3. Introductory Nuclear Physics – 1; The Liquid Drop Model

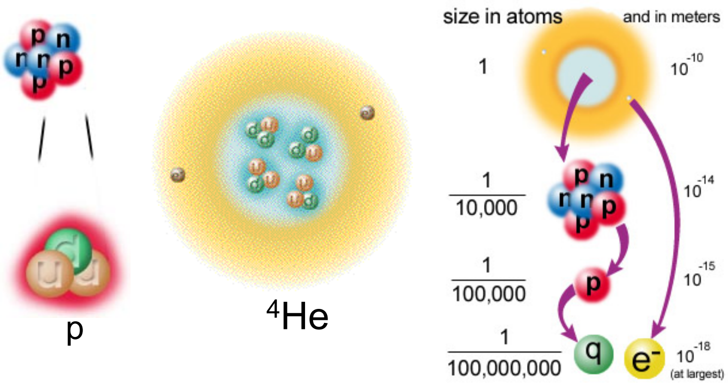
Each nucleus is a bound collection of N neutrons and Z protons. The *mass number* is A = N + Z, the *atomic number* is Z and the nucleus is written with the elemental symbol for Z

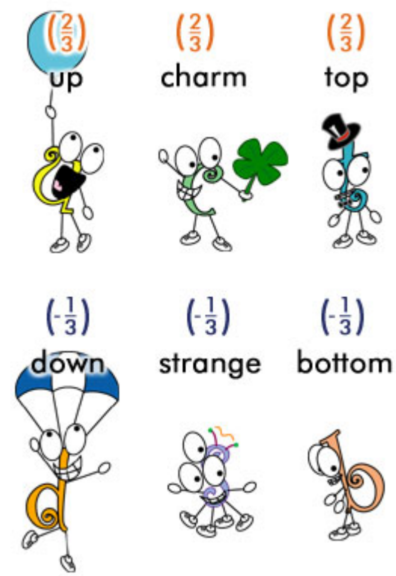
$$\boxed{^{A}[Z]}$$

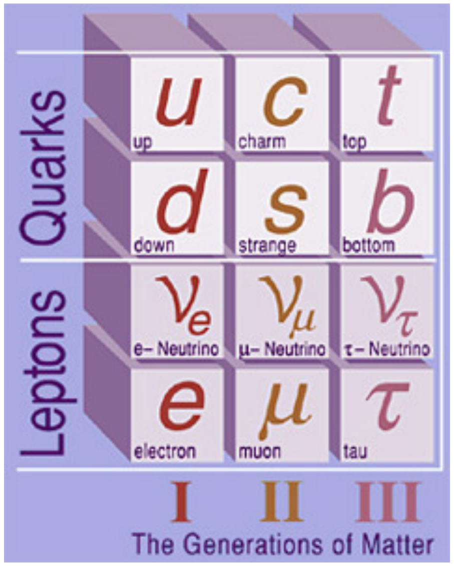
E.g. ^{12}C, ^{13}C, ^{14}C are isotopes of carbon all with Z = 6 and neutron numbers N = 6, 7, 8

The neutrons and protons are bound together by the *residual strong or color force*

In fact, the neutron and proton are themselves collections of smaller fundamental *quarks*.







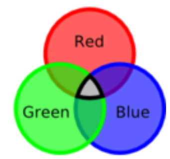
In addition there is a collection of bosons whose exchange mediates the four fundamental forces.

γ, W^{+-}, Z^0, 8 gluons, graviton

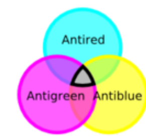
Only quarks and gluons experience the "color" force and quarks are never found in isolation

In the standard model ….

Hadrons are collections of three quarks (baryons) or a quark plus an anti-quark (mesons). This way they are able to satisfy a condition of color neutrality. Since there are three colors of quarks, the only way to have neutrality is to have one of each color, or one plus an antiparticle of the same (anti-)color.



particle



anti-particle

Quarks are bound together by *gluons* - massless bosons with spin 1, like the photon, but unlike the photon that has no charge, gluons carry color

Each gluon is a *linear combination* (given by a Hermitian, trace-free, 3x3 matrix) of color-anticolor base states (i.e., SU(3))

red antired	red antigreen	red antiblue
green antired	green antigreen	green antiblue
blue antired	blue antigreen	blue antiblue

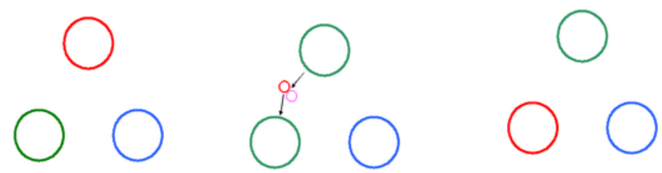
You might expect 9 combinations, but the trace free nature of the matrix means there are only 8. The combination red-antired+blue-antiblue+green-antigreen would allow interactions among hadrons that did not change color and are not observed

http://math.ucr.edu/home/baez/physics/ParticleAndNuclear/gluons.html

http://en.wikipedia.org/wiki/Gluon

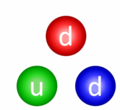
The color force only affects quarks and gluons.

