Lecture 7

Evolution of Massive Stars on the Main Sequence and During Helium Burning - Basics

Massive Stars - Key Physics and Issues

- Nuclear Physics
- Equation of state
- Opacity
- Mass loss
- Convection
- Rotation (magnetic fields)
- Binary membership
- Explosion physics

- Evolution in HR diagram
- Nucleosynthesis
- Surface abundances
- Presupernova structure
- Supernova properties
- Remnant properties
- Rotation and B-field of pulsars

Massive Stars

Generalities:

Because of the general tendency of the interior temperature of main sequence stars to increase with mass, stars of over two solar mass are chiefly powered by the CNO cycle(s) rather than the pp cycle(s). This, plus the increasing fraction of pressure due to radiation, makes their cores convective. The opacity is dominantly due to electron scattering. Despite their convective cores, the overall main sequence structure can be crudely represented as an $n = 3$ polytrope. This is especially true of the outer radiative part of the star that typically includes the majority of the mass.

For a non-degenerate gas, the entropy is given by (Clayton 2-136)

integrate the 1st law of thermo-dynamics

$T\,dS = dU + P\,dV$

$$\boxed{S = const + \frac{N_A k}{\mu} ln(\frac{T^{3/2}}{\rho}) + \frac{4a}{3}\frac{T^3}{\rho}.}$$

For an ideal gas

$$S_0 = \frac{3}{2}\ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}$$

(Reif - Statistical Physics - 7.3.6)

The electrons are included in μ and in S_0

Ideal gas (convective with negligible radiation entropy):

$$P = \text{const} \times \rho \times T \propto \rho \times \rho^{2/3} = \rho^{5/3} = \rho^{\gamma}$$

$$\text{since } \frac{T^{3/2}}{\rho} = \text{constant}$$

$$\gamma = \frac{n+1}{n} \Rightarrow n = \frac{3}{2}$$

For non relativistic, but possibly partly degenerate electrons, the electrons are given as a separate term see Clayton 2-145.

Radiation dominated gas or a gas with constant β:

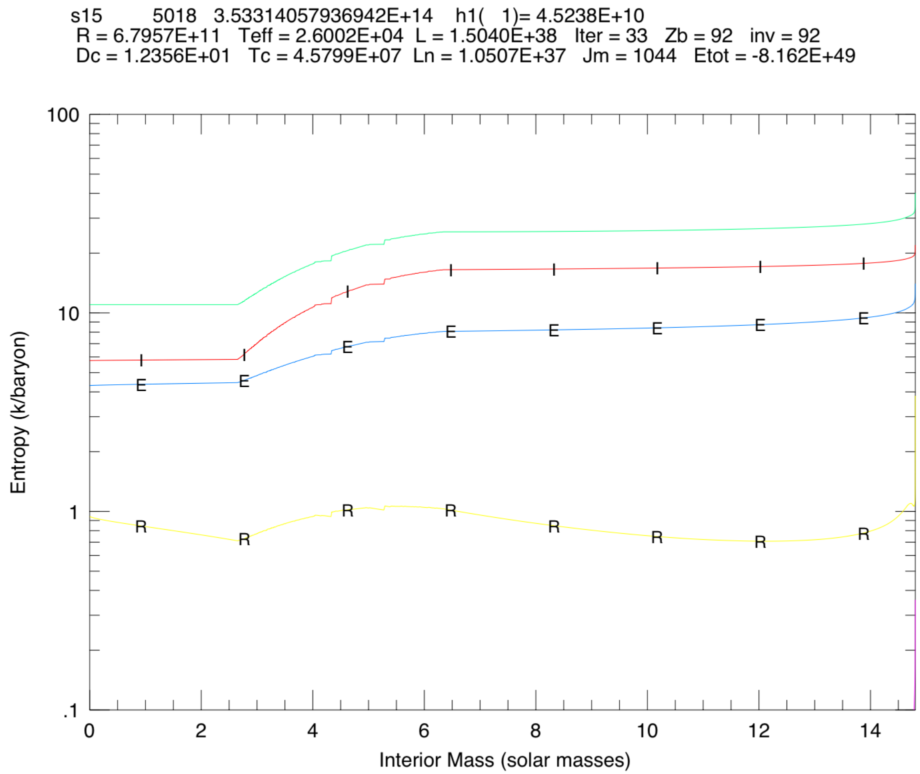
$$P = \frac{1}{3} a T^4 \propto \rho^{4/3}$$

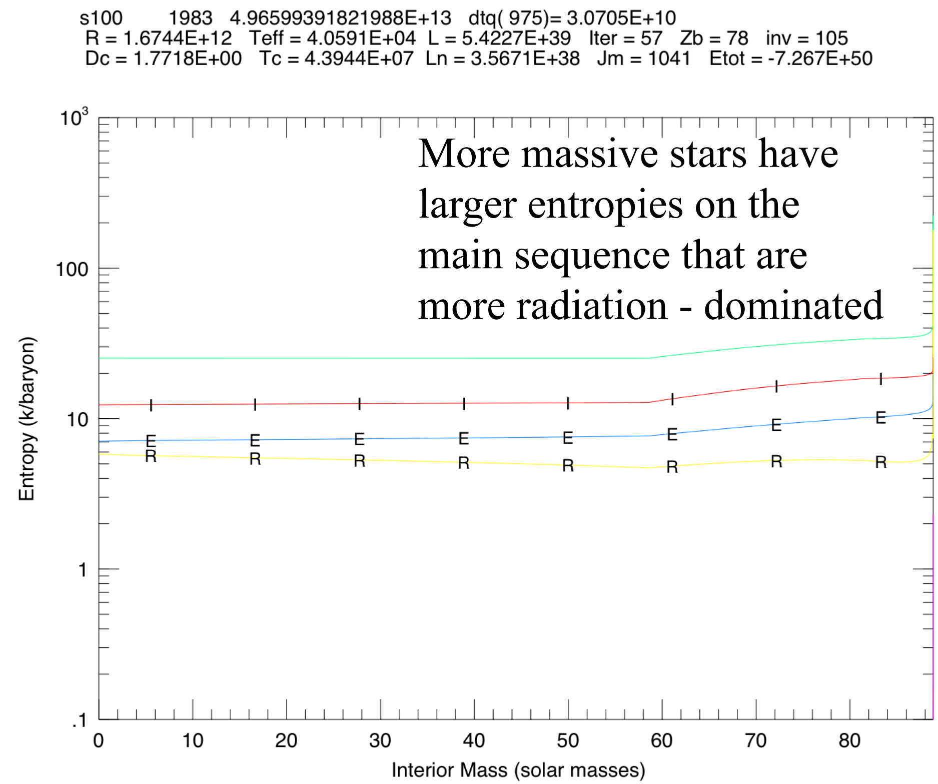
$$\text{if } \frac{T^3}{\rho} \propto \frac{P_{rad}}{P_{ideal}} = \frac{1-\beta}{\beta} = \text{constant}$$

$$\beta = \frac{P_{gas}}{P_{tot}} = \frac{P_{gas}}{P_{gas} + P_{rad}}$$

$$\gamma = \frac{n+1}{n} \Rightarrow n = 3$$

($S/N_A k$) half way through hydrogen burning 15 and 100 solar masses

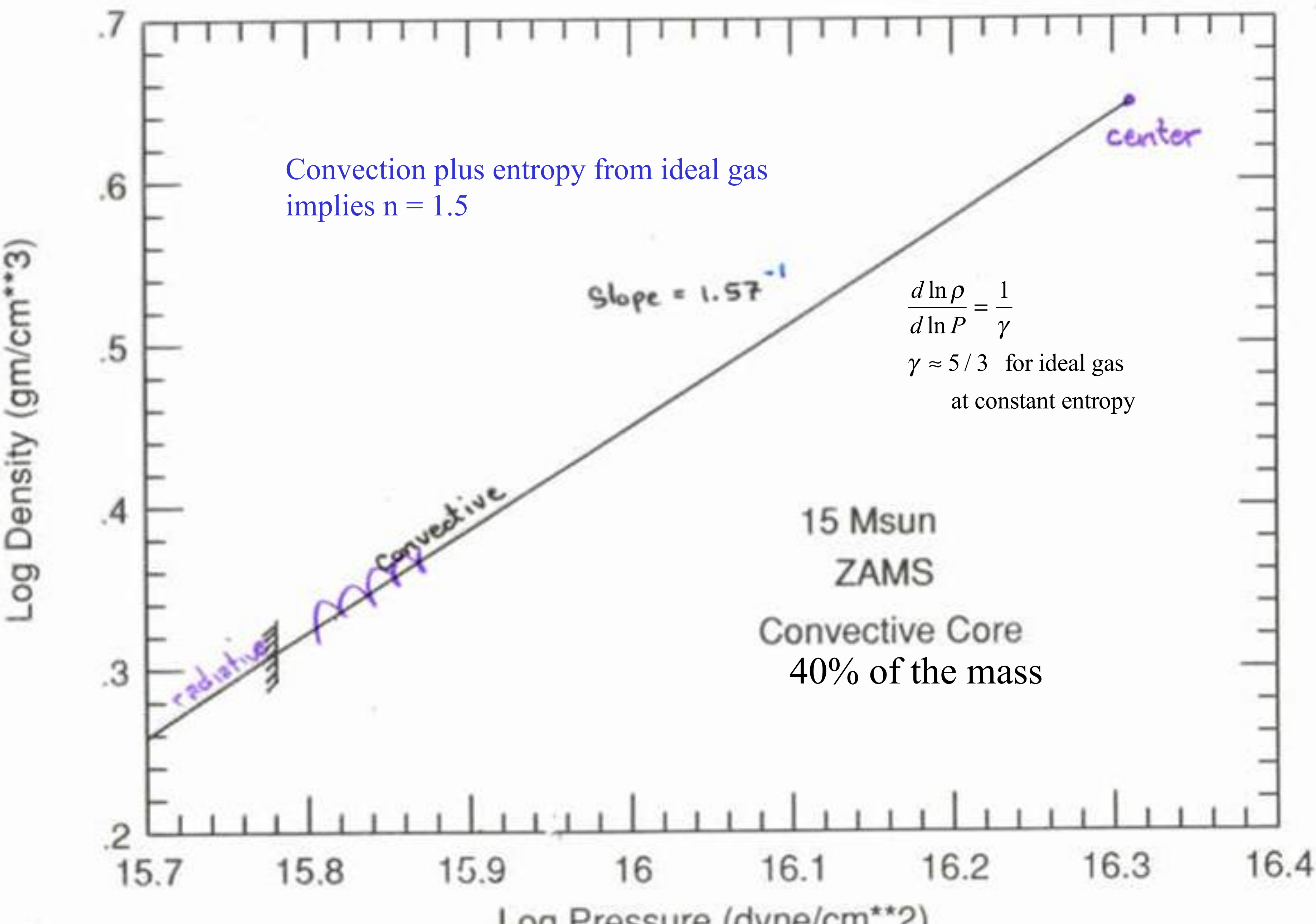




For normal massive stars, the ionic entropy always dominates on the main sequence, but for very massive stars S_{elec}, S_{rad} and S_{ionic} can become comparable.

Not surprisingly then, it turns out that massive stars are typically hybrid polytropes with their convective cores having $3 > n > 1.5$ and radiative envelopes with n approximately 3.

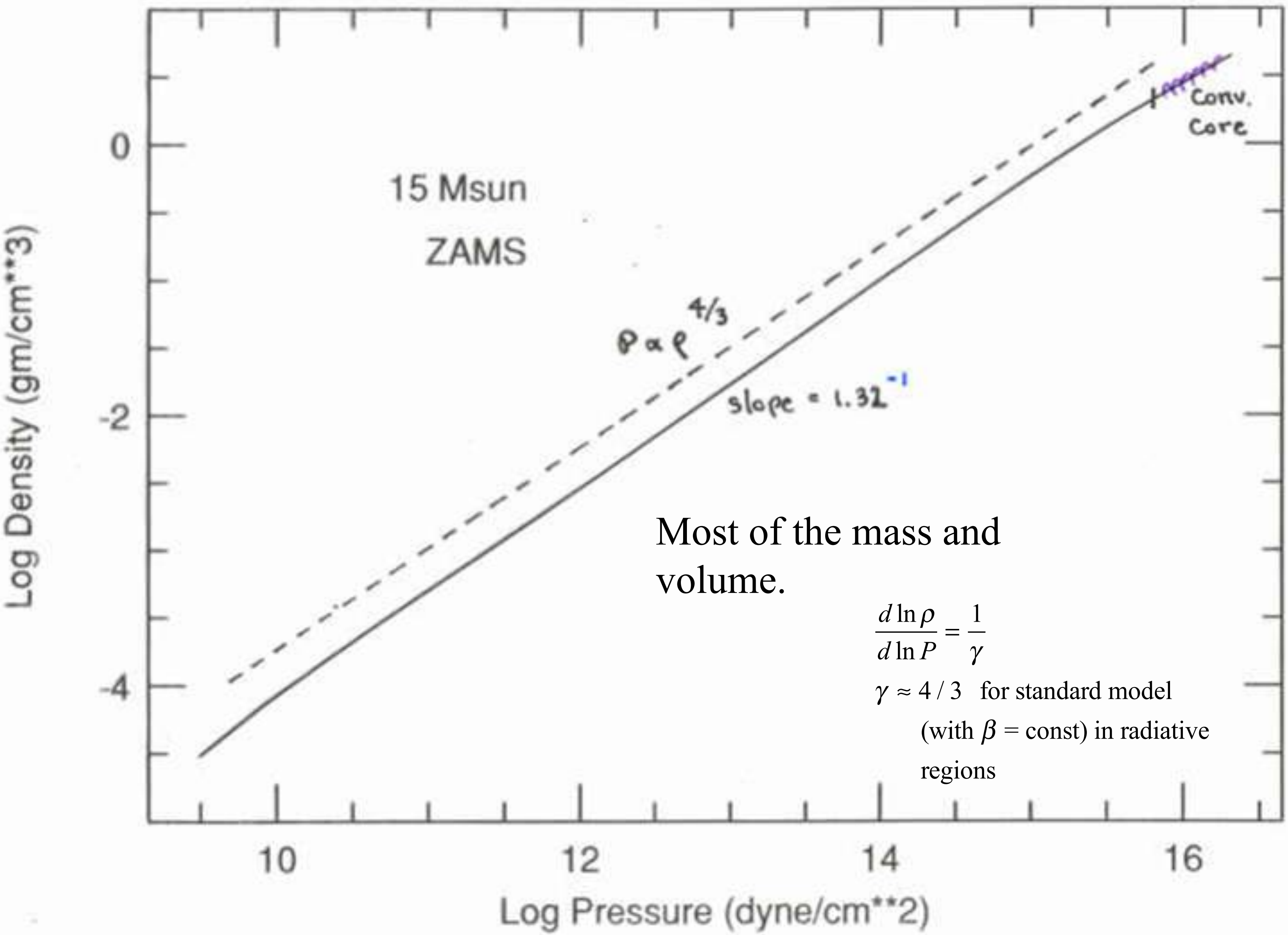
Overall $n = 3$ is not bad.


