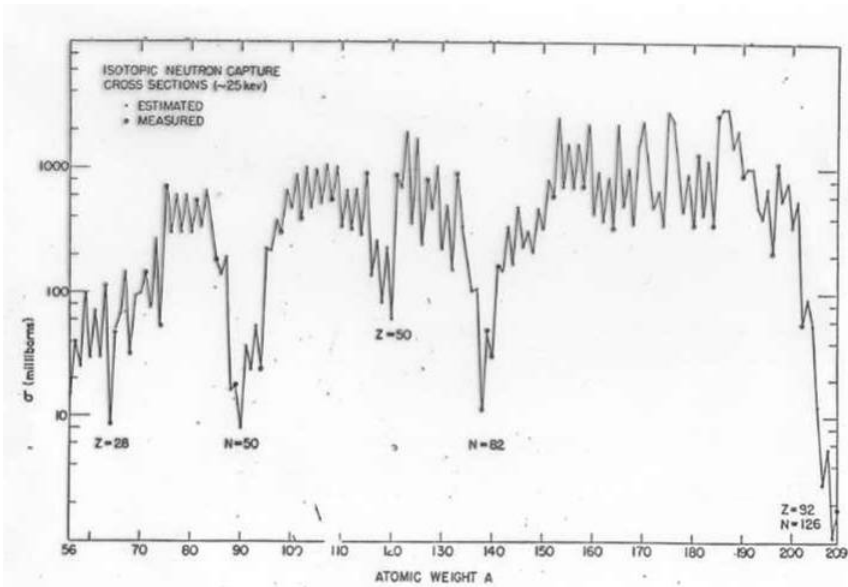


$$T_\gamma(Q_2) > T_\gamma(Q_1)$$

and as a result

$$\sigma_{n\gamma} \propto \frac{T_n T_\gamma}{T_n + T_\gamma} \approx T_\gamma$$

is larger if Q is larger



Rate Equations: Their Solutions and Implications

Assume constant density, temperature, cross section, and neutron density and ignore branching (would never assume any of these in a modern calculation). Then

$$\frac{dY(^AZ)}{dt} \equiv \frac{dY_A}{dt} = -Y_A Y_n \rho \lambda_{n\gamma}(A) + Y_{A-1} Y_n \rho \lambda_{n\gamma}(A-1)$$

and since  $n_n = \rho N_A Y_n$  and  $\lambda_{n\gamma} = N_A \langle \sigma_{n\gamma} v \rangle \approx N_A \sigma_A v_{thermal}$

defining  $\tau \equiv \int \rho N_A Y_n v_{thermal} dt = \int n_n v_{thermal} dt$ , one has

$$\frac{dn_A}{d\tau} = -n_A \sigma_A + n_{A-1} \sigma_{A-1}$$

Note that  $\tau$  has units of inverse cross section (inverse area).

If there were locations where steady state is achieved then

$$\frac{dn_A}{d\tau} \approx 0 = n_A \sigma_A - n_{A-1} \sigma_{A-1}$$

i.e.,  $\sigma n$  is locally a constant, and  $n \propto \frac{1}{\sigma}$

Attaining steady state requires a time scale longer than a few times the destruction lifetime of the species in the steady state group. One has “local” steady state because any flux that would produce, e.g., lead in steady state would totally destroy all the lighter s-process species.

*The flow stagnates at various “waiting points” along the s-process path, particularly at the closed shell nuclei.*