The color force binds the quarks in the hadrons



A red quark emits a red-antigreen gluon which is absorbed by a green quark making it red. This is going on all the time in the neutron

http://en.wikipedia.org/wiki/Color_charge



All observable particles must be color neutral but gluons are not observable. They do not have to be color neutral (and aren't in the standard model)

Mesons are quark-antiquark pairs and thus carry net spin of 0 or 1. They are Bosons while the baryons are Fermions. They can thus serve as coupling particles. Since they are made of quarks they experience both strong and weak interactions. The lightest two mesons consist only of combinations of u, d, $\bar{u}$, and $\bar{d}$ and are

The rho meson with a mass of about 770 MeV can be considered to be an excited state of the pi meson. It has the same quark content.

Name	Made of	Charge	Mass*c^2	τ (sec)
π^{o}	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	0	135 MeV	8.4(-17)
$\pi^{\pm}$	$u\bar{d},\ d\bar{u}$	± 1	139.6	2.6(-8)

$\pi^{\pm} \rightarrow \mu^{\pm}+\nu_{\mu}$ $\qquad$ $\pi^{0} \rightarrow 2\gamma$, occasionally $e^{+}+e^{-}$

There are many more mesons. Exchange of these lightest mesons give rise to a force that is complicated, but attractive. But at a shorter range, many other mesons come into play, notably the rho meson (776 MeV), and the nuclear force becomes repulsive.

The *weak interaction* allows heavier quarks and leptons to decay into lighter ones. E.g.,

$n \rightarrow p+e^{-}+\bar{\nu}_{e}$ $\qquad$ $udd \rightarrow uud$

$d(Z=-1/3) \rightarrow u(Z=2/3)+e^{-}+\bar{\nu}_{e}$

and if energy is provided

$p+e^{-} \rightarrow n+\nu_{e}$

For background on all this, read

http://particleadventure.org

"The standard model"

There are two ways of thinking of the strong force - as a residual color interaction (like a van der Waal's force) or as the exchange of mesons. Classically the latter was used.

Exchange of (pseudo)scalar mesons like the π meson give rise to an attractive force Exchange of vector mesons like the ρ and ω give rise to a repulsive force. At very short distances one has to consider the exclusion principle for the quarks

http://en.wikipedia.org/wiki/List_of_mesons

In this class we will follow a "classic" description of nuclear physics in which the nucleus is a set of nucleons bound together by the exchange of mesons. The mass of the lightest meson sets the range of the nuclear force. (nb. $\hbar c$ = 197.33 MeV fm; 1 fm = 10^{-13} cm)

$$\frac{\hbar c}{Mc^2} = \frac{197.33\text{MeV fm}}{Mc^2} \approx 1 \text{ fm}$$

$\Delta E \; \Delta t \approx \hbar$

$Mc^2 \; \ell / c \approx \hbar$

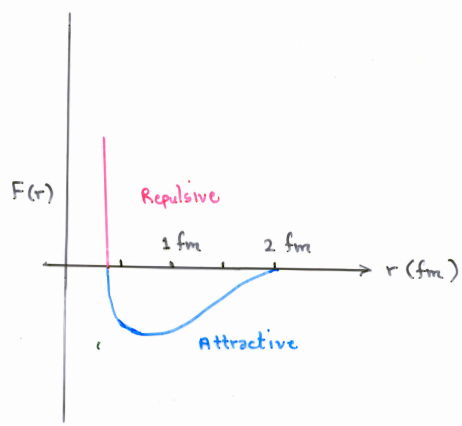
Beyond this distance the nuclear force falls abruptly to zero. The exchange of π mesons gives rise to an attractive force, but the exchange of some of the heavier ones gives a repulsive force at short range. The density of the atomic nucleus is known from a variety of experiments that constrain its radius. The density of nuclear matter is surprisingly constant, once one gets to sufficiently heavy nuclei that edge effects are not important.

$\boxed{n = 0.17 \text{ fm}^{-3} \text{ or about } 2.9 \times 10^{14} \text{ g cm}^{-3}.}$

The nuclear force at large distances is not just small, it is zero.

Repulsive at short distances.

Nuclear density nearly constant.