Convection plus entropy from ideal gas
implies n = 1.5
Slope = 1.57^-1
$$\frac{d\ln\rho}{d\ln P}=\frac{1}{\gamma}$$
$\gamma \approx 5/3$ for ideal gas
at constant entropy
center
convective
radiative
15 Msun
ZAMS
Convective Core
40% of the mass
Log Density (gm/cm**3)
Log Pressure (dyne/cm**2)
.7
.6
.5
.4
.3
.2
15.7
15.8
15.9
16
16.1
16.2
16.3
16.4

continuing farther out

centralized ϵ_{nuc} + X = const $\Rightarrow$ n=3

$P \propto \rho^{(n+1)/n}$



Most of the mass and volume.

$$\frac{d\ln\rho}{d\ln P} = \frac{1}{\gamma}$$

$\gamma \approx 4/3$ for standard model (with β = const) in radiative regions

Eddington's standard model (n=3)

Consider a star in which radiation pressure is important (though not necessarily dominant) and energy transport is by radiative diffusion

$$\frac{dP_{rad}}{dr}=\frac{d}{dr}\left(\frac{1}{3}aT^4\right)=\frac{4}{3}aT^3\frac{dT}{dr}$$

But for radiative diffusion, $\dfrac{dT}{dr}=\dfrac{3\kappa\rho}{16\pi acT^3}\dfrac{L(r)}{r^2}$ so

$$\frac{dP_{rad}}{dr}=-\frac{\kappa\rho}{4\pi c}\frac{L(r)}{r^2}$$

but hydrostatic equilibrium requires

$$\frac{dP}{dr}=-\frac{Gm\rho}{r^2}$$

Divide the 2 eqns

$$\frac{dP_{rad}}{dP}=\frac{\kappa L(r)}{4\pi Gmc}=\frac{L(r)}{L_{Edd}}$$

where $L_{Ed}=\dfrac{4\pi GMc}{\kappa}$

Define $\beta = \dfrac{P_{gas}}{P} = 1 - \dfrac{P_{rad}}{P}$ where $P = P_{gas} + P_{rad}$,

then $P_{rad} = P - P_{gas} = (1-\beta)\,P$ and

$$\frac{dP_{rad}}{dP} = (1-\beta) = \frac{\kappa L(r)}{4\pi Gmc} = \frac{L(r)}{L_{Edd}}$$

If, and it is a big IF, β (or 1-β) were a constant throughout the star, then one could write everywhere, including the surface

$$L(r) = (1-\beta)\; L_{Ed} \qquad \text{(Main sequence only)}$$

Fom polytropes (Clayton 155- 165)

$$M = -\frac{(n+1)^{3/2}}{\sqrt{4\pi}} \left.\xi_1^2 \frac{d\theta}{d\xi}\right)_{\xi_1} \left(\frac{K}{G}\right)^{3/2} \rho_c^{\frac{3-n}{2n}}$$

$$K = \left[\frac{3(N_A k)^4 (1-\beta)}{a(\mu\beta)^4}\right]^{1/3}$$

For n = 3 (β= constant), ρ_c drops out and this becomes

$$\beta = \frac{P_{gas}}{P_{total}}$$

$$M = -\frac{4}{\sqrt{\pi}} \xi_1^2 \overbrace{\left(\frac{d\theta}{d\xi}\right)_{\xi_1}}^{2.01824} \left(\frac{K}{G}\right)^{3/2} = 4.56 \left(\frac{K}{G}\right)^{3/2}$$

$$M = 4.56 \left[\frac{3(N_A k)^4 (1-\beta)}{a(\mu\beta)^4 G^3}\right]^{1/2}$$

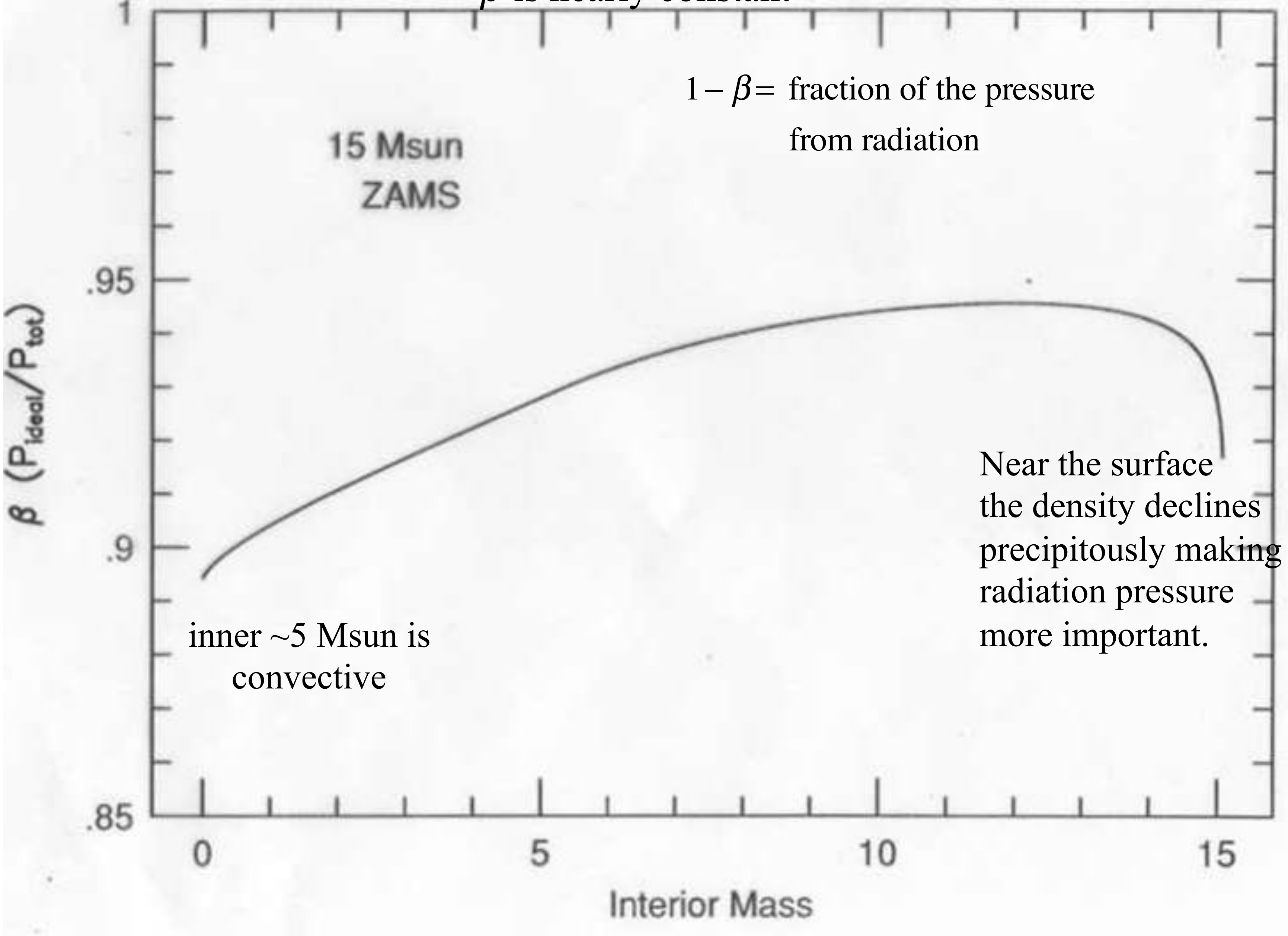
$$\lim_{\beta\to 1} M \to 0$$

$$\lim_{\beta\to 0} M \to \infty$$

Eddington's quartic equation

$$\boxed{M = \frac{18.1 M_\odot}{\mu^2} \left(\frac{1-\beta}{\beta^4}\right)^{1/2}}$$

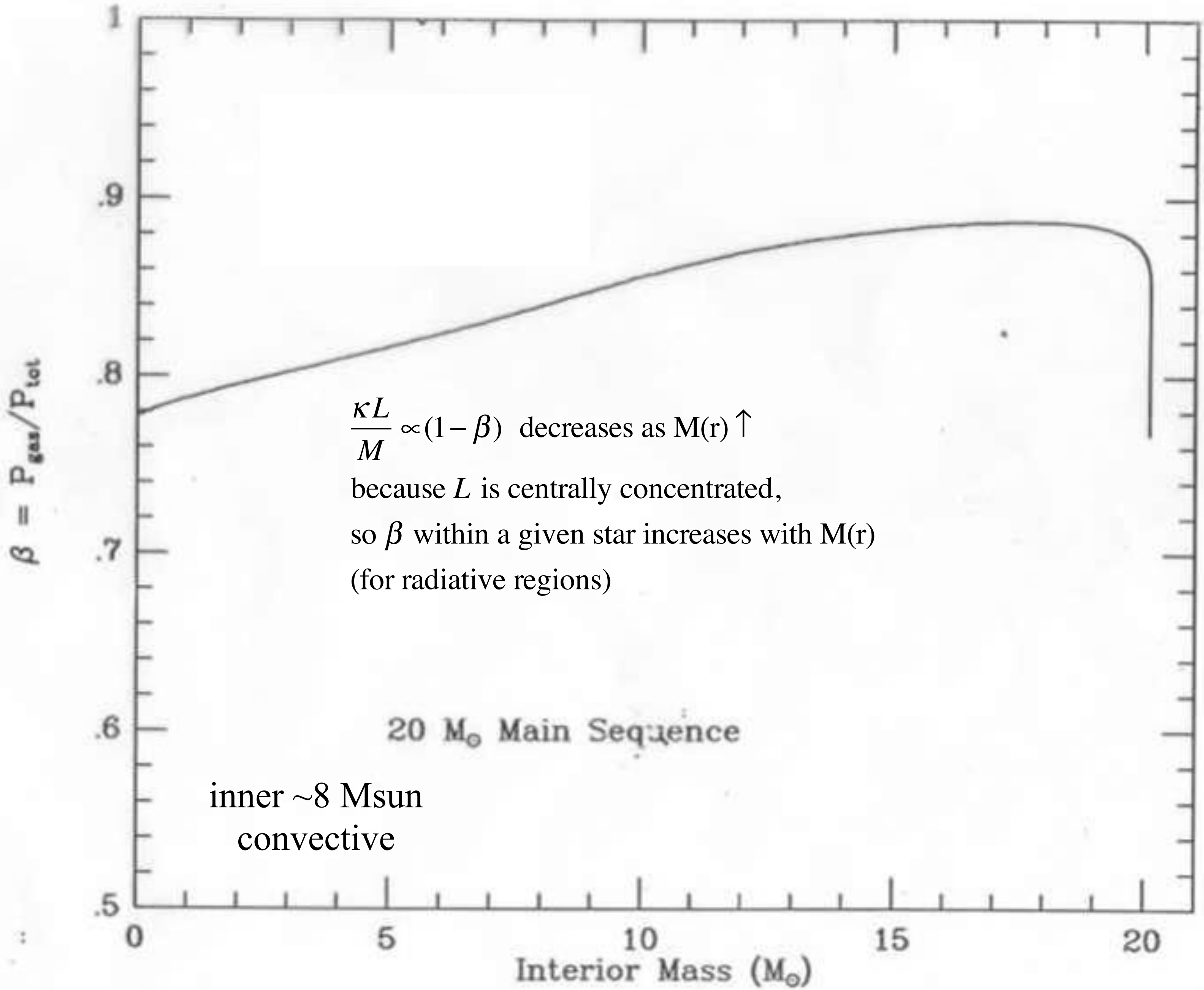
β is nearly constant



$1-\beta=$ fraction of the pressure from radiation

inner ~5 Msun is convective

Near the surface the density declines precipitously making radiation pressure more important.



$$\frac{\kappa L}{M} \propto (1-\beta)$$ decreases as M(r) ↑

because L is centrally concentrated,

so β within a given star increases with M(r)

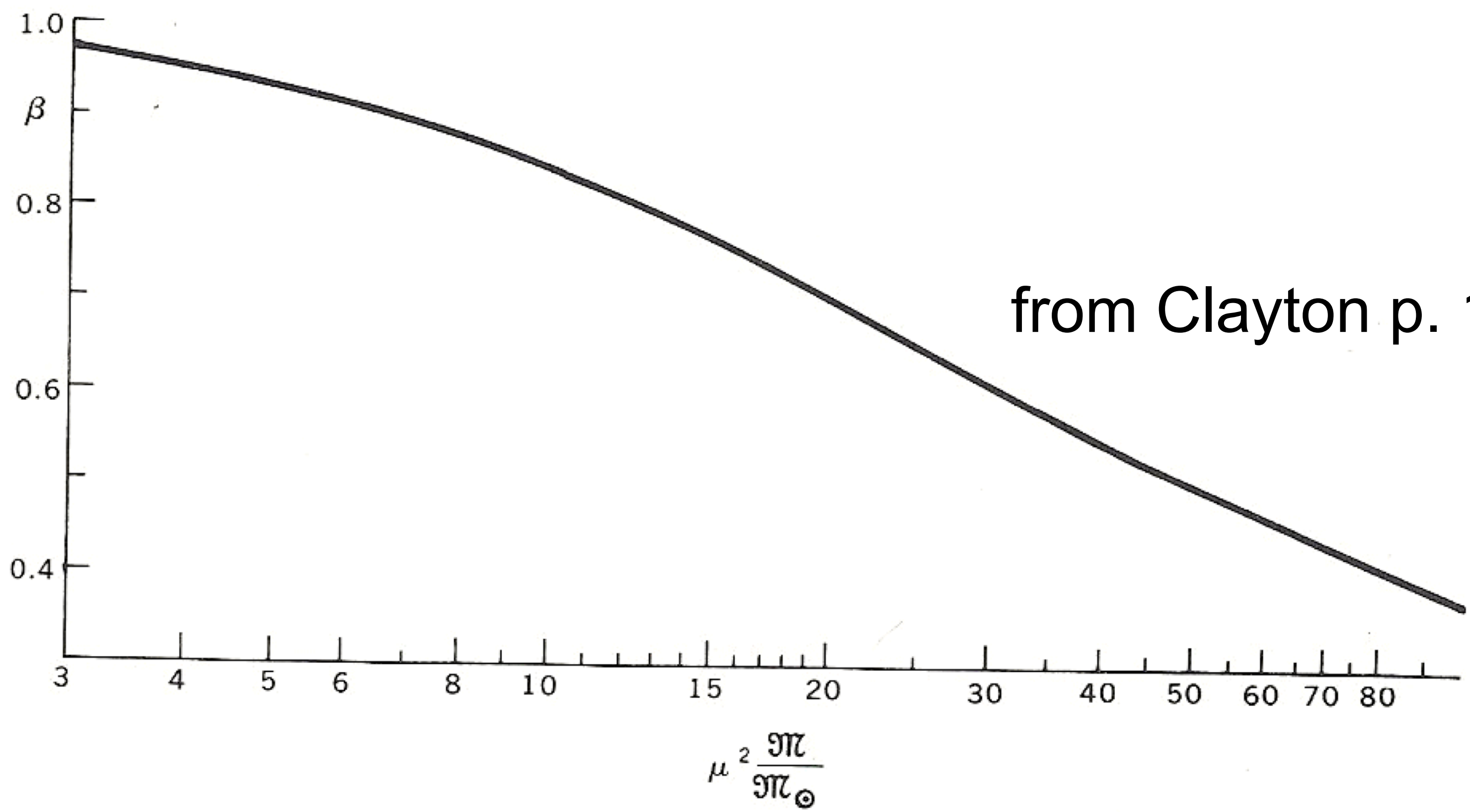
(for radiative regions)

inner ~8 Msun
convective

$$\mu = \left[\sum (1+Z_i)\ Y_i\right]^{-1} = 0.73 \text{ for } 50\% \text{ H}, 50\% \text{ He}$$

$$0.64 \text{ for } 75\% \text{ H}, 25\% \text{ He}$$

For 20 $M_\odot$ $\beta \approx 0.80 - 0.85$ $\mu^2 M \approx 11$



from Clayton p. 163

$$M = \frac{18.1\,M_\odot}{\mu^2}\left(\frac{1-\beta}{\beta^4}\right)^{1/2}$$

$$1-\beta = 4.13\times10^{-4}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\mu}{0.61}\right)^4\beta^4 \quad \text{and since}$$

$$L(r) = (1-\beta)\,L_{Edd}$$

$$L = \left(\frac{aG^3}{3(N_A k)^4}\right)\frac{\pi}{16\xi_1^4\left(d\theta/d\xi\right)^2_{\xi_1}}\mu^4\beta^4M^2\frac{4\pi Gc}{\kappa}M$$

$$= \frac{\pi^2}{12\xi_1^4\left(d\theta/d\xi\right)^2_{\xi_1}}\left(\frac{acG^4}{(N_A k)^4}\right)\left(\frac{\mu^4\beta^4}{\kappa}\right)M^3$$

$$\boxed{= 5.5\,\beta^4\left(\frac{\mu}{0.61}\right)^4\left(\frac{1\ \mathrm{cm^2\ g^{-1}}}{\kappa_{surf}}\right)\left(\frac{M}{M_\odot}\right)^3 L_\odot}$$

Mass luminosty Relation

where κ_{surf} is the value of the opacity near the surface.

