Stellar Structure and Evolution: Syllabus

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Primary Textbooks

- ***ZG:*** **Zeilik & Gregory, "Introductory Astronomy & Astrophysics" (4th edition)**
- ***CO:*** **Carroll & Ostlie, "An Introduction to Modern Astrophysics" (Addison-Wesley)**
- **also: Prialnik, "An Introduction to the Theory of Stellar Structure and Evolution"**

1. ***Observable Properties of Stars*** **(ZG: Chapters 11, 12, 13; CO: Chapters 3, 7, 8, 9)**
   - **1.1 Luminosity, Parallax (ZG: 11; CO: 3.1)**
   - **1.2 The Magnitude System (ZG: 11; CO: 3.2, 3.6)**
   - **1.3 Black-Body Temperature (ZG: 8-6; CO: 3.4)**
   - **1.4 Spectral Classification, Luminosity Classes (ZG: 13-2/3; CO: 5.1, 8.1, 8.3)**
   - **1.5 Stellar Atmospheres (ZG: 13-1; CO: 9.1, 9.4)**
   - **1.6 Stellar Masses (ZG: 12-2/3; CO: 7.2, 7.3)**
   - **1.7 Stellar Radii (ZG: 12-4/5; CO: 7.3)**
2. ***Correlations between Stellar Properties*** **(ZG: Chapters 12, 13, 14; CO: Chapters 7, 8, 13)**
   - **2.1 Mass-Luminosity Relations (ZG: 12-2; CO: 7.3)**
   - **2.2 Hertzsprung-Russell diagrams and Colour-Magnitude Diagrams (ZG: 13-3; CO: 8.2)**
   - **2.3 Globular Clusters and Open (Galactic) Clusters (ZG:13-3, 14-2; OG: 13.4)**
   - **2.4 Chemical Composition (ZG: 13-3; CO: 9.4)**
   - **2.5 Stellar Populations (ZG: 14-3; CO: 13.4)**
3. ***The Physical State of the Stellar Interior*** **(ZG: P5, 16; CO: 10)**
   - **3.1 The Equation of Hydrostatic Equilibrium (ZG: 16-1; CO: 10.1)**
   - **3.2 The Dynamical Timescale (ZG: P5-4; CO: 10.4)**
   - **3.3 The Virial Theorem and its Implications (ZG: P5-2; CO: 2.4)**
   - **3.4 The Energy Equation and Stellar Timescales (CO: 10.3)**
   - **3.5 Energy Transport by Radiation (ZG: P5-10, 16-1) and Convection (ZG: 16-1; CO: 9.3, 10.4)**
4. ***The Equations of Stellar Structure*** **(ZG: 16; CO: 10)**
   - **4.1 The Mathematical Problem (ZG: 16-2; CO: 10.5)**
     - **4.1.1 The Vogt-Russell "Theorem" (CO: 10.5)**
     - **4.1.2 Stellar Evolution**
     - **4.1.3 Convective Regions (ZG: 16-1; CO: 10.4)**
   - **4.2 The Equation of State**
     - **4.2.1 Perfect Gas and Radiation Pressure (ZG: 16-1: CO: 10.2)**
     - **4.2.2 Electron Degeneracy (ZG: 17-1; CO: 15.3)**
   - **4.3 Opacity (ZG: 10-2; CO: 9.2)**
5. ***Nuclear Reactions*** **(ZG: P5-7 to P5-9, P5-12, 16-1D; CO: 10.3)**
   - **5.1 Nuclear Reaction Rates (ZG: P5-7)**
   - **5.2 Hydrogen Burning**
     - **5.2.1 The pp Chain (ZG: P5-7, 16-1D)**
     - **5.2.2 The CN Cycle (ZG: P5-9; 16-1D)**
   - **5.3 Energy Generation from H Burning (CO: 10.3)**
   - **5.4 Other Reactions Involving Light Elements *(Supplementary)***
   - **5.5 Helium Burning (ZG: P5-12; 16-1D)**
6. ***The Evolution of Stars***
   - **6.1 The Structure of Main-Sequence Stars (ZG: 16-2; CO 10.6, 13.1)**
   - **6.2 The Evolution of Low-Mass Stars (ZG: 16-3; CO: 13.2)**
     - **6.2.1 The Pre-Main Sequence Phase**
     - **6.2.2 The Core Hydrogen-Burning Phase**
     - **6.2.3 The Red-Giant Phase**
     - **6.2.4 The Helium Flash**
     - **6.2.5 The Horizontal Branch**
     - **6.2.6 The Asymptotic Giant Branch**
     - **6.2.7 White Dwarfs and the Chandrasekhar Mass (ZG: 17-1; CO: 13.2)**