- The nuclear force is only felt among hadrons.
- At typical nucleon separation (1.3 fm) it is a very strong attractive force.
- At much smaller separations between nucleons the force is very powerfully repulsive, which keeps the nucleons at a certain average separation.
- Beyond about 1.3 fm separation, the force exponentially dies off to zero. It is greater than the Coulomb force until about 2.5 fm
- The *NN* force is nearly independent of whether the nucleons are neutrons or protons. This property is called *charge independence or isospin independence*.
- The *NN* force depends on whether the spins of the nucleons are parallel or antiparallel.
- The *NN* force has a noncentral or tensor component.

Since the nucleons are Fermions they obey FD statistics

$$p_0 = \left(\frac{3h^3}{8\pi} n\right)^{1/3}$$

where n = $\dfrac{0.17}{2}$ fm^{-3} $= 8.5\times10^{37}$ cm^{-3} is the density of n or p.

Here h = 6.626×10^{-27} erg s . This implies a speed for the nucleons of about c / 4. and a peak Fermi energy, $\varepsilon_F = \dfrac{p_0^2}{2M} = 39$ MeV.

The average Fermi energy is 3/5 of this

$$\boxed{\langle \varepsilon_F \rangle = 23 \text{ MeV per nucleon}}$$

As we shall see shortly, Coulomb energies are much smaller than this. *To zeroth order the nucleus is a degenerate gas of nucleons confined by the (residual) strong force.*

Also, because of its short range, smaller than a typical nucleus, nucleons only interact with their nearest neighbors. Experimentally, we see the effect of "saturation". Nuclear binding energy goes very nearly linearly in A, at least for A $\gtrsim$ 4. Recall that gravitational and electrical binding energy depend on M^2 and e^2 respectively. The minimum energy state is a sphere. The largest known deformation is ^{176}Lu, about 20%. The nuclear volume is also linear in A. Hence, for $A \gtrsim 12$, the radius is $\propto A^{1/3}$ and the density constant to 10%. Specifically

$$\boxed{R \propto A^{1/3}}$$

Nuclear density is a constant.

Deformation is an indication of nuclear rotation

$$\boxed{R \approx 1.12\, A^{1/3}\ \mathrm{fm}}$$

The nuclear crossing time for a typical nucleon, $\tau \sim 10R/c \sim 10^{-22}$ s sets the time scale for the shortest nuclear reactions.

The electrical force between charged nucleons is

$$E_{\mathrm{Coul}} = \frac{e^2}{R} = \frac{1.44\ \mathrm{MeV}}{R(\mathrm{fm})} << \epsilon_F$$

for any one nucleon. However, since electrical energy rises as Z^2 and nuclear binding goes only as A, for large mass nuclei, the electrical force does become non-negligible.

The nuclear force is independent of charge and is the same between neutrons and protons. However it does depend on spin and orientation. The triplet state ($\uparrow\uparrow$) [1] of two

[1] Actually the triplet state is three states - two $\uparrow\uparrow$ states and a linear combination of the two $\uparrow\downarrow$ states

nuclear force is spin dependent

nucleons has different binding (stronger) that the singlet state ($\uparrow\downarrow$). An important astrophysical example of this is the deuteron, ^{2}H. The triplet state ($J = 1^+$) is bound. The di-neutron, n$\uparrow$n$\downarrow$ and di-proton, p$\uparrow$p$\downarrow$, are not (note that p$\uparrow$p$\uparrow$ and n$\uparrow$n$\uparrow$ are forbidden by the Pauli exclusion principle).

<u>The binding energy of a nucleus:</u>

- the energy available to hold nucleus together

- For a bunch of well-separated nucleons: the binding energy is zero
- Bring them together: strong force glues them together. However, energy has to come from somewhere: binding energy must come from a **<u>reduction</u>** in nuclear mass

Formally, it is the difference between mass of component protons and neutrons and that of actual nucleus, related through $E = mc^2$:

$$\boxed{BE(A,Z) = Z\, m_p c^2 + N\, m_n c^2 - M(A,Z)\, c^2}$$

nb. BE(p) = 0
BE(n) = 0

Binding energy is a **<u>positive quantity</u>**
(even though the strong potential in which the nucleons sit is **<u>negative</u>**)

<u>Binding energy per nucleon</u>

→ the average energy state of nucleon is a sum of high energy "surface" nucleons with low energy "bulk" nucleons
→ nucleus minimizes energy by minimizing surface area – a sphere

Unless weak interactions are involved (to be discussed later), the total energy released or absorbed in a given reaction

$$Q = \sum BE(species\,out) - \sum BE(species\,in)$$

This is measured in MeV where
1 MeV = 1.6022 x 10^{-6} erg.