This was obtained with no mention of nuclear reactions.

For M not too far from $M_\odot$ β is close to 1 and L $\propto$ M^3.

At higher masses however the mass dependence of β becomes important. Eventually $\beta^4 \propto M^{-2}$ so that L $\propto$ M. In fact, the luminosity of very massive stars approaches the Eddington limit as $\beta \to 0$ (L(r) = $\left(1-\beta\right)L_{Edd}$)

$$L_{Edd} = \frac{4\pi GMc}{\kappa} = 1.47 \times 10^{38}\ \mathrm{erg\ s^{-1}} \left(\frac{M}{M_\odot}\right)\left(\frac{0.34}{\kappa}\right)$$

For n = 3 one can also derive useful equations for the central conditions based upon the original polytropic equation for mass

$$M = -4\pi\alpha^3\rho_c\,\xi_1^2 \left.\frac{d\theta}{d\xi}\right)_{\xi_1} = 2.01824\,(4\pi\alpha^3\rho_c)$$

and the definitions $\alpha = \left[\dfrac{P_c\,(n+1)}{4\pi G\rho_c^2}\right]^{1/2} = \left[\dfrac{P_c}{\pi G\rho_c^2}\right]^{1/2}$

and $P_c = \dfrac{P_{ideal}}{\beta} = \dfrac{\rho_c N_A k T_c}{\mu\beta}$ and $\dfrac{\rho_c}{\bar{\rho}} = \dfrac{4\pi R^3\rho_c}{3M} = 54.18$

$$P_c = 1.242\times 10^{17}\left(\frac{\left(M/M_\odot\right)^2}{\left(R/R_\odot\right)^4}\right)$$

$$T_c = 19.57\times 10^6\,\beta\mu\left(\frac{\left(M/M_\odot\right)}{\left(R/R_\odot\right)}\right)K$$

$$T_c = 4.62\times 10^6\,\beta\mu\left(M/M_\odot\right)^{2/3}\rho_c^{1/3}\;K$$

For the n=3 polytrope

$$T_c = 4.6 \times 10^6 \ \text{K} \ \mu\beta \left(\frac{M}{M_\odot} \right)^{2/3} \rho_c^{1/3} \qquad \text{(in general for n = 3)}$$

For stars on the main sequence and half way through hydrogen burning, $\mu \approx$ 0.84 and, unless the star is very massive, $\beta \approx$ 0.8 - 0.9. Better values are given in Fig 2-19 of Clayton replicated on the next page.

The density is not predicted from first principles since the actual radius depends upon nuclear burning, but detailed main sequence models (following page) give $\rho_c \approx 10 \left(\frac{10 \ M_\odot}{M} \right)$ gm cm^{-3},So

$$\boxed{T_c \approx 3.9 \times 10^7 \beta \left(\frac{M}{10 M_\odot} \right)^{1/3} \ \text{K}} \qquad \text{(main sequence only)}$$

M	$T_c/10^7$	ρ_C	$L/10^{37}$
9	3.27	9.16	2.8
12	3.45	6.84	7.0
15	3.58	5.58	13
20	3.74	4.40	29
25	3.85	3.73	50
40	4.07	2.72	140
60(57)	4.24	2.17	290
85(78)	4.35	1.85	510
120(99)	4.45	1.61	810

All evaluated in actual models at a core H mass fraction of 0.30 for stars of solar metallicity.

$\mu \approx 0.8$

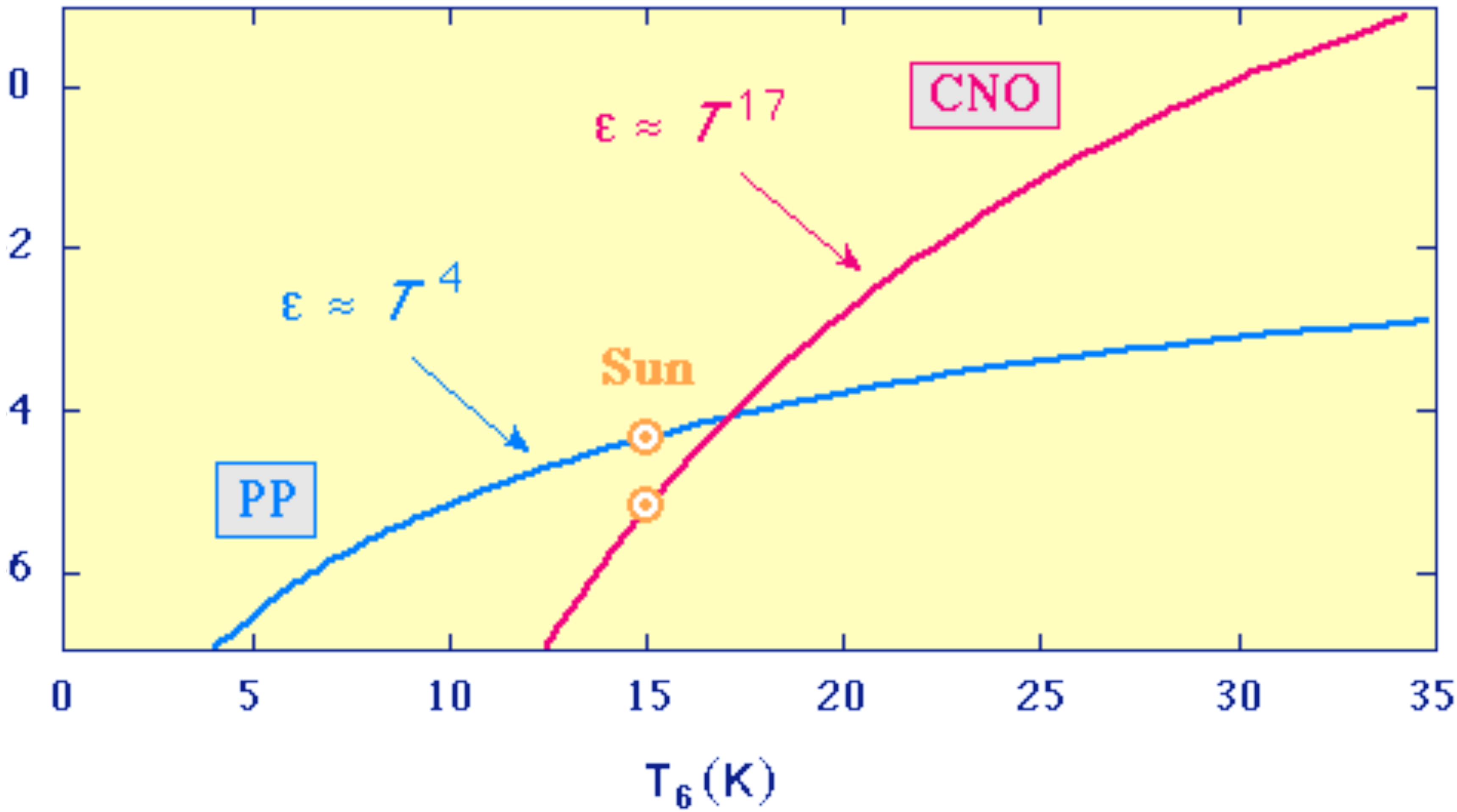
$L \propto M^{2.5}$

$L \sim (1-\beta)\, L_{Ed}$

ρ_c decreases with mass as a general consequence of the fact that

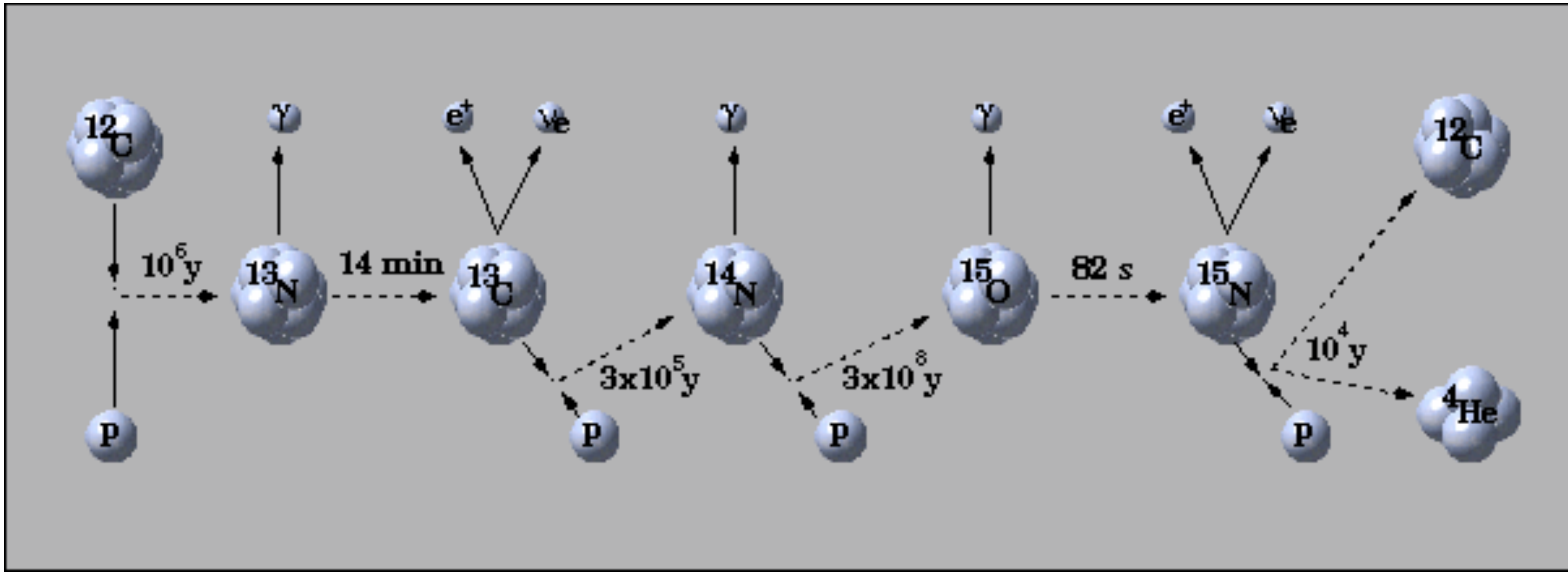
$\frac{T_c^3}{\rho_c} \propto M^2 \beta^3 \mu^3$ and H burning happens at a relatively constant temperature. Until about 40 $M_\odot$, the density decreases roughly as M^{-1}. After that it decreases more slowly. Recall $\beta \propto M^{-1/2}$ for *very* large masses

Competition between the p-p chain and the CNO Cycle



The temperature dependence of the CNO cycle is given by the sensitivity of the proton capture rate of ^{14}N. See previous lectures

The Primary CNO Cycle



In a low mass star

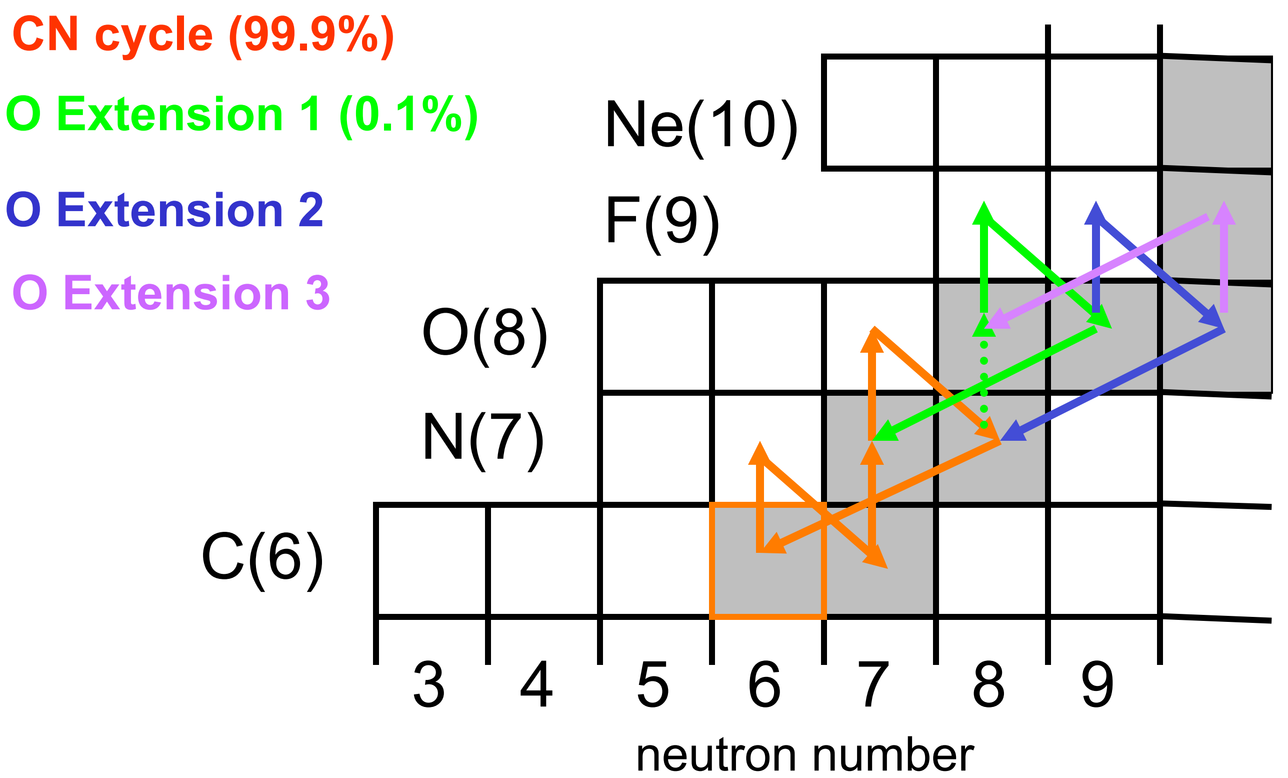
The slowest reaction is $^{14}N(p,\gamma)^{15}O$. For temperatures near 2×10^7 K.