6.3 The Evolution of Massive Stars (CO: 13.3)

6.4 Supernovae (ZG: 18-5B/C/D)

6.4.1 Explosion Mechanisms

6.4.2 Supernova Classification

6.4.3 SN 1987 A (ZG: 18-5E)

6.4.4 Neutron Stars (ZG: 17-2; CO: 15.6)

6.4.5 Black Holes (ZG: 17-3; CO: 16)

7. *Binary Stars* (ZG: 12; CO: 7, 17)

7.1 Classification

7.2 The Binary Mass Function

7.3 The Roche Potential

7.4 Binary Mass Transfer

7.5 Interacting Binaries *(Supplementary)*

Appendices (Supplementary Material)

A. Brown Dwarfs (ZG: 17-1E)

B. Planets (ZG: 7-6; CO: 18.1)

C. The Structure of the Sun and The Solar Neutrino Problem (ZG: P5-11, 10, 16-1D; CO: 11.1)

D. Star Formation (ZG: 15.3; CO: 12)

E. Gamma-Ray Bursts (ZG: 16-6; CO: 25.4)

Useful Numbers

Astronomical unit	$\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}$
Parsec	$\mathrm{pc} = 3.26\ \mathrm{ly}$
	$= 3.086 \times 10^{16}\,\mathrm{m}$
Lightyear	$\mathrm{ly} = 9.46 \times 10^{15}\,\mathrm{m}$
Mass of Sun	$\mathrm{M}_\odot = 1.99 \times 10^{30}\,\mathrm{kg}$
Mass of Earth	$\mathrm{M}_\oplus = 5.98 \times 10^{24}\,\mathrm{kg}$
	$= 3 \times 10^{-6}\,\mathrm{M}_\odot$
Mass of Jupiter	$\mathrm{M_{Jup}} = 10^{-3}\,\mathrm{M}_\odot$
Radius of Sun	$\mathrm{R}_\odot = 6.96 \times 10^{8}\,\mathrm{m}$
Radius of Earth	$\mathrm{R}_\oplus = 6380\,\mathrm{km}$
Radius of Jupiter	$\mathrm{R_{Jup}} = 10^{-3}\,\mathrm{R}_\odot$
Luminosity of Sun	$\mathrm{L}_\odot = 3.86 \times 10^{26}\,\mathrm{W}$
Effective temperature of Sun	$\mathrm{T_{eff}} = 5780\,\mathrm{K}$
Central temperature of Sun	$\mathrm{T_c} = 15.6 \times 10^{6}\,\mathrm{K}$
Distance to the Galactic centre	$\mathrm{R_0} = 8.0\,\mathrm{kpc}$
Velocity of Sun about Galactic centre	$\mathrm{V_0} = 220\,\mathrm{km\,s^{-1}}$
Diameter of Galactic disc	$= 50\,\mathrm{kpc}$
Mass of Galaxy	$= 7 \times 10^{11}\,\mathrm{M}_\odot$

Summary of Equations

Equation of Stellar Structure

Equation of Hydrostatic Equilibrium:

$$\frac{\mathrm{d}P_r}{\mathrm{d}r} = -\frac{GM_r\rho_r}{r^2} \quad \text{(page 45)}$$

Equation of Mass Conservation:

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho_r \quad \text{(page 45)}$$

Energy Conservation (no gravitational energy):

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2\rho_r\varepsilon_r \quad \text{(page 52)}$$

Energy Transport (Radiative Diffusion Equation):

$$L_r = -4\pi r^2 \,\frac{4ac}{3\kappa\rho}\,T^3\,\frac{\mathrm{d}T}{\mathrm{d}r} \quad \text{(page 55)}$$