Semi-Empirical Mass Formulae

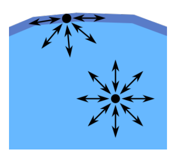
- A phenomenological understanding of nuclear binding energies as function of A, Z and N.
- Assumptions:
  - Nuclear density is constant.
  - We can model effect of short range attraction due to strong interaction by a liquid drop model.
  - Coulomb corrections can be computed using electro magnetism (even at these small scales)
  - Nucleons are fermions at T=0 in separate wells (Fermi gas model ➔ asymmetry term)
  - QM holds at these small scales ➔ pairing term
  - Nuclear force does not depend on isospin

Liquid Drop Model

- Phenomenological model to understand binding energies.
- Consider a liquid drop
  - Ignore gravity and assume no rotation
  - Intermolecular force repulsive at short distances, attractive at intermediate distances and negligible at large distances ➔ constant density.
  - *n*=number of molecules, *T*=surface tension, *BE*=binding energy *E*=total energy of the drop, *a*,*b*=free constants

$E=-an + 4\pi R^2 T$ ➔ $BE=an-bn^{2/3}$

surface area ~ $n^{2/3}$



- Analogy with nucleus
  - Nucleus has constant density
  - From nucleon-nucleon scattering experiments we know:
    - Nuclear force has short range repulsion and is attractive at intermediate distances and negligible at large distances.
  - Assume charge independence of nuclear force, neutrons and protons have same strong interactions ➔check with experiment (Mirror Nuclei!)

Volume and Surface Term

- If we can apply the liquid drop model to a nucleus
  - constant density
  - same binding energy for all constituents
- Volume term: $B_{Volume}(A) = +aA$
- Surface term: $B_{Surface}(A) = -bA^{2/3}$

a ~ 15 MeV
b ~ 17 MeV

- Since we are building a phenomenological model in which the coefficients a and b will be determined by a fit to measured nuclear binding energies we must include further terms that depend on A (or Z)

Coulomb Energy

- The nucleus is electrically charged with total charge *Ze*
- Assume that the charge distribution is spherical and homogeneous and compute the reduction in binding energy due to the Coulomb interaction

$$E_{Coulomb} = \int_0^{Ze} \frac{Q(r)}{r} dQ \qquad Q(r) = Ze(r/R)^3 \quad dQ = 3Zer^2 / R^3 dr$$

to change the integral to *dr* ; *R*=outer radius of nucleus

$$E_{Coulomb} = \int_0^R \frac{3(Ze)^2}{r} \frac{r^5}{R^6} dr = (3/5)\frac{(Ze)^2}{R}$$

includes self interaction of last proton with itself. To correct this replace Z^2 with $Z*(Z-1)$

... and remember $R=R_0A^{1/3}$

$$B_{Coulomb}(Z,A) = -c\,\frac{Z*(Z-1)}{A^{1/3}}$$

Mirror Nuclei

- Compare binding energies of mirror nuclei (nuclei with n⟷p). Eg $^7_3$Li and $^7_4$Be.
- If the assumption of isospin independence holds the mass difference should be due to n/p mass difference and Coulomb energy alone.
- From the previous page

$$\Delta E_{coulomb}(Z,Z-1) = \frac{3}{5}\frac{e^2}{R}[Z(Z-1)-(Z-1)(Z-2)] = \frac{3}{5}\frac{e^2}{R}2(Z-1)$$

$Z \sim A/2 >> 1;\ R = R_0 A^{1/3}$ to find that $\Delta E_C(Z,Z-1) \propto A^{2/3}$

- Now lets measure mirror nuclei masses, assume that the model holds and derive $\Delta E_{Coulomb}$ from the measurement.
- This should show an $A^{2/3}$ dependence

"Charge symmetry"

nn and **pp** interaction same (apart from Coulomb)

