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Research Note

Parametrization of stellar rates of mass loss as functions of the fundamental stellar parameters M, L, and R

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Abstract. We investigate the dependence of $\dot{M}$ on the three fundamental parameters mass M, radius R and luminosity L for a sample of 247 stars (number of independent data points with weight unity $=454$). "Average expected" mass-values are derived from evolutionary calculations. A simple formula, viz.

$$-\dot{M} = 9.6310^{-15}\,(L/L_\odot)^{1.42}\,(M/M_\odot)^{0.16}\,(R/R_\odot)^{0.81}\,M_\odot\,\mathrm{yr}^{-1}$$

appears to give a good representation of observed rates of mass loss over the whole Hertzsprung-Russell diagram. For luminous stars with $T_{\mathrm{eff}} > 5000\,\mathrm{K}$ the standard deviation is equal to the intrinsic error (0.37) of the rate of mass loss. The representation is comparable to or even better than that of the expressions by Reimers and Lamers, which were derived for restricted areas of the HR-diagram.

Key words: stars: atmospheres of – stars: mass of – stars: mass loss

1. Introduction

An earlier publication (De Jager et al., 1988; henceforth called Paper 1) summarizes known rates of mass loss for 247 luminous stars and derives an expression for the $\dot{M}(T_{\mathrm{eff}}, L)$ relationship. This is evidently equivalent to giving a $\dot{M}(R, L)$ relationship. However, a number of authors (cf. Reimers, 1977; Lamers, 1981) stress the importance of including the stellar mass in the relationship for cool stars (Reimers), as well as for hot stars (Lamers). Because of the far larger number of data now available, as compared to the numbers used by the earlier authors, we thought it appropriate to develop a new empirical relationship, giving the rate of mass loss as a function of the three fundamental parameters L, M, and R, in the hope to obtain a better representation. The new relation will be compared to those of Reimers and Lamers, and to our earlier 20 points representation (Paper 1).

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2. Mass determinations from evolutionary models

In order to investigate the dependence of mass loss on stellar mass, we need to know the masses of the stars. The most reliable measurements for the mass are from studies of double stars, but for virtually none of the stars in our compilation such data exist. We are therefore obliged to look elsewhere, and have thus decided – following usual practice – to use M-values derived from evolutionary calculations.

Model calculations are presently available which include the effect of overshooting of convection and which also include the effects of mass loss during the evolution of a star (Maeder and Meynet, 1988, 1989). The amount of mass lost during the evolution has been based on observational material (Paper 1). The model calculations give T_{eff}, L, and M as functions of elapsed evolutionary time, for a starting mass M_0, and given stellar abundances. Evolutionary tracks are given for zero-age masses of 0.85 to 120 $M_\odot$. Obviously, the use of another set of model calculations would result in another interpolation formula. This drawback applies to all M-parametrizations that involve stellar mass, and cannot be overcome.

To obtain the masses for a point at (T_{eff}, L) in the HR-diagram, we must invert the given relations.

There is no one-to-one relation between given $(T_{\mathrm{eff}}; L)$ values and the stellar mass, because during their evolution stars of different ZAMS masses may pass through the same $(T_{\mathrm{eff}}; L)$ point in the HR-diagram. At these points different stars may pass with different *dwell-times*, the latter being defined as the time for a star to travel over a $(\log T_{\mathrm{eff}}; \log L)$-vector of unit length in the HR-diagram. Hence, the dwell time is $\Delta t/(\Delta \log T_{\mathrm{eff}}^2 + \Delta \log L^2)^{1/2}$, where Δt is the time needed for travelling over the $(\Delta \log T_{\mathrm{eff}}; \Delta \log L)$-vector.

Let us number from 1 to N (running: n) the various subsequent rightward and leftward going parts ("subtracks") of the stellar evolutionary tracks. The total number of subtracks at a given point can range from 1 to 4, varying for the various parts of the HR-diagram. For example; the number of subtracks is unity close to the ZAMS; the value 4 does not occur very often. Let us call t_n the dwell-time at a certain $(T_{\mathrm{eff}}; L)$ point for the n^{th} subtrack, for which the stars passing through that point have masses M_n. The t_n

values were derived from the above mentioned evolution calculations. The "average expected mass" of a star at ($T_{\rm eff}$; L) is:

$$M = \sum_1^N \Psi(M_n^z) \frac{\mathrm{d}M_n^z}{\mathrm{d}\log L} t_n M_n \Big/ \sum_1^N \Psi(M_n^z) \frac{\mathrm{d}M_n^z}{\mathrm{d}\log L} t_n, \quad (1)$$

where $\Psi(M_n^z)$ is the Initial Mass Function for stars on the subtrack with index n. For these stars the ZAMS masses are M_n^z; $\mathrm{d}M_n^z/\mathrm{d}\log L$ represents the density of evolutionary tracks over an unit $\log L$-interval. We took $\Psi(M) = M^{-2.5}$ (Humphreys and McElroy, 1984; Buat et al., 1987).