Energy Transport by Convection, Convective Stability:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{\gamma-1}{\gamma}\,\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r} \quad \text{(page 57)}$$

Constitutive Relations

Equation of State, Ideal Gas:

$$P = NkT = \frac{\rho}{\mu m_{\mathrm{H}}}kT \quad \text{(page 65)}$$

Equation of State, Radiation Pressure:

$$P = \frac{1}{3}aT^4 \quad \text{(page 66)}$$

Equation of State, Electron Degeneracy ($T = 0\,K$):

$$P = K_1\left(\frac{\rho}{\mu_e m_{\mathrm{H}}}\right)^{5/3} \quad \text{(page 66)}$$

(non-relativistic degeneracy)

$$P = K_2\left(\frac{\rho}{\mu_e m_{\mathrm{H}}}\right)^{4/3} \quad \text{(page 67)}$$

(relativistic degeneracy)

Notes:

Opacity: Thomson (Electron) Scattering:

$$\kappa = 0.020\,\mathrm{m}^2\,\mathrm{kg}^{-1}\,(1+X) \quad \text{(page 69)}$$

Kramer's Opacity:

$$\kappa \propto \rho\,T^{-3.5} \quad \text{(page 69)}$$

Low-Temperature Opacity:

$$\kappa \propto \rho^{1/2}\,T^4 \quad \text{(page 69)}$$

Energy Generation Rates (Rough!)

PP Burning:

$$\varepsilon_{\mathrm{PP}} \propto \rho\,X_{\mathrm{H}}^2\,T^4 \quad \text{(page 79)}$$

CNO Burning:

$$\varepsilon_{CNO} \propto \rho\,X_{\mathrm{H}}\,X_{\mathrm{CNO}}\,T^{20} \quad \text{(page 79)}$$

Helium Burning (triple α):

$$\varepsilon_{3\alpha} \propto X_{\mathrm{He}}^3\,\rho^2\,T^{30} \quad \text{(page 82)}$$

Stellar Timescales

Dynamical Timescale:

$$t_{\mathrm{dyn}} \simeq \frac{1}{\sqrt{4G\rho}} \quad \text{(page 48)}$$

$$\sim 30\,\mathrm{min}\,\left(\rho/1000\,\mathrm{kg\,m}^{-3}\right)^{-1/2}$$

Thermal (Kelvin-Helmholtz) Timescale):

$$t_{\mathrm{KH}} \simeq \frac{GM^2}{2RL} \quad \text{(page 51)}$$

$$\sim 1.5\times10^7\,\mathrm{yr}\ (M/M_\odot)^2\ (R/R_\odot)^{-1}\ (L/L_\odot)^{-1}$$

Nuclear Timescale:

$$t_{\mathrm{nuc}} \simeq M_c/M\ \eta\ (Mc^2)/L \quad \text{(page 52)}$$

$$\sim 10^{10}\,\mathrm{yr}\ (M/M_\odot)^{-3}$$

(Radiative) Diffusion Timescale:

$$t_{\mathrm{diff}} = N\times\frac{l}{c} \simeq \frac{R_s^2}{lc} \quad \text{(page 53)}$$

Notes:

Derived Relations

Central Temperature Relation (for Ideal Gas):

$$kT_{\rm c} \simeq \frac{GM_s\,\mu m_H}{R_s} \quad \text{(page 46)}$$

Virial Theorem:

$$3(\gamma-1)U + \Omega = 0 \quad \text{(page 50)}$$

Mass–Luminosity Relation (for stars $\sim 1\,M_\odot$):

$$L \simeq L_\odot \left(\frac{M}{M_\odot}\right)^4 \quad \text{(page 85)}$$

Mass–MS Lifetime Relation (for stars $\sim 1\,M_\odot$):

$$T_{\rm MS} \simeq 10^{10}\,{\rm yr}\left(\frac{M}{M_\odot}\right)^{-3} \quad \text{(page 85)}$$

Mass–Radius Relation for White Dwarfs (non-relativistic):

$$R \propto \frac{1}{m_e}\,(\mu_e m_{\rm H})^{5/3}\,M^{-1/3} \quad \text{(page 98)}$$