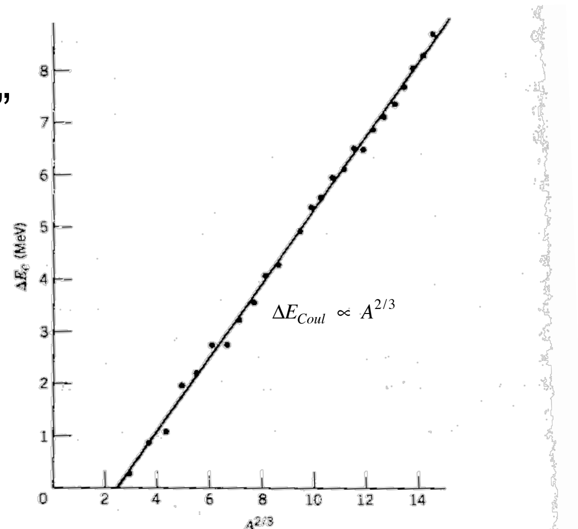
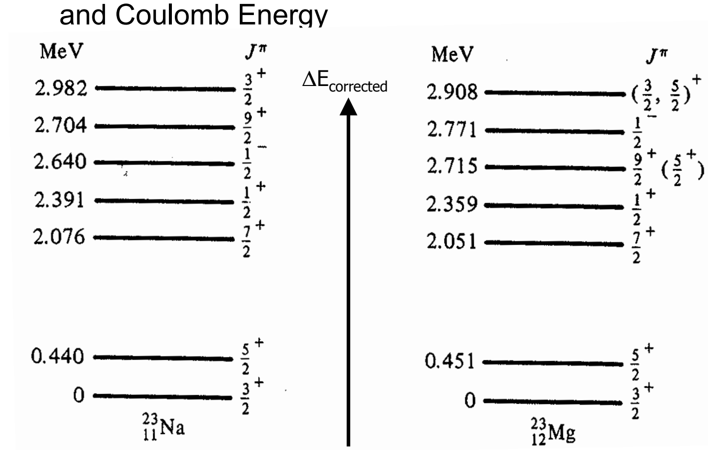


Figure 3.10 Coulomb energy differences of mirror nuclei. The data show the expected $A^{2/3}$ dependence, and the slope of the line gives R_0 = 1.22 fm.

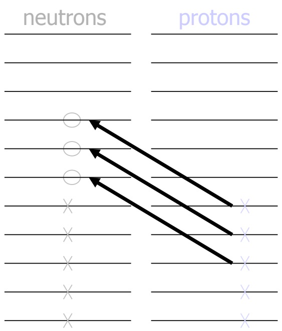
More charge symmetry

Energy Levels of two mirror nuclei for a number of excited states. Corrected for n/p mass difference and Coulomb Energy



Asymmetry Term

- Neutrons and protons are spin ½ fermions ➔ obey Pauli exclusion principle.
- If all other factors were equal nuclear ground state would have equal numbers of n & p.



Illustration

- n and p states with same spacing Δ.
- Crosses represent initially occupied states in ground state.
- If three protons were turned into neutrons
- the extra energy required would be 3×3 Δ.
- In general if there are Z-N excess protons over neutrons the extra energy is ~ ((Z-N)/2)² Δ. relative to Z=N.

$$E_{\mathrm{Sym}} \propto \frac{(Z-N)^2}{A}$$

Why is this so?

$$n(p)\,dp = \frac{2}{h^3} 4\pi p^2\,dp$$

See Clayton 2-22 for electrons.
In general, the expression on the right is multiplied by P(p), the occuparion index, i.e., the Fermo-Dirac distribution function. Here it is assumed that the gas is totally degenerate and P(p) = 1 up to p_0

$$\begin{aligned} <\epsilon_F> &= \frac{\int_0^{p_0} \frac{p^2}{2m}\frac{2}{h^3} 4\pi p^2\,dp}{\int_0^{p_0} \frac{2}{h^3} 4\pi p^2\,dp} \\ &= \frac{1}{2m}\frac{\int_0^{p_0} p^4\,dp}{\int_0^{p_0} p^2\,dp} \\ &= \frac{3}{5}\left(\frac{p_0^2}{2m}\right) \\ &= \frac{3}{5}\epsilon_F^{\max} \end{aligned}$$

Suppose we have A nucleons (an even number for now). Perturb away from the state Z = N = A/2. So

$$Z' = \frac{A}{2}(1-\lambda) \qquad N' = \frac{A}{2}(1+\lambda)$$

$$N' + Z' = A \qquad \lambda = \frac{N'-Z'}{A}$$

Treat the neutrons and protons as separate fluids.

$$\begin{aligned} \Delta E &= N' <\epsilon_F(N'\text{neutrons}> \\ &+ Z' <\epsilon_F(Z'\text{protons})> \\ &- 2A/2 <\epsilon_F(A/2)> \\ &= \frac{3}{5}\,(N'\epsilon_F^{\max}(N') + Z'\epsilon_F^{\max}(Z') \\ &\quad -2A/2\,\epsilon_F^{\max}(A/2)) \end{aligned}$$

$$\varepsilon_F^{\max} = \frac{p_0^2}{2m}$$

$$p_0 = \left(\frac{3h^3}{8\pi}n\right)^{1/3}$$

Now $\epsilon_F^{\max} = (p_0^2/2m)$, the maximum Fermi energy per nucleon, depends upon $p_0^2 \propto n^{2/3}$ and since n is the number of nucleons of a given type per unit volume and volume is $\propto A$, $n_{Z'} \propto Z'/A$ and $n_{N'} \propto N'/A$ so that