$$\varepsilon_{nuc} \propto T^n \qquad n = \frac{\tau - 2}{3} \qquad \tau = 4.248 \left(\frac{7^2 1^2 \dfrac{14 \cdot 1}{14+1}}{0.020} \right)^{1/3} = 60.0$$

$$n = 18$$

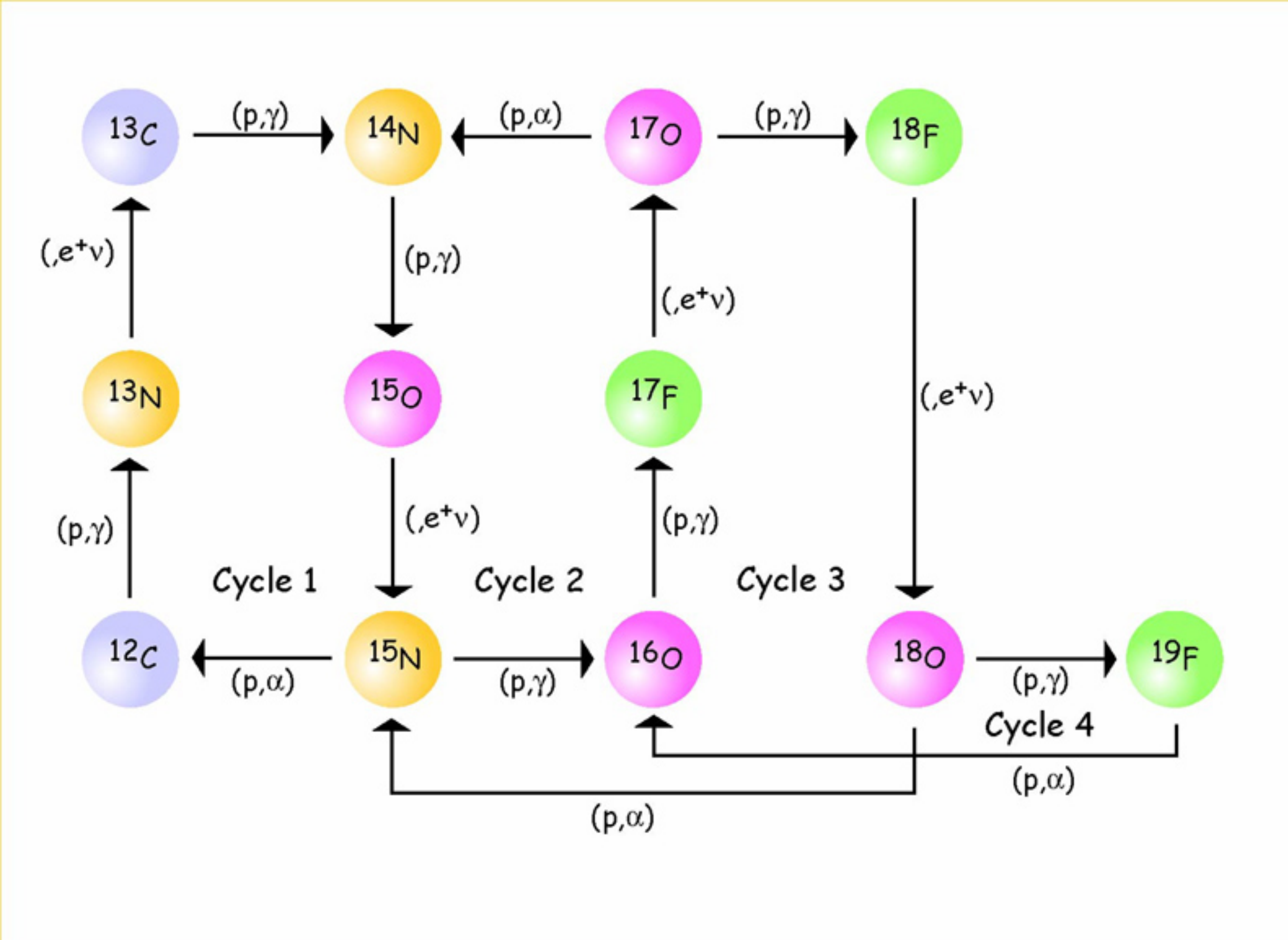
(More on nucleosynthesis later)

CNO tri-cycle



All initial abundances within a cycle serve as catalysts and accumulate at largest t

Extended cycles introduce outside material into CN cycle (Oxygen, …)



The extra loops are mainly of interest for nucleosynthesis and for bringing ^{16}O into the cycle

In general, the rates for these reactions proceed through known resonances whose properties are all reasonably well known.

There was a major revision of the rate for $^{14}N(p,\gamma)^{15}O$ in 2001 by Bertone et al., Phys. Rev. Lettr., 87, 152501. The new rate is about half as large as the old one, so the main sequence lifetime of massive stars is longer (but definitely not linear in the reciprocal rate). Mainly affected globular cluster ages (0.7 to 1 Gy increase in lifetime due to the importance of the CNO cycle at the end of the MS life and during thick H shell burning).

Equation of state

Well defined if tedious to calculate up to the point of iron core collapse.

Ions - ideal gas - $P = \frac{\rho}{\mu} N_A k T$

Radiation $P = \frac{1}{3} a T^4$

Electrons - the hard part - can have arbitrary relativity and degeneracy (solve Fermi integrals or use fits or tables).
At high T must include electron-positron pairs.

Beyond 10^{11} g cm^{-3} - neutrino trapping, nuclear force, nuclear excited states, complex composition, etc.

Opacity

In the interior on the main sequence and within the helium core for later burning stages, electron scattering dominates.

In its simplest form:

$$\kappa_e = \frac{n_e \sigma_{\mathrm{Th}}}{\rho} = \frac{\rho \mathrm{N_A} Y_e \sigma_{\mathrm{Th}}}{\rho} = Y_e(\mathrm{N_A}\, \sigma_{\mathrm{Th}})$$

$$\sigma_{\mathrm{Th}} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2$$

$$\kappa_e = 0.40\ Y_e\ \mathrm{cm^2\ gm^{-1}}$$

Recall that for 75% H, 25% He, $Y_e = 0.875$, so $\kappa_\mathrm{e} = 0.35$

For He and heavier elements $\kappa_e \approx 0.20$.

There are correction terms that must be applied to κ_{es} especially at high temperature and density

1) The *electron-scattering cross section* and Thomson cross section differ at high energy. The actual cross section is smaller.

Klein-Nishina

$$\sigma_{KN} = \sigma_{Th}\left[1 - \left(\frac{2h\nu}{m_e c^2}\right) + \frac{26}{5}\left(\frac{h\nu}{m_e c^2}\right)^2 + \ldots\right]$$

$$h\nu << 10^{20}\ Hz$$

2) *Degeneracy* – at high density the phase space for the scattered electron is less. This decreases the scattering cross section.

3) *Incomplete ionization* – especially as the star explodes as a supernova. Use the Saha equation.

4) *Electron positron pairs* may increase κ at high temperature.

Effects 1) and 2) are discussed by

Chin, *ApJ*, **142**, 1481 (1965)
Flowers & Itoh, *ApJ*, **206**, 218, (1976)
Buchler and Yueh, *ApJ*, **210**, 440, (1976)
Itoh et al, *ApJ*, **382**, 636 (1991) and references therein

Electron conduction is not very important in massive stars but is important in white dwarfs and therefore the precursors to Type Ia supernovae

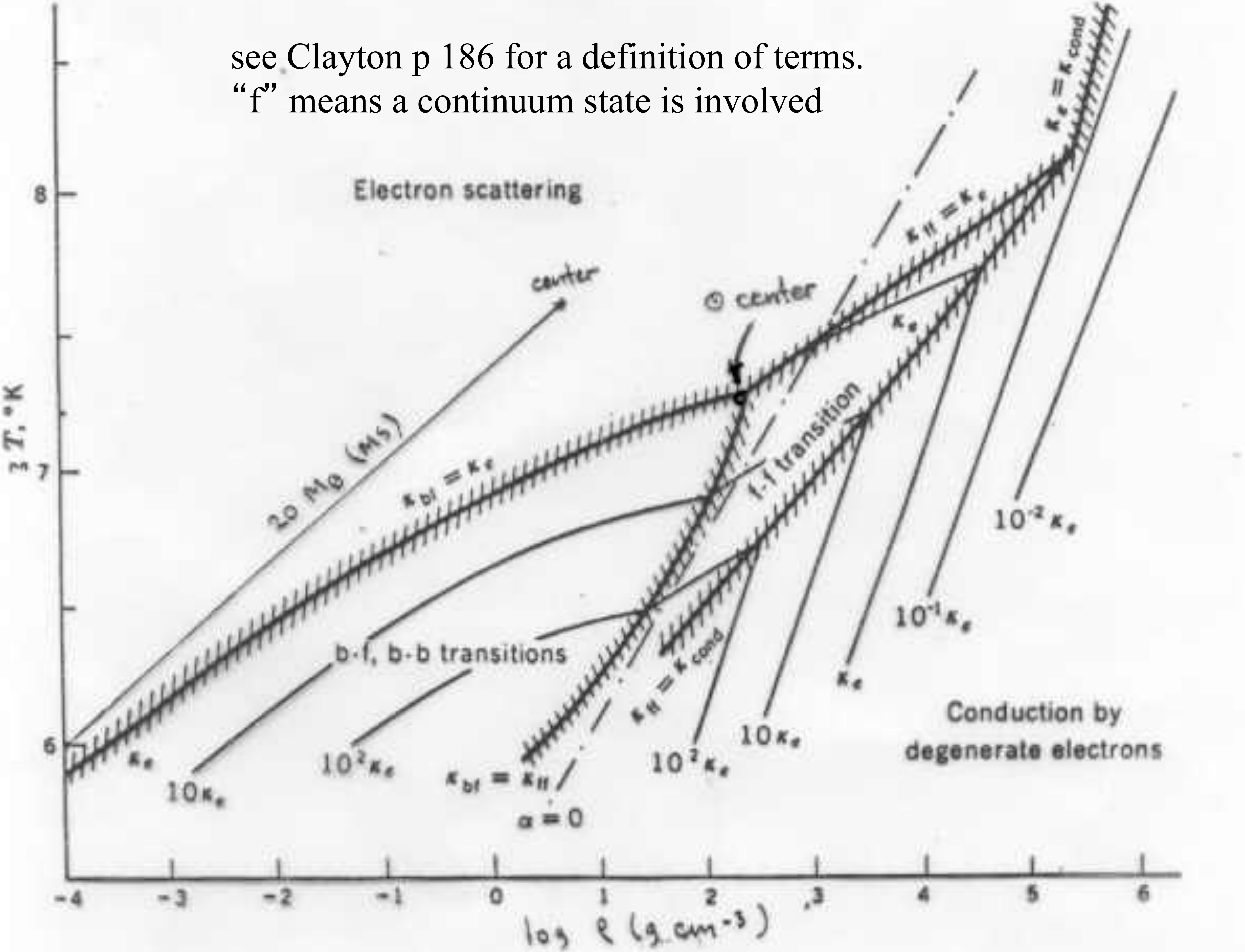
Itoh et al, *ApJ*, **285**, 758, (1984) and references therein

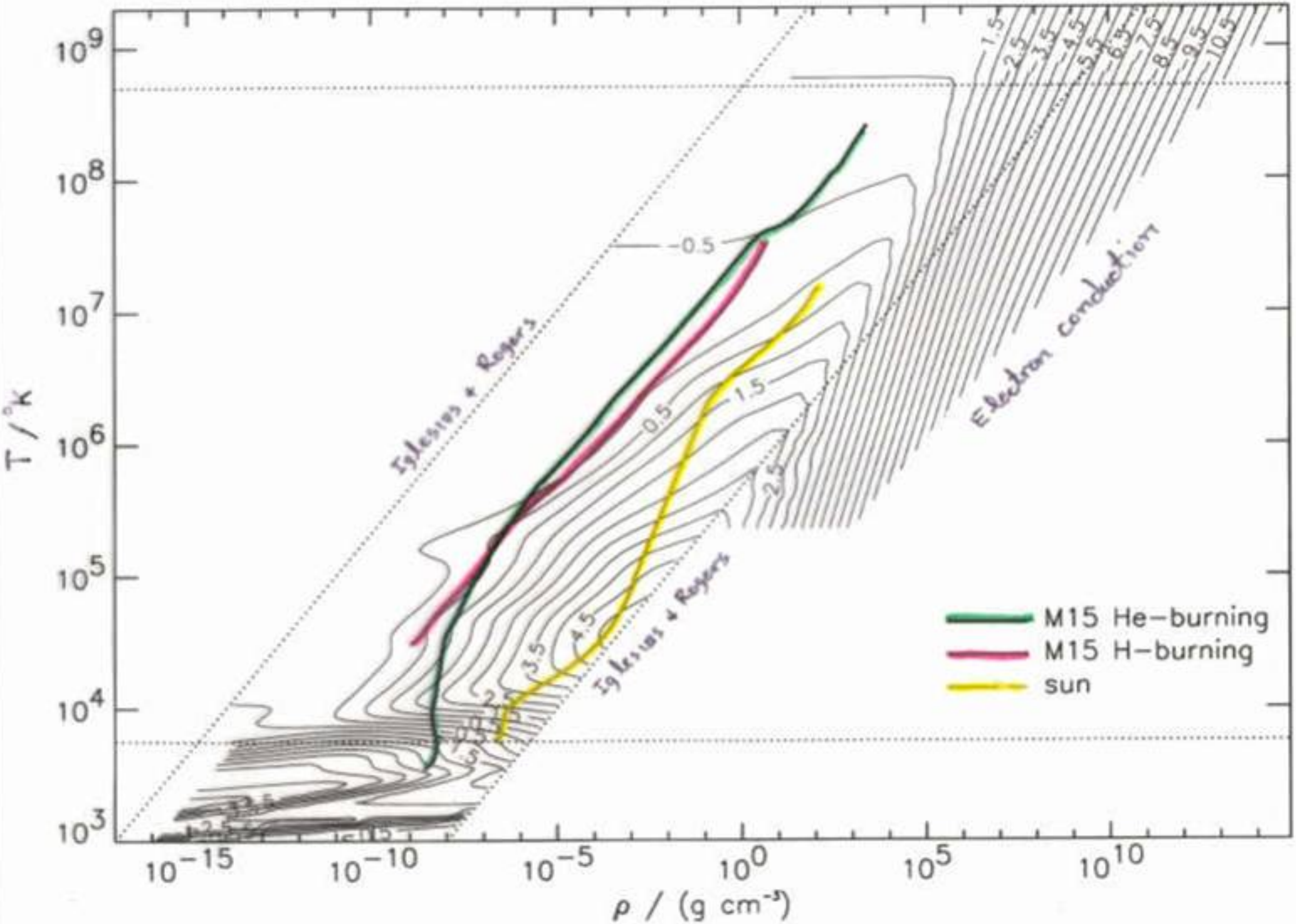
For radiative opacities other than κ_{es}, in particular κ_{bf} and κ_{bb},