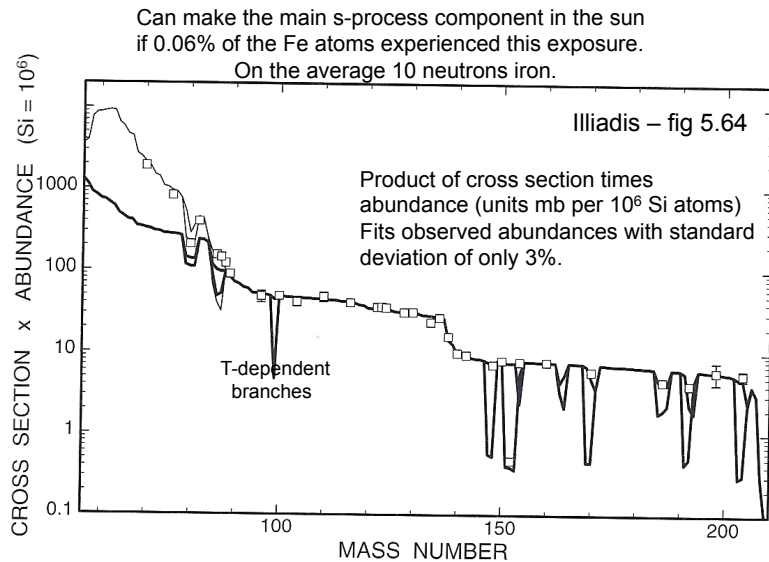
Eg.,  $n_n \sim 10^8 \text{ cm}^{-3} \Rightarrow \rho Y_n \sim n_n / N_A \sim 5 \times 10^{-17}$

$\lambda_{n\gamma}$  experimentally at helium burning temperatures is  $10^5 - 10^8$

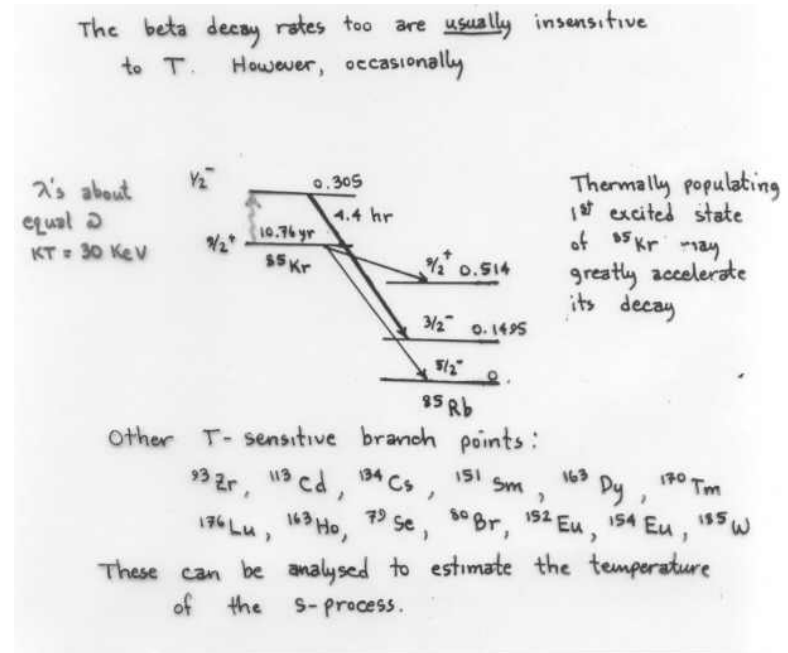
$$\tau_{n\gamma} = (\rho Y_n \lambda_{n\gamma})^{-1} = \left( \frac{d \ln Y_A}{dt} \right)^{-1} \sim 10 - 10^4 \text{ years}$$

This can be greatly lengthened in a massive star by convection.

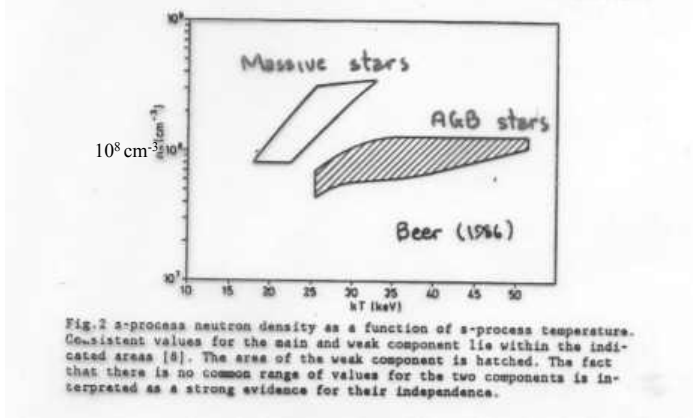
As a result nuclei with large cross sections will be in steady state while those with small ones are not. This is especially so in He shell flashes in AGB stars where the time scale for a flash may be only a few decades.