$$\begin{aligned} \Delta E &\propto Z' n_{Z'}^{2/3} + N' n_{N'}^{2/3} - 2(\frac{A}{2}) n_{A/2}^{2/3} \\ &\propto \frac{A}{2}[(1-\lambda)\left(\frac{(1-\lambda)A}{2A}\right)^{2/3} \\ &+ (1+\lambda)\left(\frac{(1+\lambda)A}{2A}\right)^{2/3} - 2(\frac{A}{2A})^{2/3}] \\ &\propto \frac{A}{2}\left[(1-\lambda)^{5/3}) + (1+\lambda)^{5/3} - 2\right]. \end{aligned}$$

$$Z' = \frac{A}{2}(1-\lambda)$$

$$N' = \frac{A}{2}(1+\lambda)$$

Since λ is a small number we can use the binomial expansion theorem to get an approximate answer. Note that quadratic terms must be included.

$$\begin{aligned} \Delta E &\propto A\lambda^2 \\ &\propto \frac{(N'-Z')^2}{A} \end{aligned}$$

$$(1+\lambda)^n \approx 1 + n\lambda + \frac{n(n-1)}{2!}\lambda^2 + \ldots$$

$$\lambda = \frac{N'-Z'}{A}$$

The proportionality constant is about 28 MeV

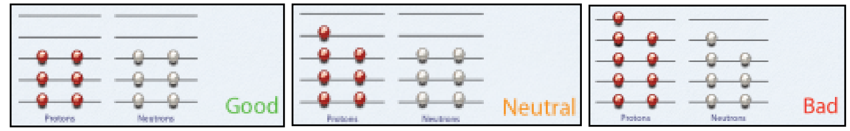
So far we have

$$BE = a\,A - b\,A^{2/3} - c\frac{(N-Z)^2}{A} - d\frac{Z^2}{A^{1/3}}$$

Spin pairing in the liquid drop model:

Spin pairing favours pairs of fermionic nucleons (similar to electrons in atoms)
i.e. a pair with opposite spin have lower energy than pair with same spin

Best case: even numbers of both protons and neutrons
Worst case: odd numbers of both protons and neutrons
Intermediate cases: odd number of protons, even number of neutrons or vice versa



→ Subtract small energy d required to decouple nucleons from binding energy:

$$\begin{aligned}\delta &= +a_p A^{-1/2} && \text{for both N \& Z odd}\\ &= 0 && \text{for N even, Z odd / Z even, N odd}\\ &= -a_p A^{-1/2} && \text{for both N \& Z even}\end{aligned}$$

→ a_p collects constants, $A^{-1/2}$ dependence provides best empirical fit to data

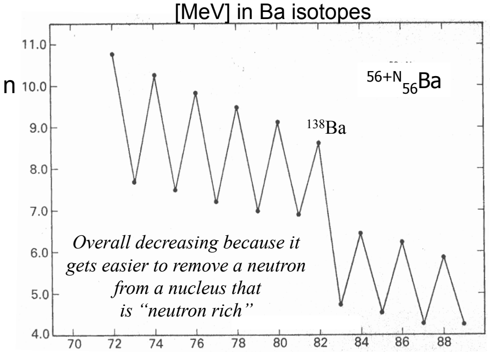
→ subtracting δ **reduces** BE for N and Z both odd

→ subtracting δ **adds** small amount to BE for N and Z both even

Pairing Term

- Nuclei with even number of n or even number of p more tightly bound then with odd numbers.
- Only 4 stable o-o nuclei but 153 stable e-e nuclei.
- But note also closed shell effect at N = 82