Chandrasekhar Mass for White Dwarfs:

$$M_{\rm Ch} = 1.457\left(\frac{2}{\mu_e}\right)^2 M_\odot \quad \text{(page 99)}$$

Schwarzschild Radius (Event Horizon) for Black Holes:

$$R_{\rm S} = \frac{2GM}{c^2} \simeq 3\,{\rm km}\left(\frac{M}{M_\odot}\right) \quad \text{(page 112)}$$

Notes:

Miscellaneous Equations

Distance Modulus:

$$(m-M)_V = 5\,\log\left(D/10{\rm pc}\right) \quad \text{(page 12)}$$

Absolute V Magnitude:

$$M_V = -2.5\,\log\,L/L_\odot + 4.72 + B.C. + A_V \quad \text{(page 12)}$$

Salpeter Initial Mass Function (IMF):

$$f(M)\,{\rm d}M \propto M^{-2.35}\,{\rm d}M \quad \text{(page 15)}$$

Black-Body Relation:

$$L = 4\pi\,R_s^2\,\sigma T_{\rm eff}^4 \quad \text{(page 17)}$$

Kepler's Law:

$$a^3\left(\frac{2\pi}{P}\right)^2 = G(M_1+M_2) \quad \text{(page 25)}$$

Notes:

STELLAR STRUCTURE AND EVOLUTION

1. OBSERVABLE PROPERTIES OF STARS

Basic large-scale observable properties:

Luminosity
Surface temperature
Radius
Mass

Further observable:

Spectrum ... **yields information about surface chemical composition and gravity**

Evidence from:

- **Individual stars**
- **Binary systems**
- **Star clusters....these reveal how stars evolve with time**
- **Nuclear physics...energy source, synthesis of heavy elements**

No direct information about physical conditions in stellar interiors (except from helioseismology and solar neutrinos)
No direct evidence for stellar evolution......typical timescale $10^6 - 10^9$ years.......(except for a few very unusual stars and supernovae)

Notes:

1.1 LUMINOSITY (ZG: 11; CO: 3.1)
('power', [J/s=W])

$$\boxed{L_s = \int_0^\infty L_\lambda d\lambda = 4\pi R_s^2 \int_0^\infty F_\lambda d\lambda}$$

where F_λ is the *radiative flux* at wavelength λ at the stellar surface, R_s the stellar radius. Energy may also be lost in the form of neutrinos or by direct mass loss (generally unobservable).

Astronomers measure:

$$f_\lambda = (R_s/D)^2 F_\lambda \qquad \text{at Earth's surface}$$

- **To obtain L_λ we must know the star's *distance D* and correct for:**
  - ▷ ***absorption in the Earth's* atmosphere (standard methods)**
  - ▷ ***absorption in interstellar space* (negligible for nearby stars)**
- **Measurements from the *Hipparcos satellite* (1989–1993) have yielded *parallaxes* accurate to 0.002 arcsec for about 100,000 stars. The largest stellar parallax (Proxima Centauri) is 0.765 arcsec.**

Notes:

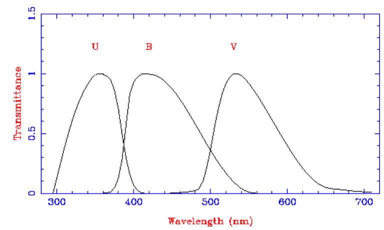
1.2 STELLAR MAGNITUDES (ZG: 11; CO: 3.2, 3.6)

- measure ***stellar flux*** (i.e. $f = L/4\pi D^2$, L: luminosity, D: distance)
  - ▷ for ***Sun:*** $L_\odot = 3.86 \times 10^{26}\,W$, $f = 1.360 \times 10^3\,W\,m^{-2}$ ***(solar constant)***
  - ▷ luminosity measurement requires distance determination ($1 A.U. = 1.50 \times 10^{11}\,m$)
- define ***apparent magnitudes*** of two stars, m_1, m_2, by $m_1 - m_2 = 2.5 \log f_2/f_1$
- zero point: ***Vega*** (historical) → $m_\odot = -26.82$
- to measure ***luminosity*** define ***absolute magnitude M*** to be the apparent magnitude of the object if it were at a distance 10 pc (1 pc = 3.26 light years = $3.09 \times 10^{16}\,m$)
- define ***bolometric magnitude*** as the absolute magnitude corresponding to the luminosity integrated over all wavebands; for the Sun $M_\odot^{bol} = 4.72$
- in practice, the total luminosity is difficult to measure because of atmospheric absorption and limited detector response
- define magnitudes over limited wavelength bands