The symbols are s-only nuclei. The solid lines are the model results for a standard (exponential) set of exposure strengths. Below A = 90 there is evidence for a separate additional s-process component.







$$\frac{n_{new}(A) - n_{old}(A)}{d\tau} = -n_{new}(A)\sigma_A + n_{new}(A-1)\sigma_{A-1}$$

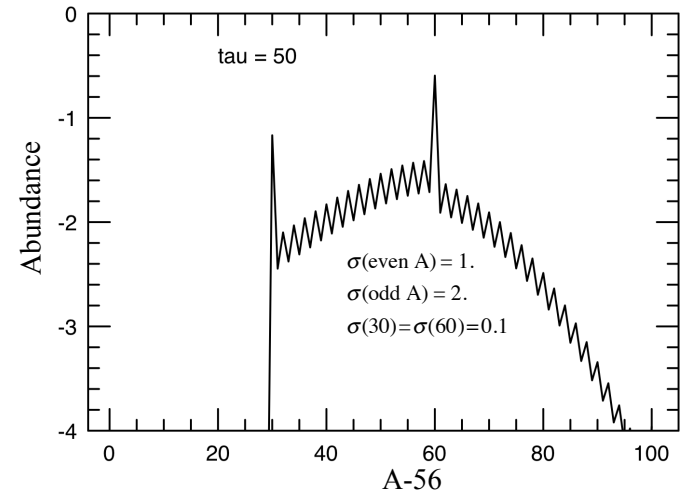
$n_{new}(A-1)$  known from previous step in recursion relation