Neutron separation energy [MeV] in Ba isotopes



Neutron number

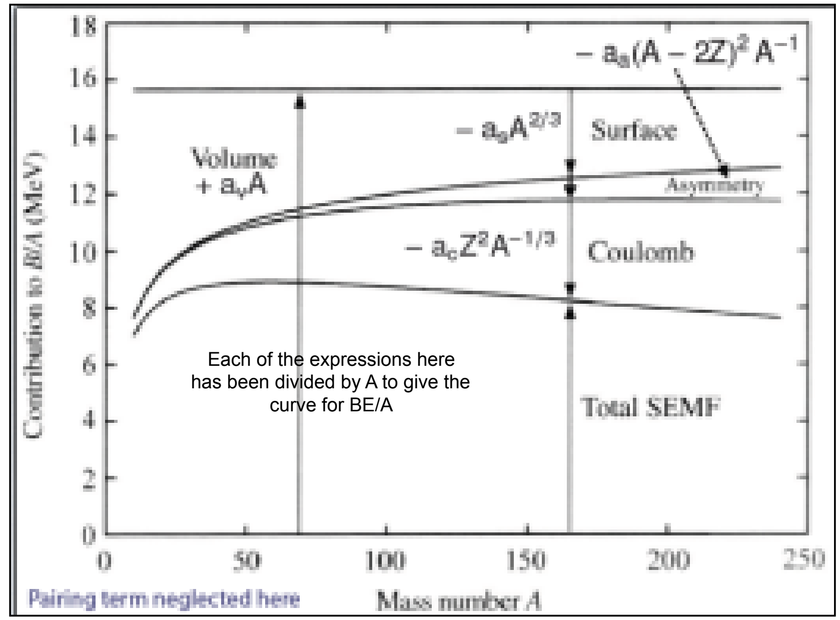
Similar to atoms, we shall find that nuclei have closed shells that are particularly sable, the nuclear equivalent of the noble gases. One such stable nucleus is ^{4}He; another is ^{16}O. Removing a nucleon from one of these "magic nuclei" with a closed shell takes more energy than a "valence" nucleon. E.g., it is easier to remove a neutron from ^{17}O than from ^{16}O (even accounting for the odd-even effect).

The pairing and shell corrections are purely empirical quantum mechanical corrections (for now) to the liquid drop model

Putting it all together:

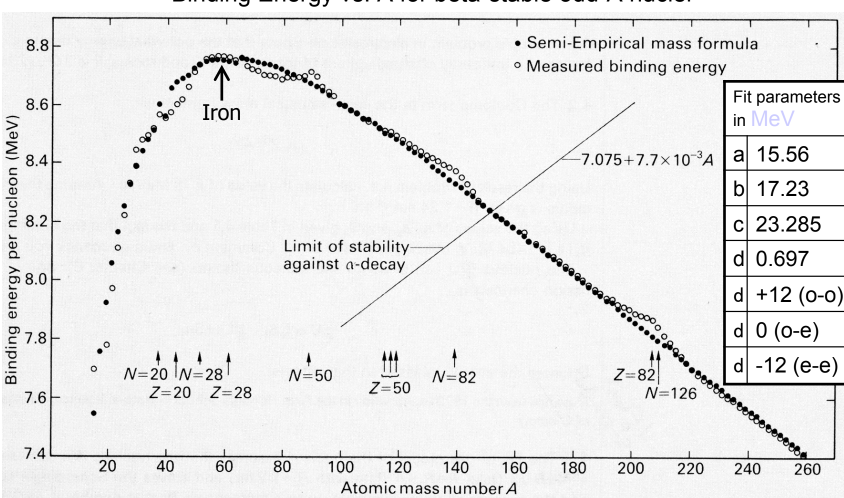
$$BE \approx a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(Z-N)^2}{A} - \delta(A) - S(A)$$

where terms 1 through 4 are the volume, surface, Coulomb, and symmetry energies respectively, $\delta(A)$ is the pairing correction, and $S(A)$ is the shell correction. Without the last two terms which are strictly quantum mechanical, this is known as the "liquid drop model" or the Bethe-Weizsäcker semi-empirical mass formula. Empirically, and crudely from fitting to known binding energies (deShalit and Feshbach, *Theoretical Nuclear Physics*, p. 126), we have $a_1 = 15.68$ MeV, $a_2 = 18.56$ MeV, $a_3 = 0.717$ MeV, and $a_4 = 28.1$ MeV. Also $\delta(A) = +34/A^{3/4}$, 0, $-34/A^{3/4}$ MeV for odd-Z, odd-N; odd-A; and even-Z, even-N nuclei respectively. S(A) is complicated.



Semi Empirical Mass Formula

Binding Energy vs. A for beta-stable odd-A nuclei



Fit parameters in MeV	
a	15.56
b	17.23
c	23.285
d	0.697
d	+12 (o-o)
d	0 (o-e)
d	-12 (e-e)

Utility of Mass Law

- Only makes sense for A greater than about 20
- Good fit for large A (<1% in most instances)
- Deviations are interesting - shell effects
- Explains the "valley of beta-stability"
- Explains energetics of nuclear reactions
- Incomplete consideration of QM effects (energy levels not all equally spaced)

More accurate values of mass and a modern mass law can be found at http://t2.lanl.gov/nis/data/astro/molnix96/massd.html

$$BE = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N-Z)^2}{A}$$

2.1.2 Most Tightly Bound Nuclei

For a given A what will be the most tightly bound nucleus, that is what will be its Z and N?