We have developed a computer program for the calculation of M following Eq. (1). This program is available on request.

3. Mass loss rate as a function of *L*, *R* and *M*

For each of the stars in Table 1 of our Paper 1, we have determined the average expected masses, as described above, and we considered the representation:

$$-\dot{M} = K M^a L^b R^c.$$

We found that the expression

$$\log(-\dot{M}) = -14.02 + 1.24 \log\left(\frac{L}{L_\odot}\right) + 0.16 \log\left(\frac{M}{M_\odot}\right) + 0.81 \log\left(\frac{R}{R_\odot}\right) \quad (2)$$

has a σ-value over the whole upper part of the HR-diagram of $\sigma = 0.50$. It is gratifying that this value is only slightly larger than the σ-value for our 20-terms representation in T and L (Paper 1) which we have calculated anew, and which we have found to be 0.44; the differences being due to the way the weights of the data points have been included.

Since use of $T_{\rm eff}$ is useful in practice, Eq. (2) is rewritten logarithmically as

$$\log(-\dot{M}) = -7.93 + 1.64 \log\left(\frac{L}{L_\odot}\right) + 0.16 \log\left(\frac{M}{M_\odot}\right) - 1.61 \log T_{\rm eff}. \quad (3)$$

We also investigated the accuracy of fit in five different parts of the HR-diagram by calculating the standard deviations σ of the fits. Table 1 summarizes the results. For comparison we also calculated the σ-value for the 20-points representation (Paper 1). A very good fit is found for stars with $T_{\rm eff} > 5000$ K and $\log(L/L_\odot) > 5.0$ where σ is 0.36. This is equal to the *intrinsic* deviation of a measurement of unit weight, which is 0.37 (Paper 1). A better representation can obviously not be offered.

Table 1. The accuracy of the four-points fit (3) as compared to that of our previous 20-points representation formula. $\sum\omega$ denotes the weighted number of datapoints, read from Table 1 (Paper 1) with the proviso that the weight of individual objects has not been allowed to exceed 5 (= maximum number of individual measurements)

Range		$\sum\omega$	σ-value	
$\log L/L_\odot$	$\log T$		20-points	4-points
All	All	454	0.44	0.50
All	>4.0	326	0.38	0.39
>5.0	>3.7	314	0.35	0.36
>5.0	<3.7	39	0.51	0.68
<5.0	>3.7	25	0.65	0.78
<5.0	<3.7	74	0.54	0.74

4. Other representations

In the literature a number of formulae have been given, describing rates of mass loss in terms of L, M, and R or other related parameters. Since only two of these are generally used, we restrict our comparison to these two expressions:

1. From a study of 41 hot stars Lamers (1981) derived a formula which, after some algebra, is rewritten in logarithmic form as:

$$\log(-\dot{M}) = -8.20 + 1.72 \log\left(\frac{L}{L_\odot}\right) - 0.99 \log\left(\frac{M}{M_\odot}\right) - 1.21 \log T_{\rm eff}. \quad (4)$$

2. For cool stars Reimers's (1977) expression, based on a dozen of stars, is often used. After inserting constants his expression becomes

$$\log(-\dot{M}) = -4.74 + 1.50 \log\left(\frac{L}{L_\odot}\right) - \log\left(\frac{M}{M_\odot}\right) - 2.00 \log T_{\rm eff}. \quad (5)$$

We verified the goodness-of-fit of these two expressions for the relevant parts of our data set, and we calculated the σ-values for our 20-point representation, for the 4-points fit, and for expression (4) or (5) respectively.

Table 2. A comparison of the accuracy of the 20-points representation, and that of Eq. (3) with that of expressions of Lamers (4) and Reimers (5)

Eq.	Range		$\sum\omega$	σ-value		
	$\log L/L_\odot$	$\log T$		20-points	Eq. (3)	Eq. (4) or Eq. (5)
(4)	all	>4.0	326	0.38	0.39	0.46
(5)	>5.0	<3.7	39	0.51	0.68	0.83
(5)	<5.0	<3.7	74	0.54	0.74	0.68

The results are given in Table 2, from which we read that the 20-points representation is in all cases the best. Our new expression (3) is more accurate than that of Lamers, as well as that for Reimers $\log(L/L_{\odot}) > 5.0$, but it is slightly worse than Reimers' expression for stars with $\log(L/L_{\odot}) < 5.0$.

We conclude that for most cases the use of Eq. (3) can be recommended for the whole of the HR-diagram. However, the best precision can be obtained with our 20-points representation.

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