### Implicit solution:

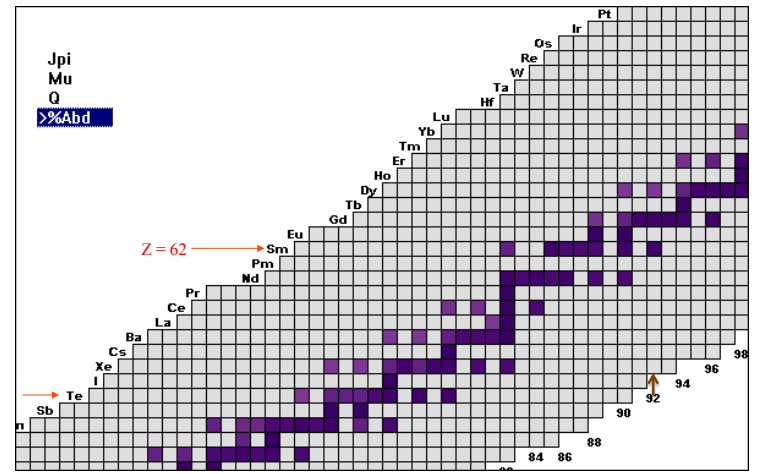
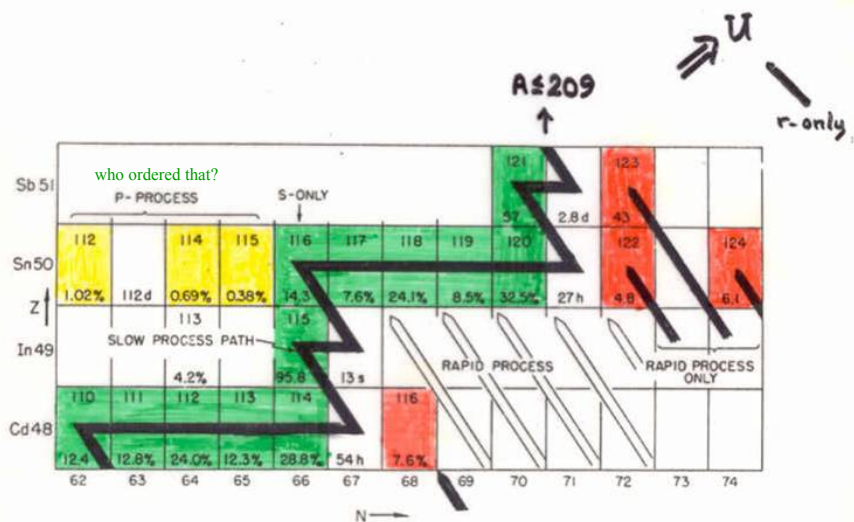
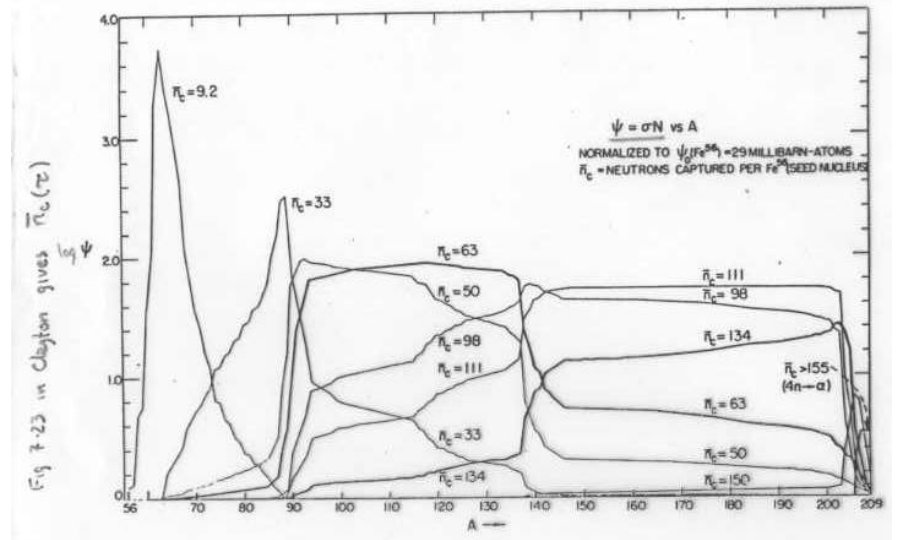
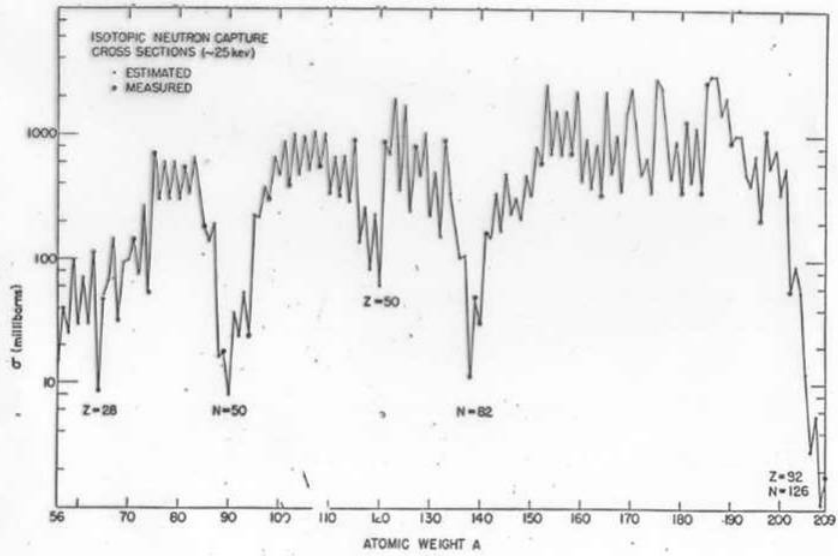
Assuming no flow downwards from  $A+1$  and greater to  $A$  and below.

$$n_{new}(A) = \frac{n_{old}(A)/d\tau + n_{new}(A-1)\sigma_{n\gamma}(A-1)}{1/d\tau + \sigma_{n\gamma}(A)}$$

This works because in the do loop,  $n_{new}(A-1)$  is updated to its new value before evaluating  $n_{new}(A)$ . Matrix inversion reduces to a recursion relation.



Sample output from toy model code micros2.f



e.g.,  $^{117}\text{Sn}$ ,  $^{118}\text{Sn}$ ,  $^{119}\text{Sm}$ , and  $^{120}\text{Sn}$  are s,r isotopes. Sn is not a good place to look for s n = const though because it is a closed shell.