Given A, what is the most tightly bound Z? Volume and surface energy are constant.

$$\left(\frac{\partial BE}{\partial Z}\right)_A = 0 - 0 - \frac{\partial}{\partial Z}\left(a_3 \frac{Z^2}{A^{1/3}}\right) - \frac{\partial}{\partial Z}\left(a_4 \frac{(A-2Z)^2}{A}\right)$$

N = A-Z
N-Z = A-2Z

$$= 0 = -\frac{2a_3 Z}{A^{1/3}} - \frac{a_4(2)(A-2Z)(-2)}{A}$$

Only the Coulomb and pairing terms contained Z explicitly

So that

$$Z_{\text{stable}} = \frac{2a_4 A}{a_3 A^{2/3} + 4a_4}$$

$a_3 = 0.717$
$a_4 = 28.1$

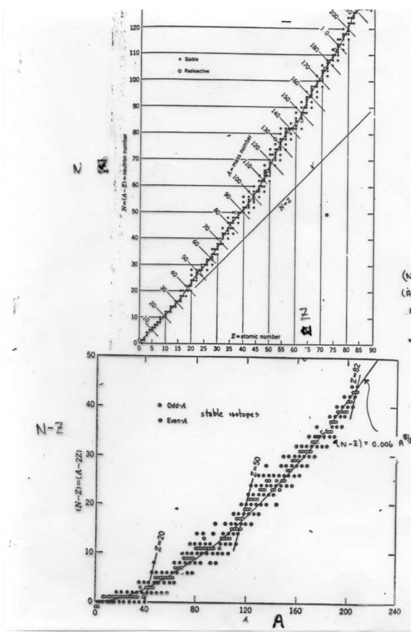
where a_3 measures Coulomb forces and a_4 measures symmetry. If A is small, ~~$a_3 A^{2/3} \ll 4a_4$~~ $a_3\, A^{2/3} << 4a_4$, then $Z_{\text{stable}} = N_{\text{stable}} = A/2$. As A increases however, $Z_{\text{stable}} < A/2 < N_{\text{stable}}$ So, nuclei like ^{12}C, ^{14}N, ^{16}O, ^{28}Si, ^{40}Ca are most tightly bound (and most abundant in nature), but the most tightly bound isotope of ion is ^{56}Fe with 26 protons and 30 neutrons. At larger A there is a trade off between the increased nuclear repulsion from a larger number of protons and the extra symmetry energy required to accommodate an excess number of neutrons. Numerically, using the values from our sample mass law,

symmetry dominates

$$\boxed{Z_{\text{stable}} = \left(\frac{56.2}{0.717 A^{2/3} + 112.4}\right) A.}$$

A	Z_s
20	9.6
40	18.6
60	27.3

The path in the Z vs. N diagram (physicist's



$$(N-Z)_{stable} = (A-2Z)_{stable}$$
$$= A\left(1 - \frac{4a_4}{a_3 A^{2/3} + 4a_4}\right)$$
$$= A\left(\frac{a_3 A^{2/3}}{a_3 A^{2/3} + 4a_4}\right)$$
$$\approx \frac{a_3}{4a_4} A^{5/3} = 0.0064\ A^{5/3}$$

$a_3 = 0.717 \quad a_4 = 28.1$

so $a_3 A^{2/3} << 4a_4$

Evans - The Atomic Nucleus - 3.4

periodic chart) is known as the "valley of beta-stability". It is the locus of the stable elements since all other isotopes can liberate energy by decaying back to this line.

What nucleus will be the most tightly bound of all, that is have the greatest BE/A? For simplicity, consider only Z = N = A/2 and neglect shell and pairing corrections.

$$\frac{BE}{A} = a_1 - \frac{a_2}{A^{1/3}} - \frac{a_3 A^2}{4A^{4/3}}$$
$$\frac{\partial}{\partial A}(BE/A) = 0 = \frac{1}{3}\frac{a_2}{A_*^{4/3}} - \frac{a_3}{6A_*^{1/3}}$$
$$A_* = \frac{2a_2}{a_3} = \frac{(2)(18.56)}{0.717} = 52$$

1 is volume
2 is surface
3 is Coulomb

The actual answer owing to shell effects is, for Z = N, $A_* = 56$, i.e., ^{56}Ni. If $Y_e \approx 0.46$ as characterizes the valley of beta stability in the iron group, then ^{56}Fe is most tightly bound. However the most tightly bound nucleus of all, in terms of BE/A, is ^{62}Ni, again because of shell effects (Z = 28 is magic).

Nucleus	BE/A	Y_e
^{56}Ni	8.643	0.500
^{56}Fe	8.790	0.464
^{62}Ni	8.794	0.452