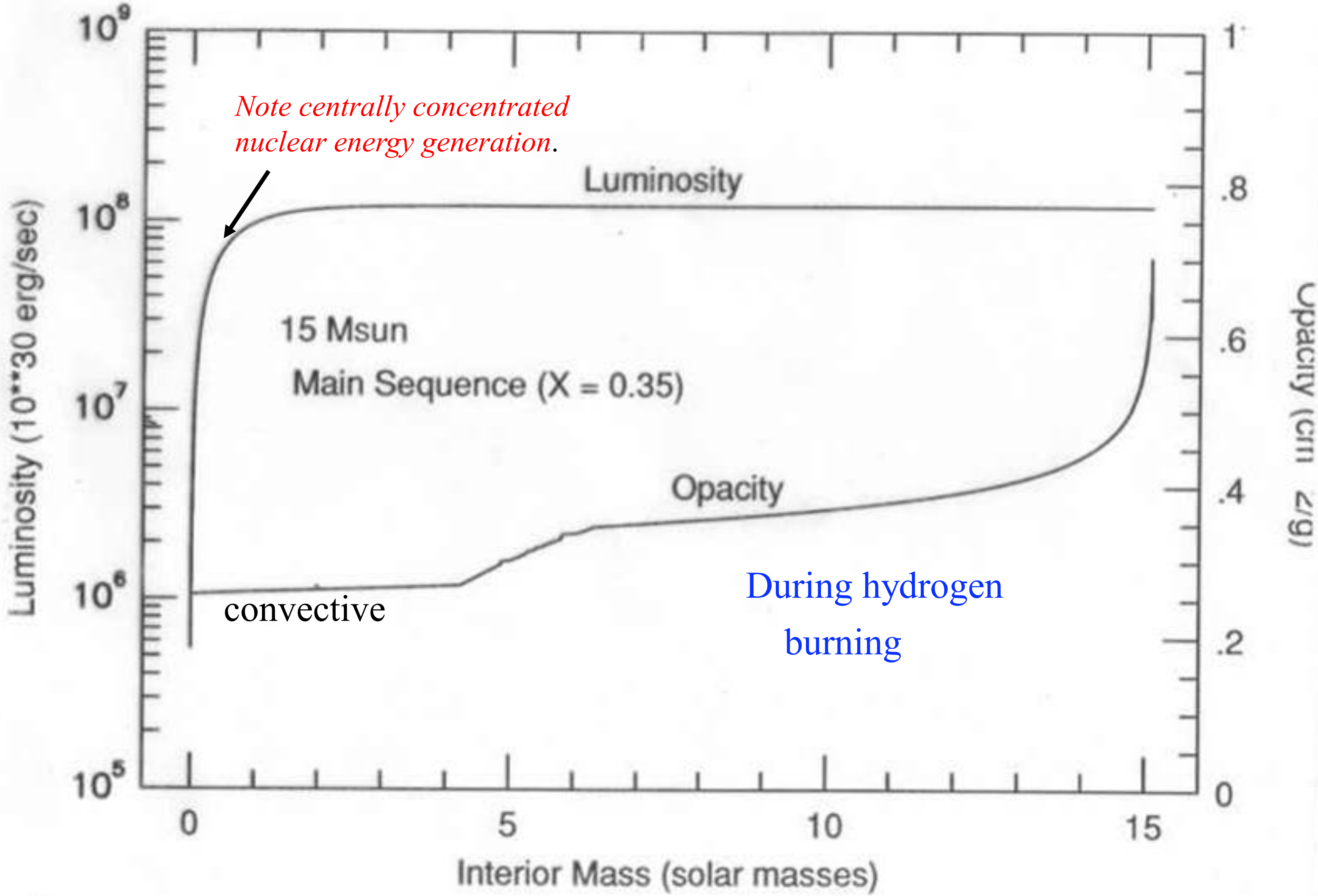
Iglesias and Rogers, *ApJ*, **464**, 943 (1996)

Rogers, Swenson, and Iglesias, *ApJ,* **456**, 902 (1996)

see Clayton p 186 for a definition of terms.
"f" means a continuum state is involved




T / °K
ρ / (g cm⁻³)
Iglesias & Rogers
Iglesias & Rogers
Electron conduction
M15 He-burning
M15 H-burning
sun


Note centrally concentrated nuclear energy generation.
Luminosity
15 Msun
Main Sequence (X = 0.35)
Opacity
convective
During hydrogen burning
Luminosity (10**30 erg/sec)
Opacity (cm^2/g)
Interior Mass (solar masses)
10^9
10^8
10^7
10^6
10^5
0
5
10
15
1
.8
.6
.4
.2
0

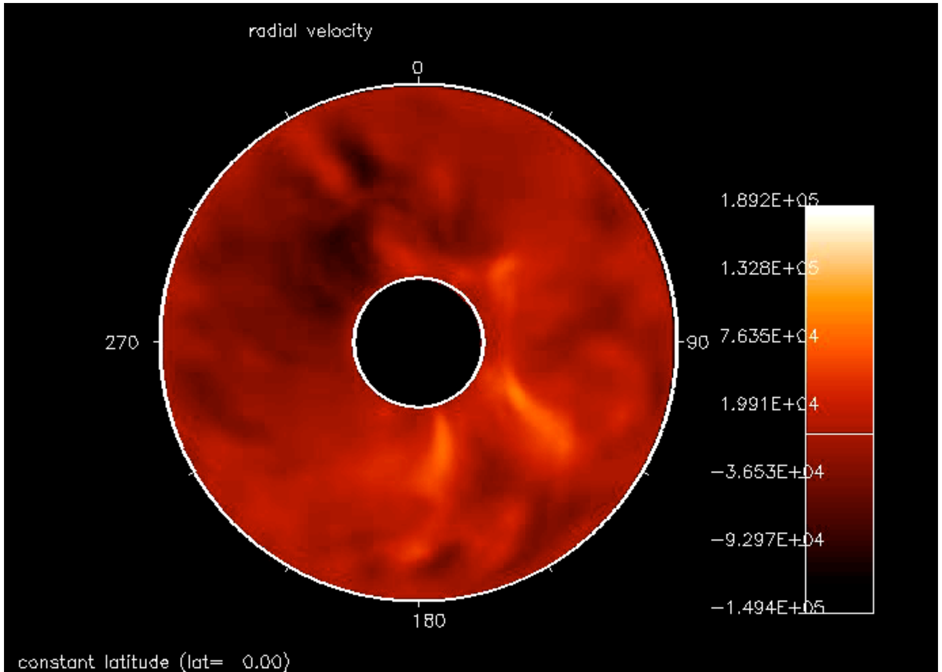
Convection

All stellar evolution calculations to date, except for brief snapshots, have been done in one-dimensional codes.

In these convection is universally represented using some variation of mixing length theory.

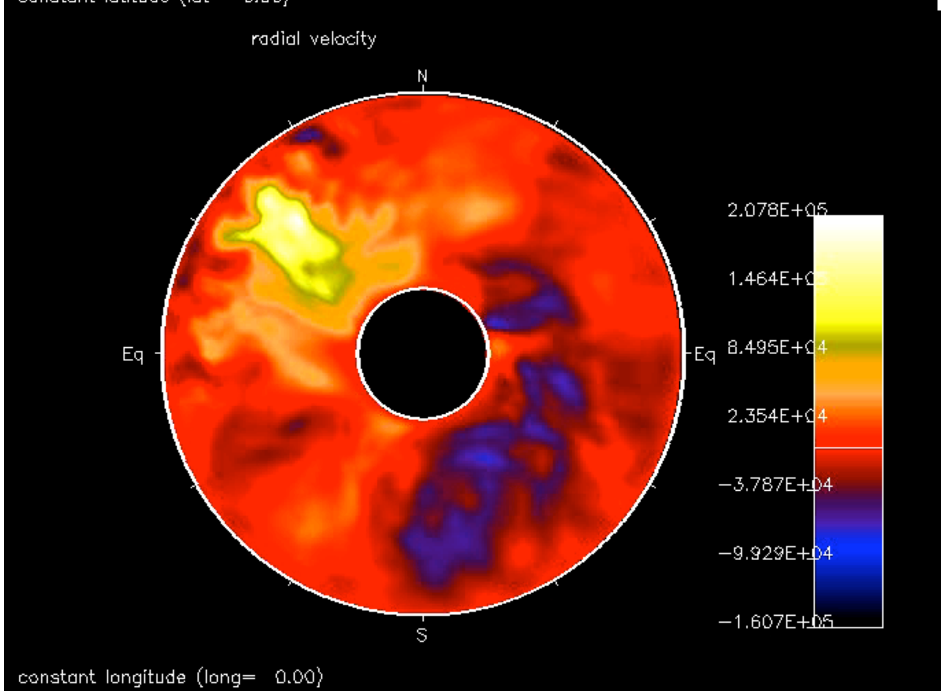
Caveats and concerns:

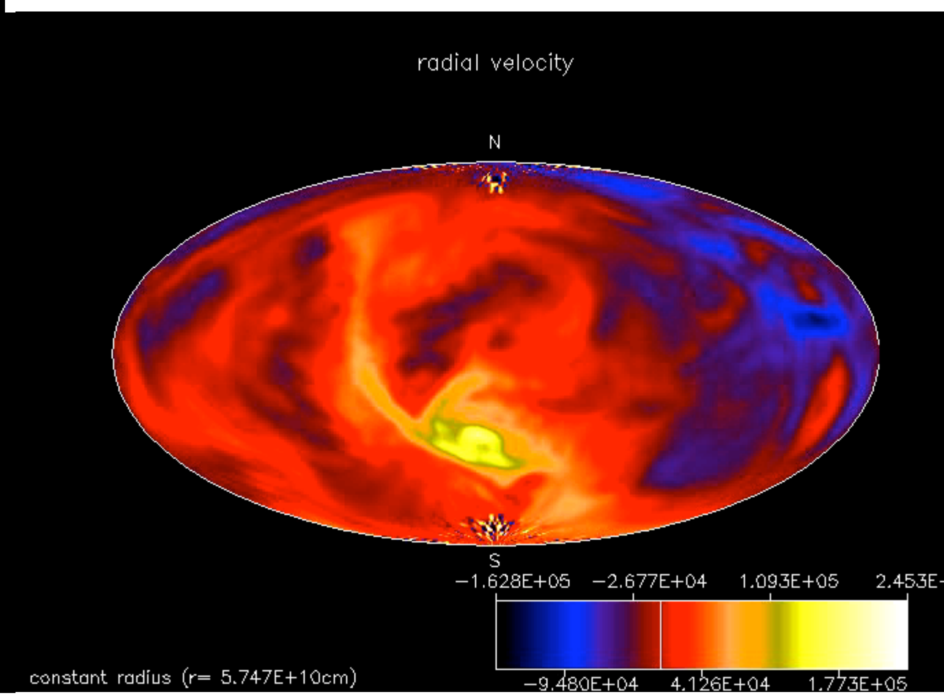
- The treatment must be time dependent
- Convective overshoot and undershoot (next lecture)
- Semiconvection (next lecture)
- Convection in parallel with other mixing processes, especially rotation
- Convection in situations where evolutionary time scales are not very much longer than the convective turnover time.



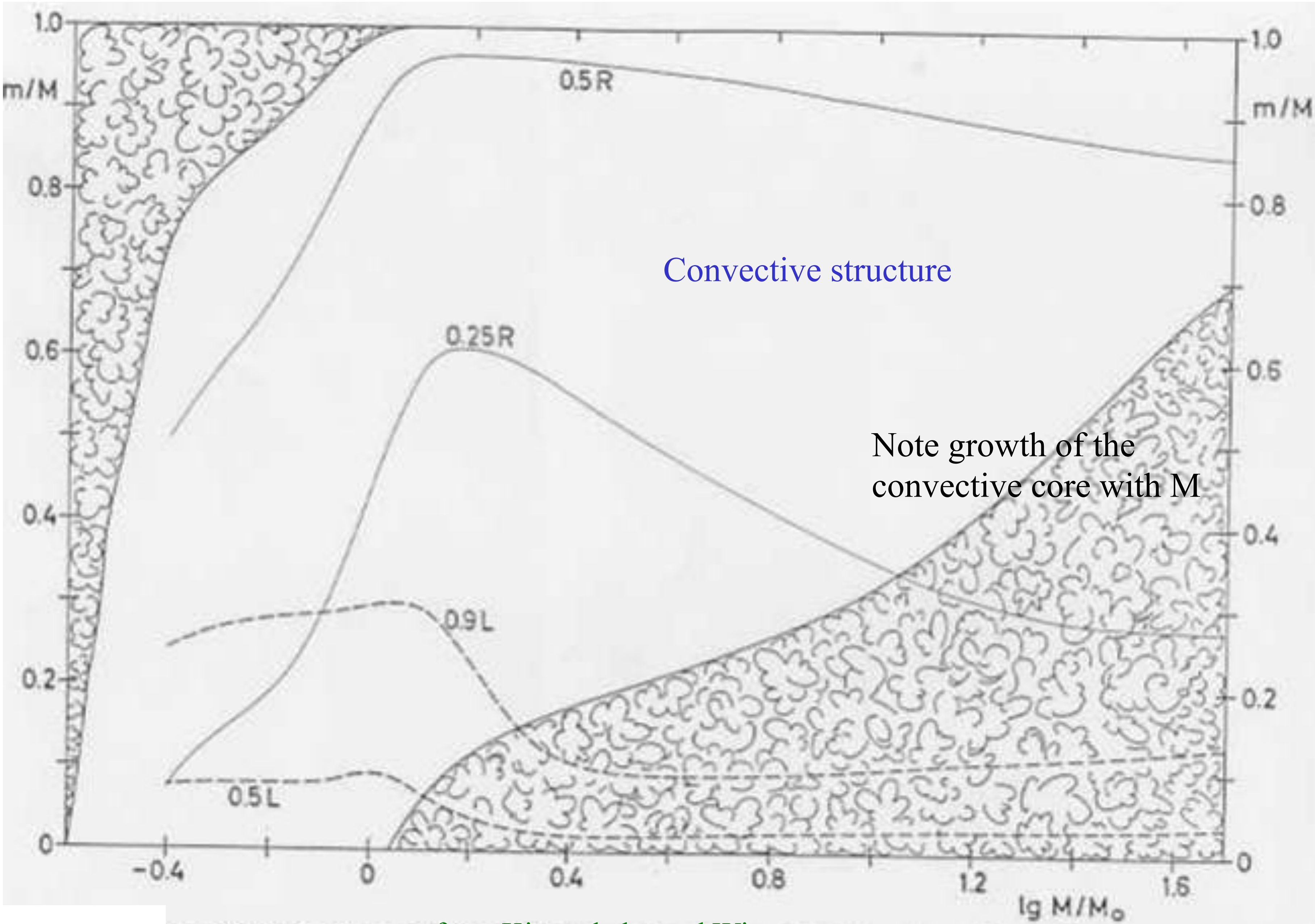
Kuhlen, Woosley, and Glatzmaier explored the physics of stellar convection using 3D anelastic hydrodynamics.
See also Meakin and Arnett (2007)
Gilet et al (ApJ, 2013)

The model shown is a 15 solar mass star half way through hydrogen burning. For now the models are not rotating. Mixing length theory is not a bad description of the overall behavior.