Notes:

THE UBV SYSTEM

- the UBV system (ultraviolet, blue, visual) which can be extended into the red, infrared (RI)



approximate region	notation for magnitudes apparent	absolute	solar value
ultraviolet	U or m_U	M_U	5.61
blue	B or m_B	M_B	5.48
visual (near yellow)	V or m_V	M_V	4.83

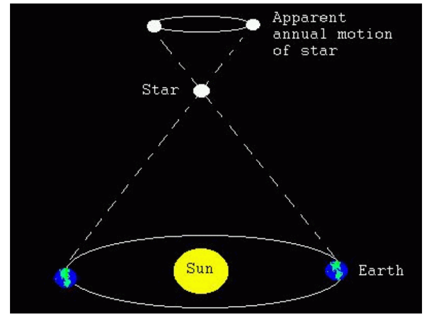
- ***colours (colour indices):*** relative magnitudes in different wavelength bands, most commonly used: B – V, U – B
- define ***bolometric correction:*** $B.C. = M_{bol} - M_V$ (usually tabulated as a function of B – V colour)
- ***visual extinction*** A_V***:*** absorption of visual star light due to extinction by interstellar gas/dust (can vary from ∼ 0 to 30 magnitudes [Galactic centre])
- ***distance modulus:*** $(m - M)_V = 5 \times \log D/10pc$
- summary: $M_V = \underbrace{-2.5 \log L/L_\odot + 4.72}_{M_{bol}} - B.C. + A_V$

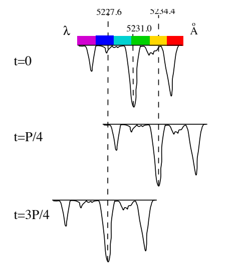
Notes:

Nearby Stars to the Sun (from Norton 2000)

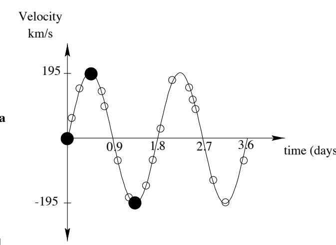
Common Name (Scientific Name)	Distance (light year)	Magnitudes apparent	Magnitudes absolute	spectral type
Sun		-26.8	4.8	G2V
Proxima Centauri (V645 Cen)	4.2	11.05 (var)	15.5	M5.5V
Rigel Kentaurus (Alpha Cen A)	4.3	-0.01	4.4	G2V
(Alpha Cen B)	4.3	1.33	5.7	K1V
Barnard's Star	6.0	9.54	13.2	M3.8V
Wolf 359 (CN Leo)	7.7	13.53 (var)	16.7	M5.8V
(BD +36 2147)	8.2	7.50	10.5	M2.1V
Luyten 726-8A (UV Cet A)	8.4	12.52 (var)	15.5	M5.6V
Luyten 726-8B (UV Cet B)	8.4	13.02 (var)	16.0	M5.6V
Sirius A (Alpha CMa A)	8.6	-1.46	1.4	A1V
Sirius B (Alpha CMa B)	8.6	8.3	11.2	DA
Ross 154	9.4	10.45	13.1	M4.9V

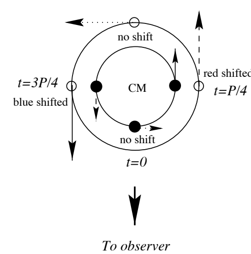
Notes:





Doppler shifted absorption line spectra





Notes:

- **Accurate information about *relative luminosities* has been obtained from measuring relative apparent brightnesses of *stars within clusters.***
- **Some wavelengths outside the visible region are completely absorbed by the *Earth's atmosphere.* Hence we must use theory to estimate contributions to L_s from obscured spectral regions until *satellite measurements* become available.**
- **Observations of clusters show that *optical luminosities of stars* cover an enormous range:**

$$10^{-4}\ L_\odot < L_s < 10^{6}\ L_\odot$$

- **By direct measurement:**

$$L_\odot = (3.826 \pm 0.008) \times 10^{26}\ \mathrm{W}.$$

- **The *luminosity function* for nearby stars shows the overwhelming preponderance of intrinsically faint stars in the solar neighbourhood. Highly luminous stars are very rare: the majority of nearby stars are far less luminous than the Sun.**
- ***Initial mass function (IMF):* distribution of stellar masses (in mass interval dM)**

$$f(M)\,dM \propto M^{-\gamma}\,dM \quad \gamma \simeq 2.35\ \text{[Salpeter] to 2.5}$$

(good for stars more massive than $\gtrsim 0.5\,M_\odot$).

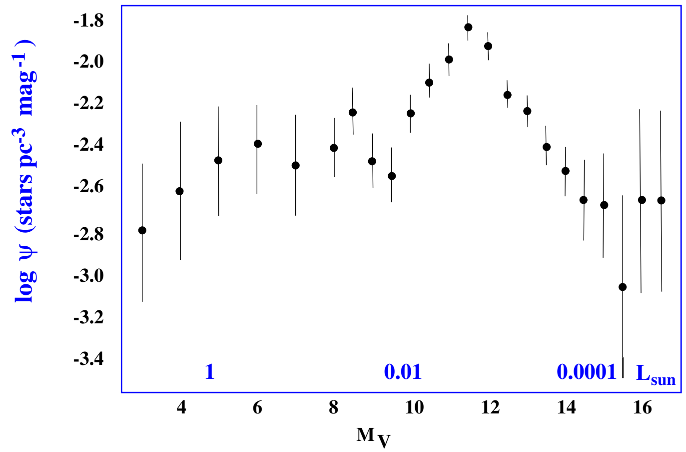
$\rightarrow$ **most of the mass in stars is locked up in low-mass stars (brown dwarfs?)**

$\triangleright$ ***but* most of the luminosity comes from massive stars.**

Notes:

Luminosity Function

(after Kroupa)



Notes:

1.3 STELLAR SURFACE TEMPERATURES
(ZG: 8-6; CO: 3.4)

Various methods for ascribing a temperature to the stellar photosphere:

1. ***Effective temperature,*** T_{eff} (equivalent black-body temperature):

$$\boxed{L_s = 4\pi R_s^2 \int F_\lambda d_\lambda = 4\pi R_s^2 \sigma T_{eff}^4}$$

   Direct determination of T_{eff} not generally possible because R_s is not measurable except in a few cases. T_{eff} can be derived indirectly using model atmospheres.

2. *Colour temperature*

   ▷ Match shape of observed continuous spectrum to that of a *black body,*

$$\Phi(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}.$$

   ▷ An *empirical relationship* between *colour temperature* and *B-V* has been constructed (B and V are magnitudes at λ_B and λ_V respectively).

Notes:

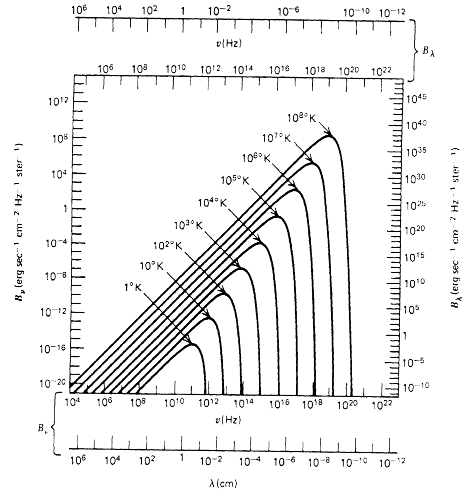


Figure 1.11 Spectrum of blackbody radiation at various temperatures (taken from Kraus, J. D. 1966, Radio Astronomy, McGraw-Hill Book Company)