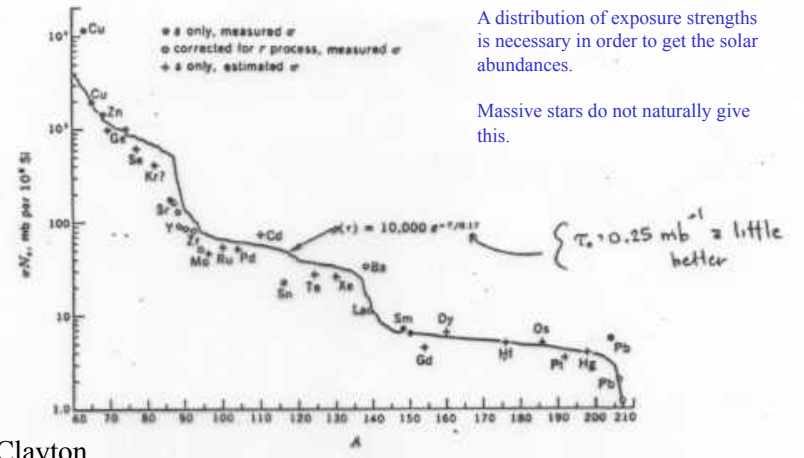
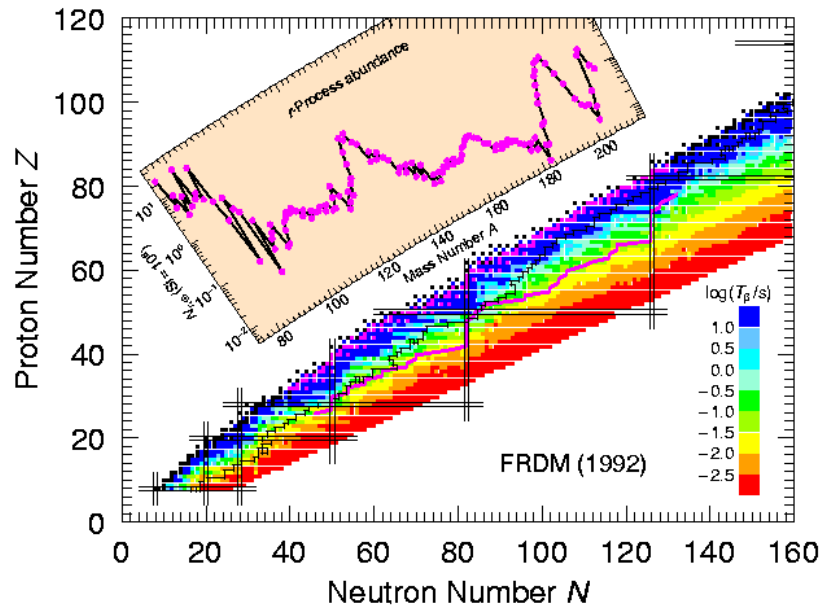
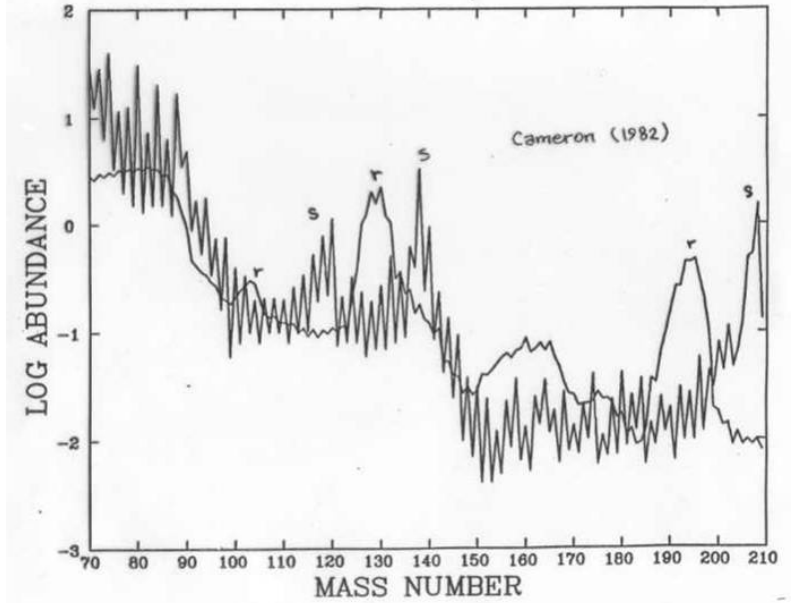
See