Convective structure



Note growth of the convective core with M

from Kippenhahn and Wiegert

The (Swartzschild) adiabatic condition can be written in terms of the temperature as

$$\frac{dP}{P} + \frac{\Gamma_2}{1-\Gamma_2}\frac{dT}{T} = 0$$

This defines Γ_2 (see Clayton p 118)

For an ideal gas $\Gamma_2 = 5/3$, but if radiation is included the expression is more complicated

Convective instability is favored by a large fraction of radiation pressure, i.e., a small value of β.

$$\left(\frac{dT}{dr}\right)_{\text{star}} > \left(1-\frac{1}{\Gamma_2}\right)\frac{T}{P}\left(\frac{dP}{dr}\right) \Rightarrow \text{convection}$$

$$\Gamma_2 = \frac{32-24\beta-3\beta^2}{24-18\beta-3\beta^2} \qquad \frac{4}{3} < \Gamma_2 < \frac{5}{3} \qquad \text{(Clayton 2-129)}$$

For $\beta=1$, $\left(1-\frac{1}{\Gamma_2}\right)=0.4$; for $\beta = 0$, $\left(1-\frac{1}{\Gamma_2}\right) = 0.25$

for $\beta = 0.8$ $\left(1-\frac{1}{\Gamma_2}\right) = 0.294$

So even a 20% decrease in β causes a substantial decrease in the critical temperature gradient necessary for convection. Since β decreases with increasing mass, convection becomes more extensive.

Also more massive stars are generating a lot more energy in a star whose physical dimension is not much larger.

M	$T_c/10^7$	ρ_C	$L/10^{37}$	$Q_{conv\ core}$
9	3.27	9.16	2.8	0.26
12	3.45	6.84	7.0	0.30
15	3.58	5.58	13	0.34
20	3.74	4.40	29	0.39
25	3.85	3.73	50	0.43
40	4.07	2.72	140	0.53
60(57)	4.24	2.17	290	0.60
85(78)	4.35	1.85	510	0.66
120(99)	4.45	1.61	810	0.75

All evaluated at a core H mass fraction of 0.30 for stars of solar metallicity.

The convective core shrinks during hydrogen burning

During hydrogen burning the mean atomic weight is increasing from near 1 to about 4. The ideal gas entropy is thus decreasing.

Also convection is taking entropy out of the central regions and depositing it farther out in the star.

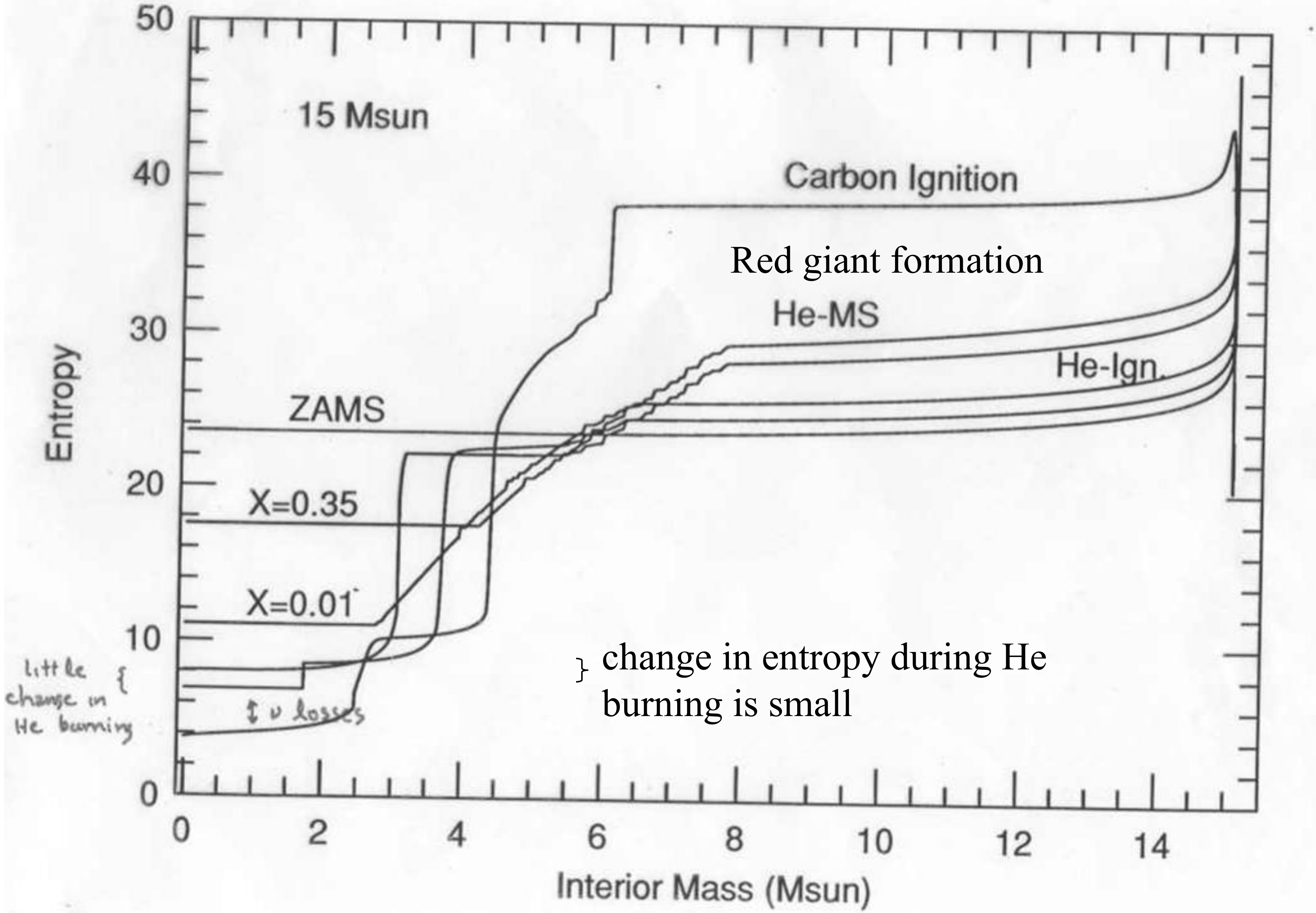
As the central entropy decreases compared with the outer layers of the star it becomes increasingly difficult to convect through most of the star's mass.

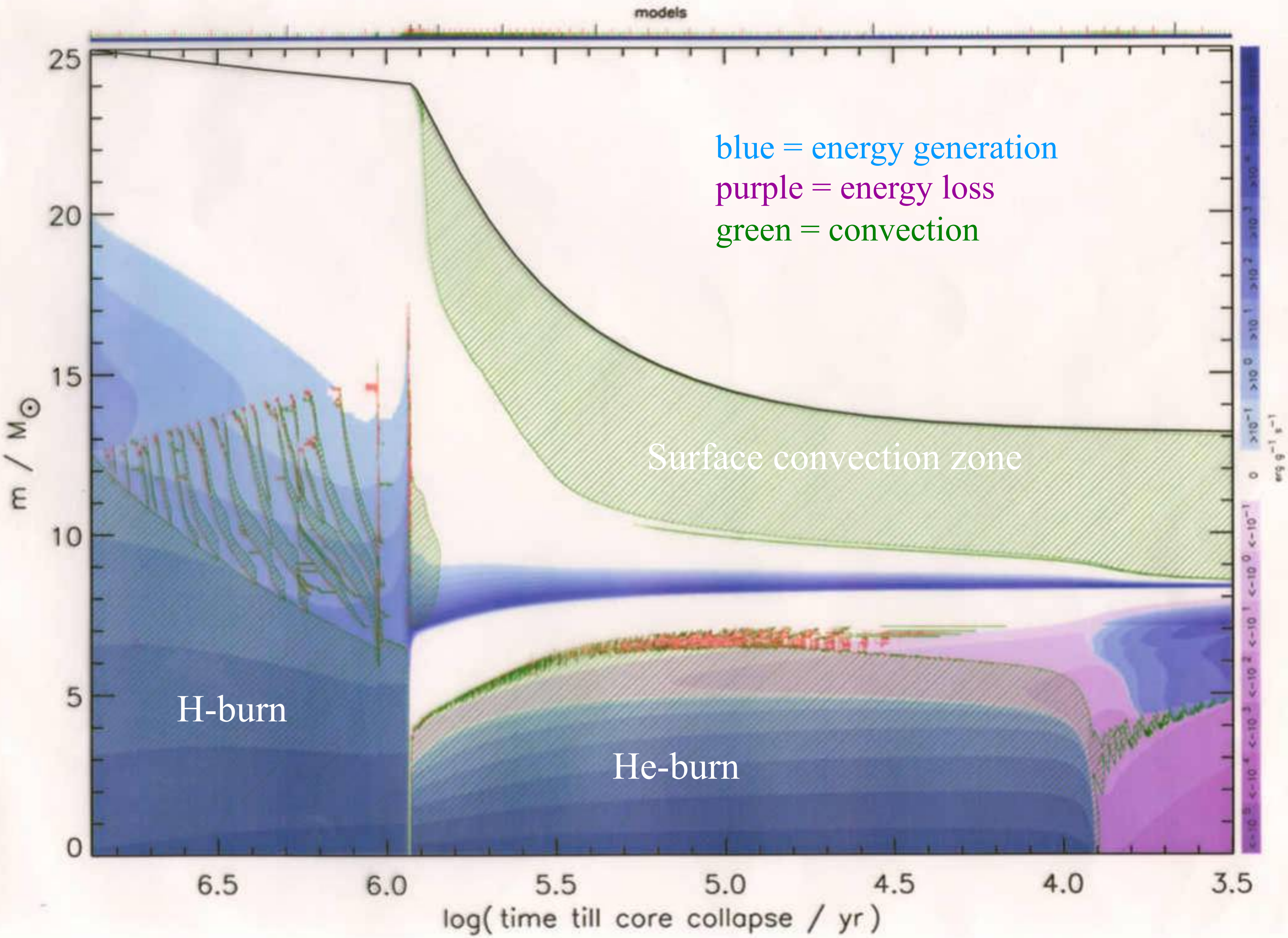
For an ideal gas plus radiation:

(see Clayton p. 123)

$$S = const + \frac{N_A k}{\mu} ln(\frac{T^{3/2}}{\rho}) + \frac{4a}{3}\frac{T^3}{\rho}.$$

$\mu = \frac{1}{2}$ for pure hydrogen; $\frac{4}{3}$ for pure helium


15 Msun
Carbon Ignition
Red giant formation
He-MS
He-Ign.
ZAMS
X=0.35
X=0.01
change in entropy during He burning is small
little change in He burning
ν losses
Entropy
Interior Mass (Msun)
0
2
4
6
8
10
12
14
10
20
30
40
50


models
blue = energy generation
purple = energy loss
green = convection
Surface convection zone
H-burn
He-burn
m / M⊙
log(time till core collapse / yr)
0
5
10
15
20
25
6.5
6.0
5.5
5.0
4.5
4.0
3.5

The convective core grows during helium burning.

During helium burning, the convective core grows, largely because the mass of the helium core itself grows. This has two effects:

a) As the mass of the core grows so does its luminosity, while the radius of the convective core stays nearly the same (density goes up). For a 15 solar mass star:

	He mass fraction	Radius conv core	Lum conv core	Lum star
3.0 $M_\odot$	1	0.87×10^{10} cm	3.2×10^{37} erg s^{-1}	2.16×10^{38} erg s^{-1}
3.6 $M_\odot$	0.5	1.04×10^{10} cm	6.8×10^{37} erg s^{-1}	2.44×10^{38} erg s^{-1}

The rest of the luminosity is coming from the H shell..

b) As the mass of the helium core rises its β decreases.

$$\frac{T^3}{\rho} \sim M^2 \beta^3 \qquad \frac{1/3aT^4}{\rho N_A kT} \sim \frac{1-\beta}{\beta} \sim M^2 \beta^3$$

This decrease in β favors convection.

The entropy during helium burning also continues to decrease, and this would have a tendency to diminish convection, but the β and L effects dominate and the helium burning convective core grows until near the end when it shrinks both due to the decreasing central energy generation.

This growth of the helium core can have two interesting consequences:

- Addition of helium to the helium convection zone at late time increases the O/C ratio made by helium burning

- In very massive stars with low metallicity the helium convective core can grow so much that it encroaches on the hydrogen shell with major consequences for stellar structure and nucleosynthesis.

METALLICITY

Metallicity affects the evolution in four distinct ways:

- Mass loss
- Energy generation
- Opacity
- Initial H/He abundance

lower main sequence: homology – see appendix

κ decreases if Z decreases $\quad L \sim \kappa_o^{-16/13}\,\varepsilon_o^{1/13}$

$$T_c \sim (\kappa_o \varepsilon_o)^{-2/15}$$

For example, 1 $M_\odot$ at half hydrogen depletion

	Z = 0.02	Z = 0.001
$\log T_c$	7.202	7.238
L	$L_\odot$	$2.0\,L_\odot$

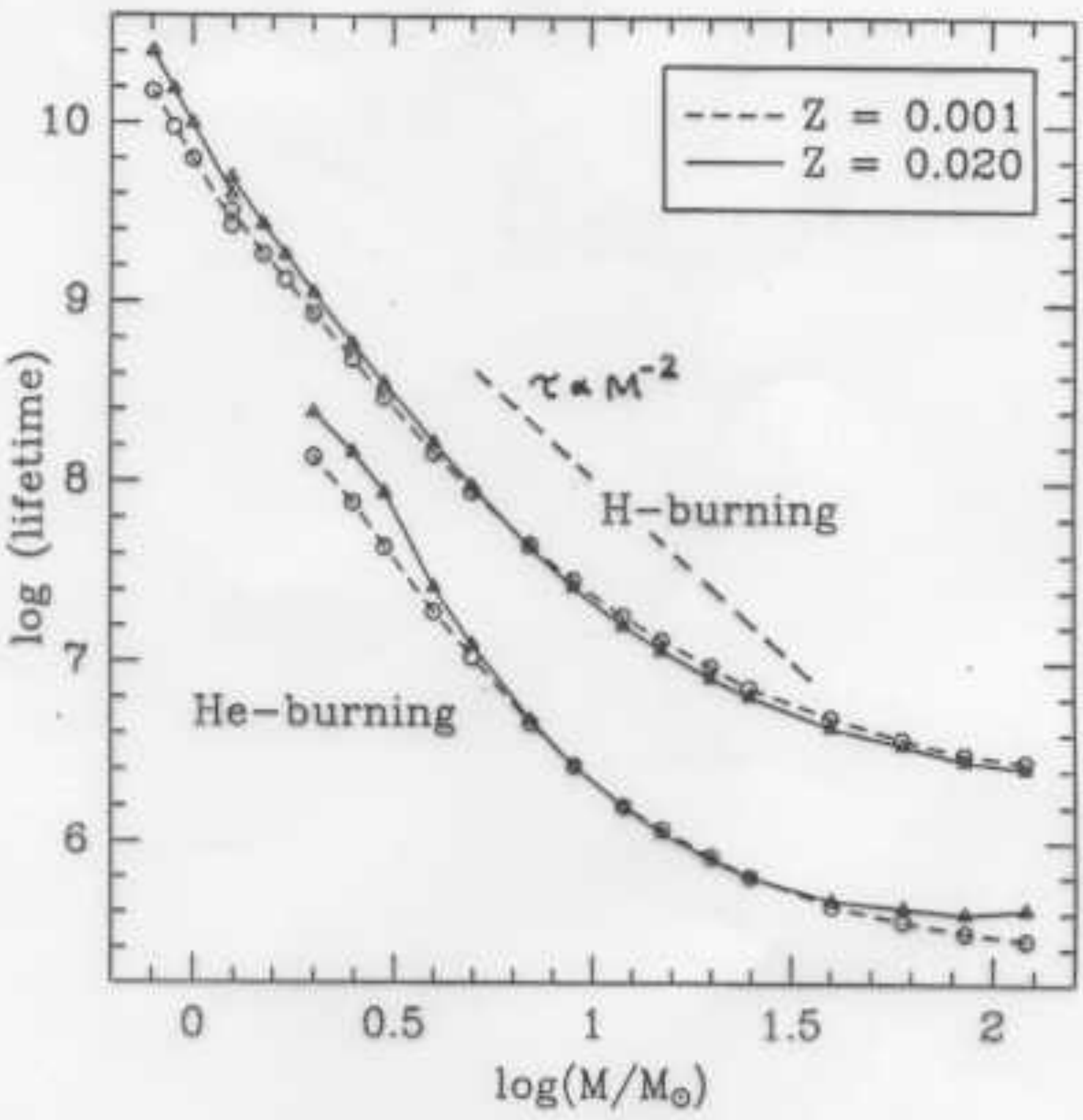
Because of the higher luminosity, the lifetime of the lower metallicity star is shorter (it burns about the same fraction of its mass). But this is the sun, it's opacity is not due to electron scattering and so depends on Z

Upper main sequence:

The luminosities and ages are very nearly the same because the opacity is, to first order, independent of the metallicity. The central temperature is a little higher at low metallicity because of the decreased abundance of ^{14}N to catalyze the CNO cycle.

For example in a 20 solar mass star at $X_H = 0.3$

	$Z = 0.02$	$Z = 0.001$
$\log T_c$	7.573	7.647
$\log L / L_\odot$	4.867	4.872
Q_{cc}	0.390	0.373
$M / M_\odot$	19.60	19.92 (mass loss)



Z = 0.001
Z = 0.020
τ ∝ M⁻²
H–burning
He–burning
log (lifetime)
log(M/M☉)
0
0.5
1
1.5
2
6
7
8
9
10

There is a slight difference in the lifetime on the upper main sequence though because of the different initial helium abundances. Schaller et al. used Z = 0.001, Y = 0.243, **X=0.756** and Z = 0.02, Y = 0.30, **X = 0.68.**

So for the higher metallicity there is less hydrogen to burn.

But there is also an opposing effect, namely mass loss. For higher metallicity the mass loss is greater and the star has a lower effective mass and lives longer.

Both effects are small unless the mass is very large.

For helium burning, there is no effect around 10 solar mases, but the higher masses have a longer lifetime with higher metallicity because mass loss decreases the mass.

For lower masses, there is a significant metallicity dependence for the helium burning lifetime. The reason is not clear. Perhaps the more active H-burning shell in the solar metallicity case reduces the pressure on the helium core.

Zero and low metallicity stars may end their lives as compact blue giants – depending upon semiconvection and rotationally induced mixing

For example, $Z = 0$, presupernova, full semiconvection

a) 20 solar masses

$R = 7.8 \times 10^{11}$ cm $T_{eff} = 41{,}000$ K

b) 25 solar masses

$R=1.07 \times 10^{12}$cm $T_{eff} = 35{,}000$ K

$Z = 0.0001\ Z_{\odot}$

a) 25 solar masses, little semiconvection

$R = 2.9 \times 10^{12}$ cm $T_{eff} = 20{,}000$ K

b) 25 solar masses, full semiconvection

$R = 5.2 \times 10^{13}$ cm $T_{eff}= 4800$ K

Caveat: Primary ^{14}N production

Table 1: Presupernova Radii as f(Z)

BSG

Mass	Z = 0	10^{-4} $Z_\odot$	0.01 $Z_\odot$	0.1 $Z_\odot$	1/3 $Z_\odot$
12	0.140	2.64	2.79	[illegible].83	2.55
13	0.0898	0.694	2.98	3.01	2.80
15	0.0674	0.181	0.709	3.44	3.35
18	0.0696	0.144	0.309	1.79	4.17
20	0.0795	0.215	0.866	3.42	4.88
22	0.0890	0.186	0.617	4.76	5.19
25	0.0916	0.239	0.809	2.14	5.91
30	0.113	0.316	1.72	6.89	6.87
35	0.174	0.563	5.35	8.50	7.78
40	0.196	0.625	7.96	9.52	(28)

RSG

[a] All radii in units of 10^{13} cm

Caution - rotationally induced mixing changes these results

Very massive stars

As radiation pressure becomes an increasingly dominant part of the pressure, β decreases in very massive stars.

This implies that the luminosity approaches Eddington.
But in a 100 solar mass star β is still 0.55.

Recall for n = 3

$$L(r) = (1-\beta)\frac{4\pi GMc}{\kappa} . = (1-\beta)\ L_{Ed}$$

For very massive stars L is proportional to M and approaces the Eddington luminosity

In fact, except for a thin region near their surfaces, such stars will be entirely convective and will have a total binding energy that approaches zero as b approaches zero. But the calculation applies to those surface layers which must stay bound.

Completely convective stars with a luminosity proportional to mass have a constant lifetime, which is in fact the shortest lifetime a (main sequence) star can have.

$$L_{Edd} = \frac{4\pi G M c}{\kappa} = 1.47 \times 10^{38} \text{ erg s}^{-1} \left(\frac{M}{M_\odot}\right)\left(\frac{0.34}{\kappa}\right)$$

$$q_{nuc} = 4.8 \times 10^{18} \text{ erg/g}$$

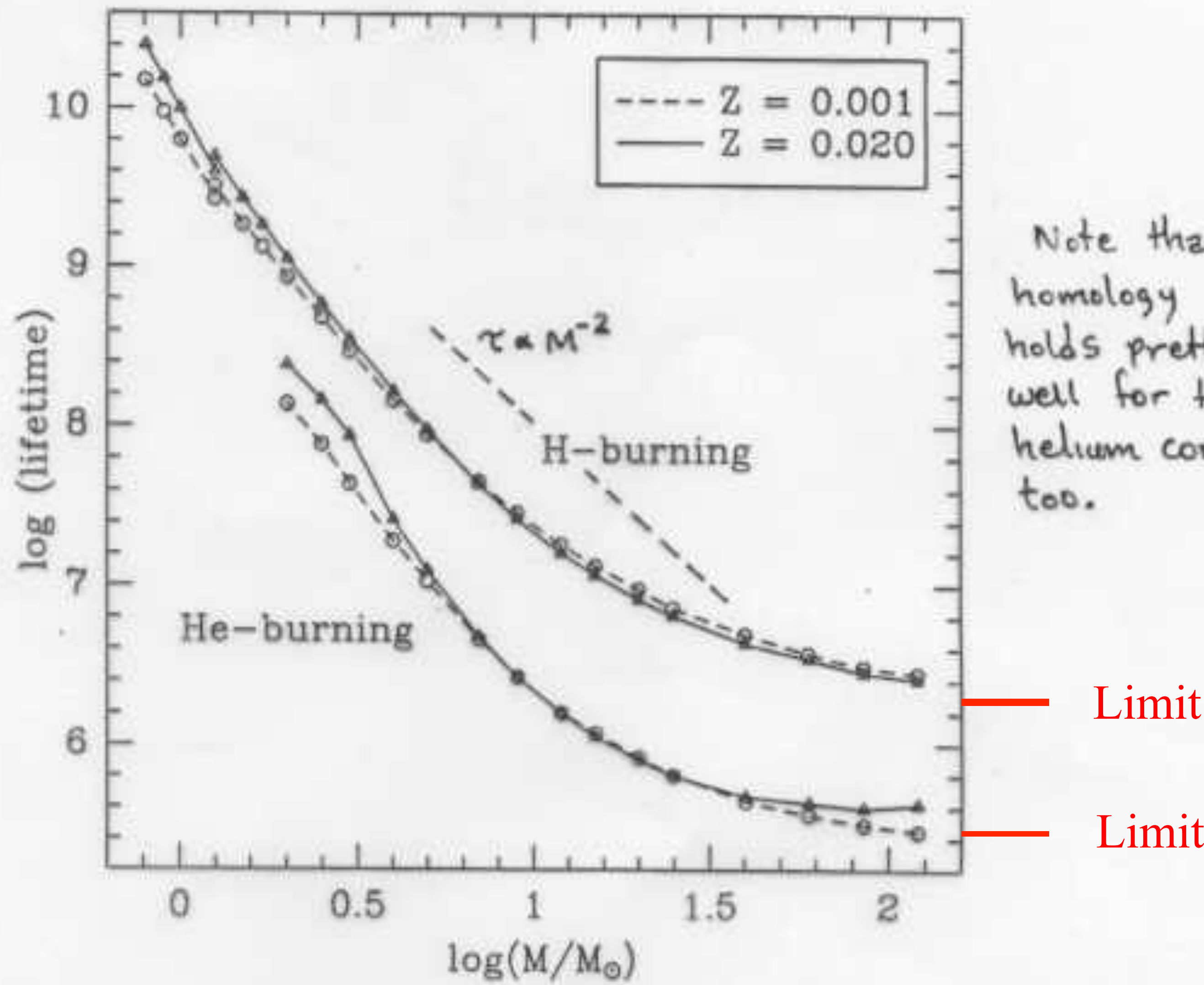
$$\tau_{\mathrm{MS}} = q_{nuc} M / L_{Edd} = 2.1 \text{ million years}$$

(exception *super*massive stars over 10^5 solar masses – post-Newtonian gravity renders unstable on the main sequence)

Similarly there is a lower bound for helium burning. The argument is the same except one uses the q-value for helium burning to carbon and oxygen.

One gets 7.3×10^{17} erg g^{-1} from burning 100% He to 50% each C and O.

Thus the minimum (Eddington) lifetime for helium burning is about 300,000 years.



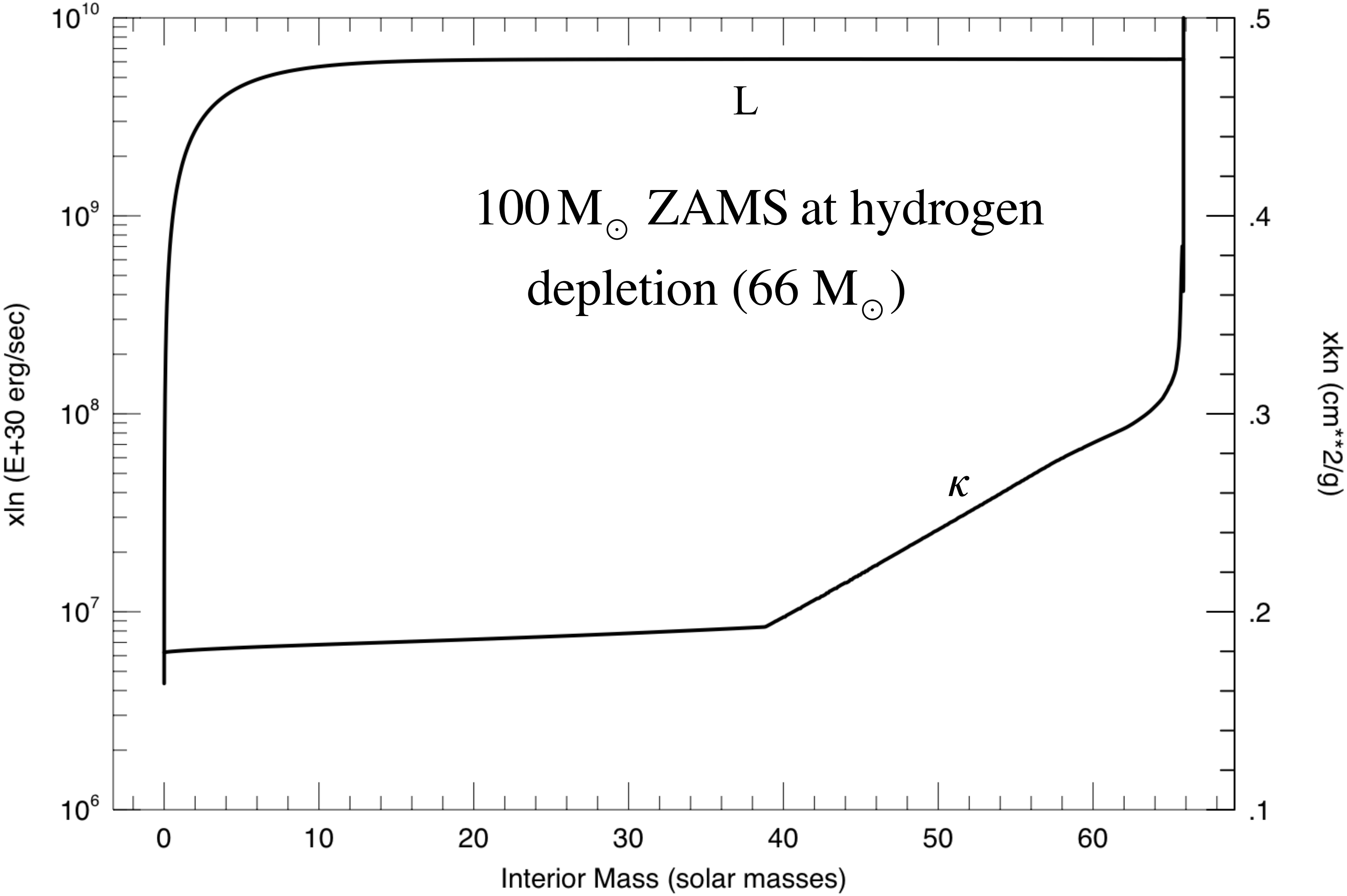
Note that homology holds pretty well for the helium cores too.

VERY HIGH MASS STARS

Since $\Gamma \sim 4/3$, such stars are loosely bound (total energy much less than gravitational or internal energy) and are subject to large amplitude pulsations. These can be driven by either opacity instabilities (the κ mechanism) or nuclear burning instabilities (the ε mechanism). β is less than 0.5 for such stars on the main sequence, but ideal gas entropy still dominates.

For solar metallicity it has long been recognized that such stars (well over 100 solar masses) would pulse violently on the main sequence and probably lose much of their mass before dying.