<http://www.nndc.bnl.gov/chart/>

similarly for Sm	$\sigma$ (mb)	$\eta$	$\sigma\eta$
144Sm p	92 ± 9	7.42(-3)	0.68
147Sm r, s	1000 ± 100	3.71(-2)	37 ± 4
148Sm s	267 ± 12	2.70(-2)	7.2 ± 0.3 ←
149Sm r, s	1494 ± 66	3.32(-2)	49.3 ± 2.2
150Sm s	447 ± 26	1.75(-2)	7.8 ± 0.5 ←
152Sm s, r	375 ± 23	6.41(-2)	24.2 ± 1.5
154Sm r	253 ± 19	5.45(-2)	16.0 ± 1.0

$\tau_{1/2}(^{151}\text{Sm}) = 90 \text{ years}; \tau_{1/2}(^{153}\text{Sm}) = 46 \text{ hours}$

Based upon the abundances of s-only isotopes and the known neutron capture cross sections one can subtract the s-portion of s,r isotopes to obtain the r- and s-process yields separately.



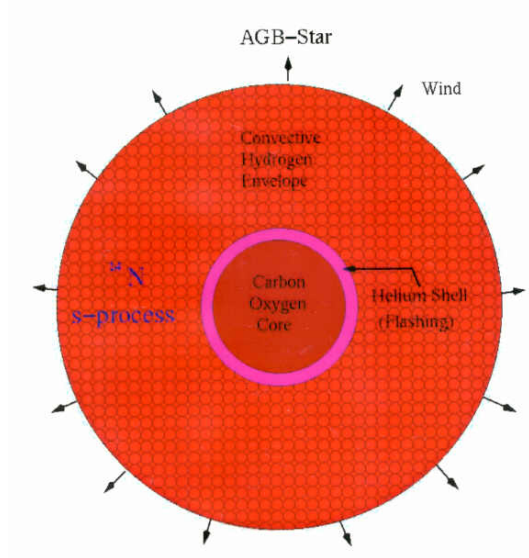
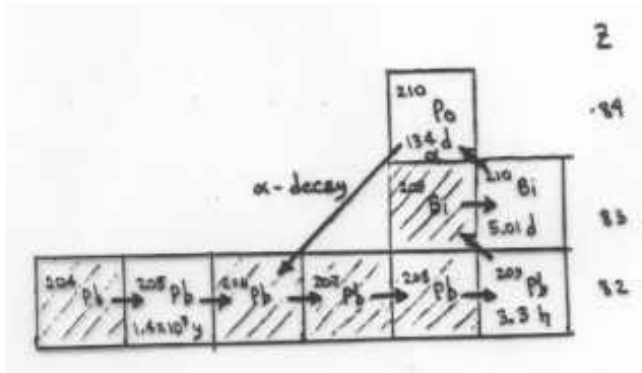
A distribution of exposure strengths is necessary in order to get the solar abundances.

Massive stars do not naturally give this.

Clayton

Fig. 7-29 The solar-system  $\nu N$  curve. The product of the neutron-capture cross sections for  $E\gamma = 30 \text{ keV}$  times the nuclide abundance per  $10^{21}$  silicon atoms is plotted versus the atomic mass number  $A$ . The solid curve is the calculated result of an exponential distribution of neutron exposures. [P. A. Seeger, W. A. Fowler, and D. D. Clayton, *Astrophys. J. Suppl.*, 11:121 (1963). By permission of The University of Chicago Press. Copyright 1965 by The University of Chicago.]

Termination of the s-Process



Formation of an AGB star - after core helium burning in a star lighter than about 8 solar masses

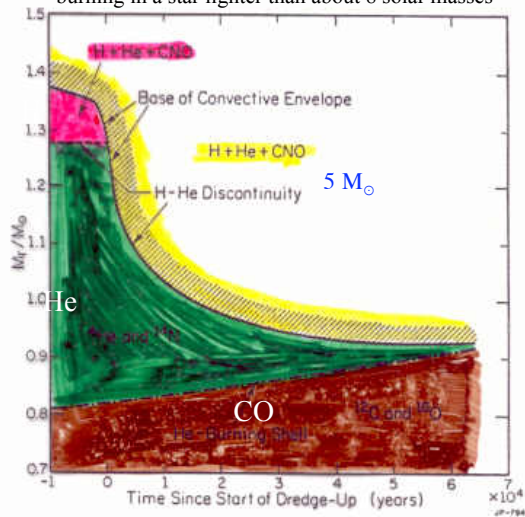


Figure 1 The variation with time of the location (in mass) of the base of the convective envelope and of the center of the helium-burning shell during the second dredge-up phase in a model of mass  $5 M_{\odot}$  and initial composition  $(Y, Z) = (0.28, 0.001)$ .