Ledoux, *ApJ,* **94**, 537, (1941)
Schwarzschild & Harm, *ApJ*, 129, 637, (1959)
Appenzeller, *A&A*, **5**, 355, (1970)
Appenzeller, A&A, **9**, 216, (1970)
Talbot, *ApJ*, **163**, 17, (1971)
Talbot, *ApJ*, **165**, 121, (1971)
Papaloizou, *MNRAS*, **162**, 143, (1973)
Papaloizou, *MNRAS*, **162**, 169, (1973)



L
100 M☉ ZAMS at hydrogen depletion (66 M☉)
κ
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
.5
.4
.3
.2
.1
0
10
20
30
40
50
60
Interior Mass (solar masses)
xln (E+30 erg/sec)
xkn (cm**2/g)

Eddington luminosity for 66 solar masses

$$L_{Edd} = \frac{4\pi GMc}{\kappa} = 9.6 \times 10^{39} \text{ erg s}^{-1} \left(\frac{0.34}{\kappa}\right)$$

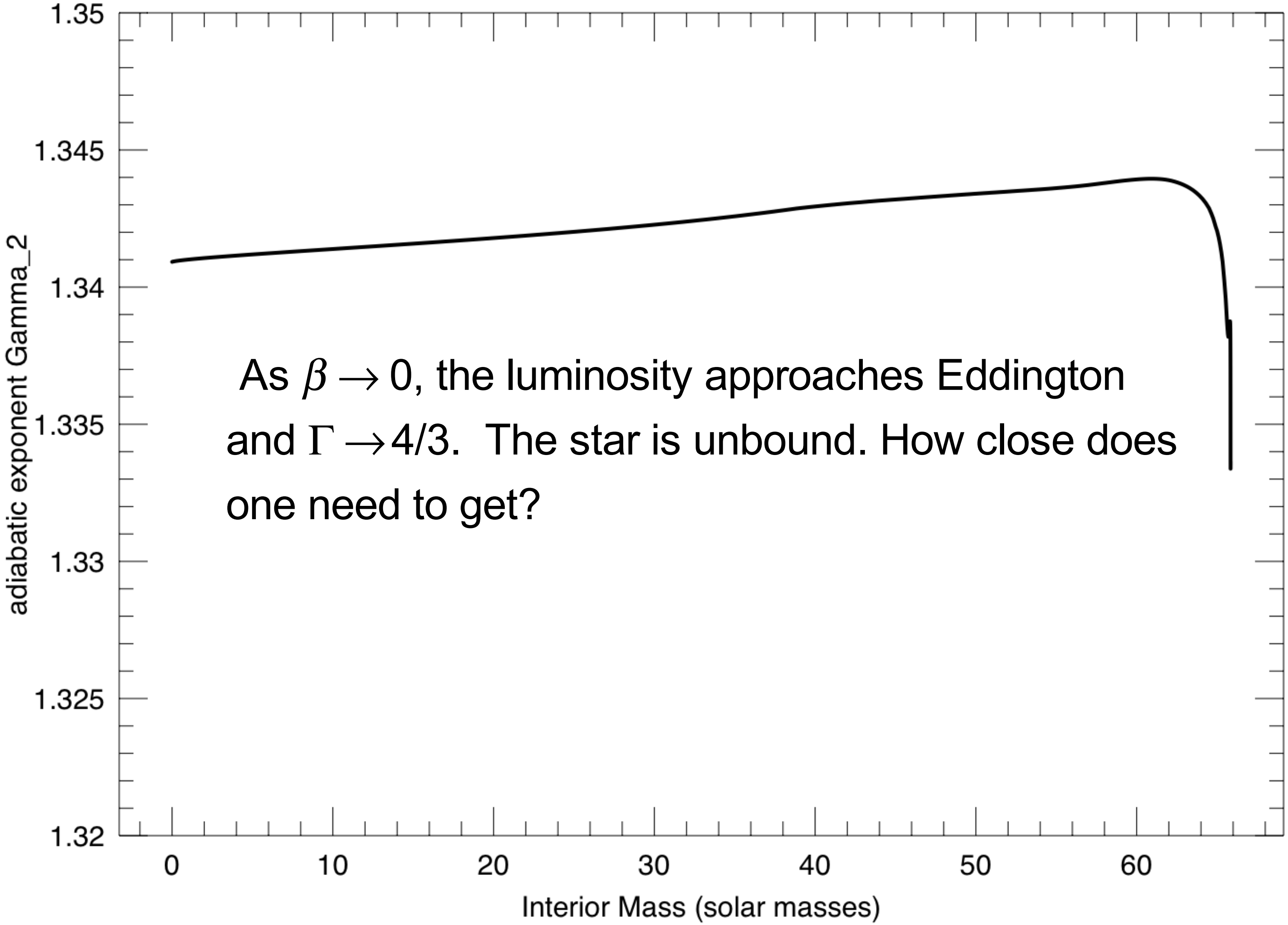
So model is about ~2/3 Eddington luminosity for electron scattering opacity near the surface.

As the star contracts and ignites helium burning its luminosity rises from 6 to 8 x 10^{39} erg s^{-1} and its mass continues to decrease by mass loss. Super-Eddington mass ejection?

$$\dot{M} \sim \frac{\Delta L_{excess} R}{GM} \sim 0.001 M_{\odot}\, y^{-1} \quad \Delta L_{38}\, R_{13}\, M_{100}^{-1}$$

ΔL_{excess} depends on κ which is at least as large as electron scattering.

Luminous blue variable stars?



As $\beta \to 0$, the luminosity approaches Eddington and $\Gamma \to 4/3$. The star is unbound. How close does one need to get?

Upper mass limit: theoretical predictions

Ledoux (1941)	radial pulsation, e- opacity, H	100 $M_\odot$
Schwarzchild & Härm (1959)	radial pulsation, e- opacity, H and He, evolution	65-95 $M_\odot$
Stothers & Simon (1970)	radial pulsation, e- and atomic	80-120 $M_\odot$
Larson & Starrfield (1971)	pressure in HII region	50-60 $M_\odot$
Cox & Tabor (1976)	e- and atomic opacity Los Alamos	80-100 $M_\odot$
Klapp et al. (1987)	e- and atomic opacity Los Alamos	440 $M_\odot$
Stothers (1992)	e- and atomic opacity Rogers-Iglesias	120-150 $M_\odot$

Upper mass limit: observation

R136	Feitzinger et al. (1980)	250-1000 $M_\odot$
Eta Car	various	120-150 $M_\odot$
R136a1	Massey & Hunter (1998)	136-155 $M_\odot$
Pistol Star	Figer et al. (1998)	140-180 $M_\odot$
Eta Car	Damineli et al. (2000)	~70+? $M_\odot$
LBV 1806-20	Eikenberry et al. (2004)	150-1000 $M_\odot$
LBV 1806-20	Figer et al. (2004)	130 (binary?) $M_\odot$
HDE 269810	Walborn et al. (2004)	150 $M_\odot$
WR20a R139	Bonanos et al. (2004) Rauw et al. (2004) Taylor et al (2011)	82+83 $M_\odot$ 78+66 $M_\odot$ binaries
R136 NGC3603	Crowther et al (2010)	~300 $M_\odot$

Figer (2005)
Nature, **434**, 192
Arches cluster
$M < 150\ M_O$

Calculations suggested that strong non-linear pulsations would grow, steepening into shock in the outer layers and driving copious mass loss until the star became low enough in mass that the instability would be relieved.

But what about at low metallicity?

Ezer and Cameron, *Ap&SS*, **14**, 399 (1971) pointed out that $Z = 0$ stars would not burn by the pp-cycle but by a high temperature CNO cycle using catalysts produced in the star itself, $Z \sim 10^{-9}$ to 10^{-7}.

Maeder, *A&A*, **92**, 101, (1980) suggested that low metallicity might raise M_{upper} to 200 solar masses.

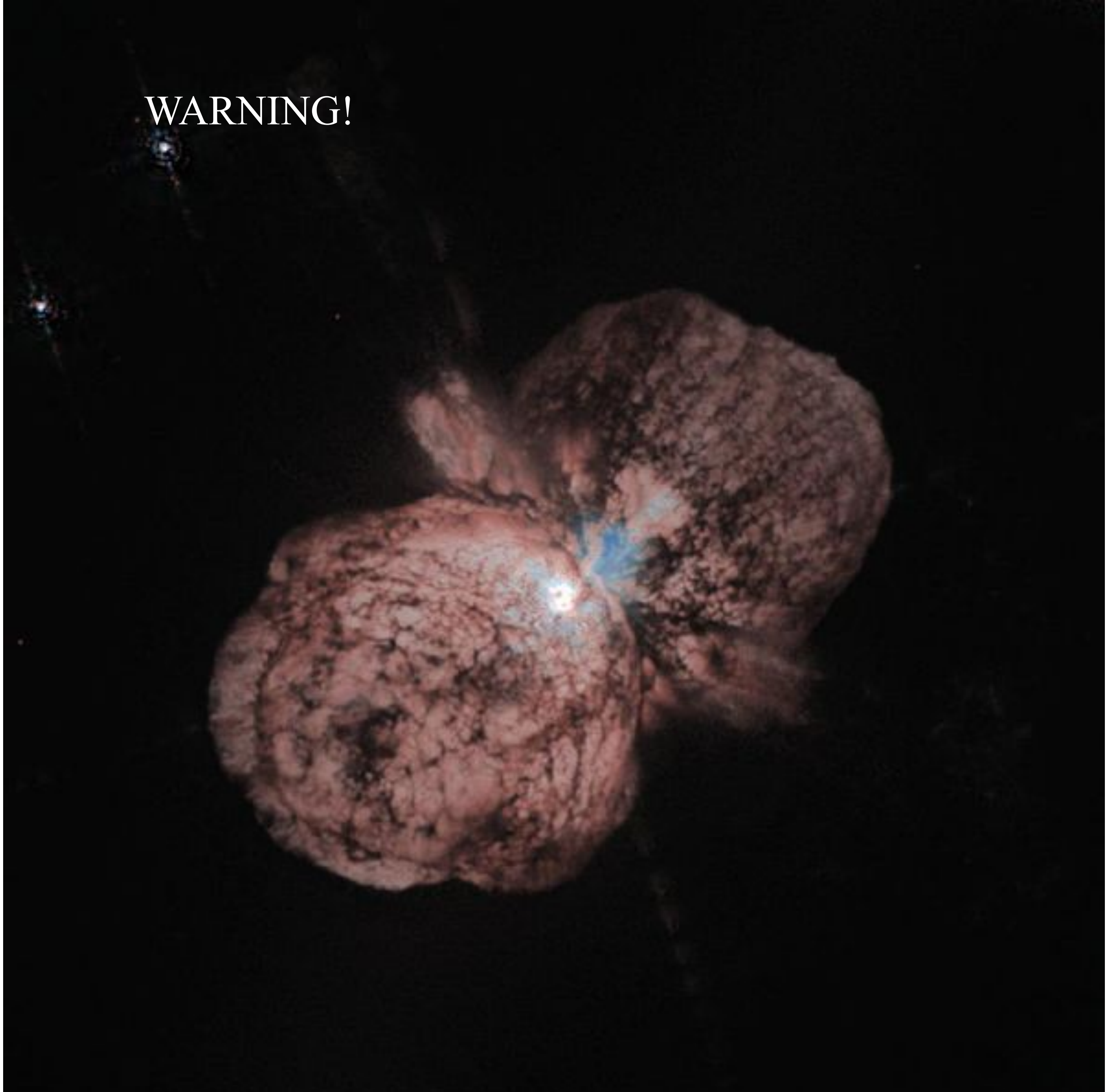
Baraffe, Heger, and Woosley, *ApJ*, **550**, 890, (2001) found that zero metallicity stars (Pop III) are stable on the main sequence up to several hundred solar masses. This only concerns the main sequence though. Mass may be lost later as a giant, especially if nitrogen is produced by dredge up of carbon from the helium burning core.

If very massive stars keep their large masses until their death the resulting supernovae and nucleosynthesis is special.

These stars may also play an important role in reionizing the universe (even if only a tiny fraction of the matter forms into such stars).

$$\frac{8\,\text{Mev/nucleon}}{13\,\text{eV/nucleon}} \sim 10^6$$

There are really three important distinct issues: What masses were the early stars born with, what were there typical masses on the main sequence, and with what mass did they die?


WARNING!

Freezing out of convection?

$$L_{\max} = 4\pi r^2 \rho v_{conv} f \varepsilon$$

with r the radius

ρ the density

v_{conv} the convection speed << c_{sound}

f < 1

ε the internal energy of the zone at radius r

As $\rho \downarrow$ in the RSG envelope c_{sound} decreases

Currently being explored

Appendix

Homology

Homology

$$\frac{dP}{dr} = -\frac{m(r)G}{r^2}\rho;$$

$$\frac{P}{R} \sim \frac{MG}{R^2} \cdot \frac{M}{R^3}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon;$$

$$\frac{L}{R} \sim R^2 \frac{M}{R^3} \epsilon \sim R^2 \frac{M}{R^3} \epsilon_0 \frac{MT^n}{R^3}$$

$$L = \frac{4\pi r^2}{\rho} \frac{c}{\kappa} \frac{d}{dr}\left(\frac{1}{3} a T^4\right);$$

$$L \sim R^2 \frac{1}{\rho\kappa} \frac{T^4}{R} \sim R^2 \left(\frac{R^3}{M}\right)^2 \frac{T^{7/2}}{\kappa_0} \frac{T^4}{R}$$

$$P = \frac{\rho k T}{\mu m_H};$$

$$P \sim \frac{M}{R^3} \frac{T}{\mu} \qquad \text{(gas pressure)}$$

$$P = \frac{1}{3} a T^4;$$

$$P \sim T^4 \qquad \text{(radiation pressure)}$$

Energy generation rates:

$\epsilon = \epsilon_0 \rho T^n$ $\qquad$ n ~ 18

Opacity: $\qquad$ Kramer's opacity law (bound-free)

$\kappa = \kappa_0 \rho T^{-7/2}$ $\qquad$ κ = *const* if eleectron scattering

$\beta =$ constant would imply that the star was an n=3 polytrope!

$$\mathrm{P} = \frac{P_{rad}}{(1-\beta)} = \frac{aT^4}{3(1-\beta)} \Rightarrow T = \left[\frac{3P(1-\beta)}{a}\right]^{1/4}$$

$$P = \frac{P_{gas}}{\beta} = \frac{N_A k}{\mu\beta}\rho T \Rightarrow P = \frac{N_A k}{\mu\beta}\rho\left[\frac{3P(1-\beta)}{a}\right]^{1/4}$$

hence $\mathrm{P}^{3/4} = \frac{N_A k}{\mu\beta}\rho\left[\frac{3(1-\beta)}{a}\right]^{1/4}$ and

$$\mathrm{P} = \left[\frac{3(N_A k)^4(1-\beta)}{a(\mu\beta)^4}\right]^{1/3} \rho^{4/3}$$

If β = constant thoughout the star, this would be the equation for an n = 3 polytrope and the multiplier of $\rho^{4/3}$ is K.

Aside:

For an ideal, non-degenerate gas our (and Clayton's) equations suggest that the electronic entropy is proportional to Y_e (i.e., the number of electrons) and the ionic entropy to $1/\bar{A}$ (the number of ions). For hydrogen burning composition (75% H, 25% He) $Y_e = .875$ and $1/\bar{A} = 0.81$ (Lecture 1)

This suggests that the entropy of the electrons and ions should be about equal in the envelopes on the previous page. Our equation for the entropy is too simple and contains only the T and rho dependent terms for an ideal gas plus radiation. There are additive constants that depend on the mass of the particle

For an ideal gas

$$S_0 = \frac{3}{2}\ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}$$

(Reif - Statistical Physics - 7.3.6)

$\frac{3}{2}\ln(1836) = 11.3$

so in H envelope

$S_i \approx S_e + 10$

Different equation for electrons when they are degenerate – Fermi integrals