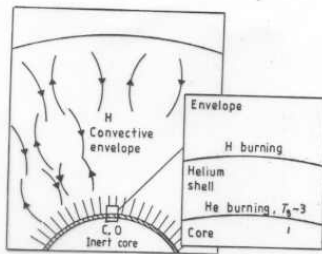
$$\frac{dP}{dm} = \frac{GM(R)}{4\pi R^4}$$

**Thin shell instability:**

If a shell is sufficiently thin, its pressure is set by the gravitational potential in which it rests. During a flash, this does not change. Burning raises the temperature but does not change the pressure so the density in the shell goes down. But the energy generation is very sensitive to the temperature and continues to go up. Finally sufficient burning occurs to cause enough overall expansion to reduce the pressure. This instability tends to happen at the edges of compact objects where the burning shells are quite thin compared with the radius of the core and the gravitational potential is thus constant.

Schwarzschild & Harm (1965) and Weigert (1966), Yoon, Langer and van der Sluys, *A&A*, **425**, 207 (2004)

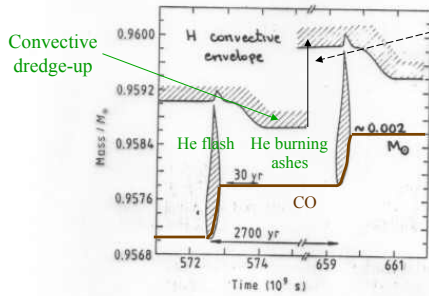
Models by Icko Iben



Schematic structure of a  $7M_{\odot}$  mass star during shell burning. For illustration, the helium shell is greatly exaggerated. Helium shell burning occurs in pulses, which are triggered when the CNO cycle at the bottom of the envelope has produced a critical amount of helium.

*Part of the ashes of each helium shell flash are incorporated into the fuel for the next flash.*

*This naturally gives a power law distribution of exposures*

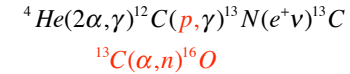


H-shell burning extends helium layer in preparation for the next flash

*During each flash have a mixture of He, C,  $^{13}\text{C}$  or  $^{22}\text{Ne}$ , new seed nuclei and s-process from prior flashes*

Important nuclear physics modification:

s-process giants derived from AGB stars in the solar neighborhood do not show the large  $^{26}\text{Mg}$  excesses one would expect if the neutron source were  $^{22}\text{Ne}(a,n)^{25}\text{Mg}$  [as it surely is in massive stars]. Moreover these stars are too low in mass for  $^{22}\text{Ne}(a,n)^{25}\text{Mg}$  to function efficiently. A different way of making neutrons is required. Probably



with the protons coming from mixing between the helium burning shell and the hydrogen envelope.

Each  $p$  mixed in becomes an  $n$ .

McWilliam and Lambert, *MNRAS*, **230**, 573 (1988) and  
 Malaney and Boothroyd, *ApJ*, **320**, 866 (1987)  
 Hollowell and Iben, *ApJL*, **333**, L25 (1988); *ApJ*, **340**, 966, (1989)  
 many more since then

The metallicity history of the s-process can be quite complicated.

In the simplest case in massive stars with a  $^{22}\text{Ne}$  neutron source it is independent of metallicity until quite low values of  $Z$ .

At very low  $Z$  things can become complicated because of the effect of neutron poisons,  $^{12}\text{C}$  and  $^{16}\text{O}$ . In AGB stars the mixing between H and He shells is  $Z$  dependent. Some metal poor stars are actually very s-process rich. Cristallo et al (*ApJ*, 696, 797 (2009)).