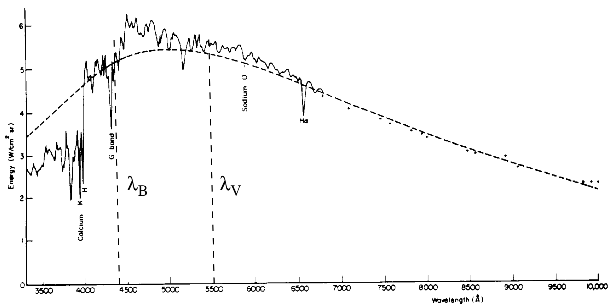
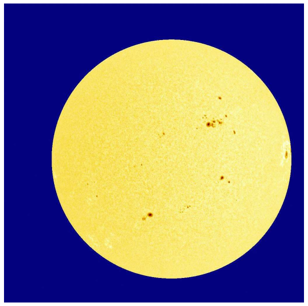
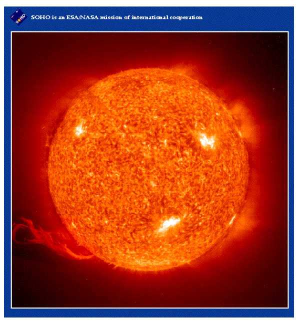
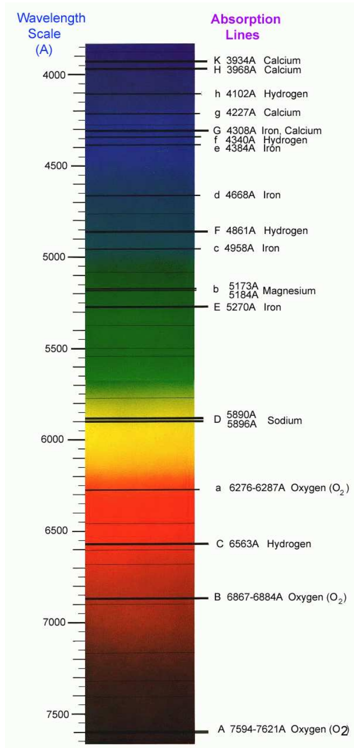


Figure 9.5 The spectrum of the Sun. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)

Notes:







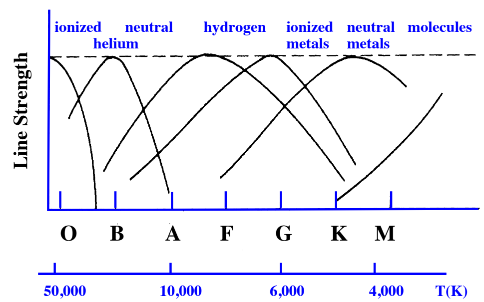
Notes:

1.4 SPECTRAL CLASSIFICATION (ZG: 13-2/3; CO: 5.1, 8.1, 8.3)

- Strengths of spectral lines are related to *excitation temperature* and *ionization temperature* of photosphere through Boltzmann and Saha equations.
- An *empirical relation* between *spectral class and surface temperature* has been constructed (e.g. Sun: G2 → 5,800 K).
- Different properties yield different temperatures. Only a full *model atmosphere calculation* can describe all spectral features with a unique T_{eff}: not available for most stars. Normally astronomers measure V and B − V and use an empirical relation based on model atmosphere analysis of a limited number of stars to convert V to L_s and B − V to T_{eff}.
- L_s *and* T_{eff} are the key quantities output by stellar structure model calculations.
- *Range of* T_{eff}*:* $2000\,K < T_{eff} < 100,000\,K$

Notes:

Spectral Classification



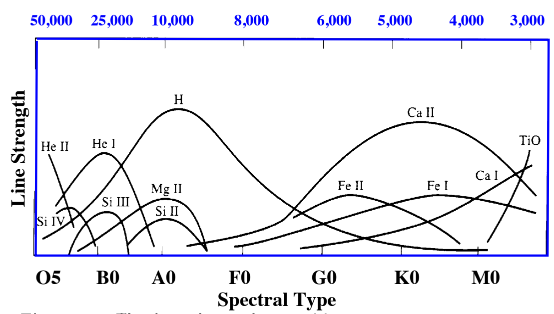


Figure 8.9 The dependence of spectral line strengths on temperature.

Notes:

Luminosity Classes

Class	Type of Star
Ia	Luminous supergiants
Ib	Less Luminous supergiants
II	Bright giants
III	Normal giants
IV	Subgiants
V	Main-sequence stars (Dwarfs)
VI, sd	Subdwarfs
D	White Dwarfs

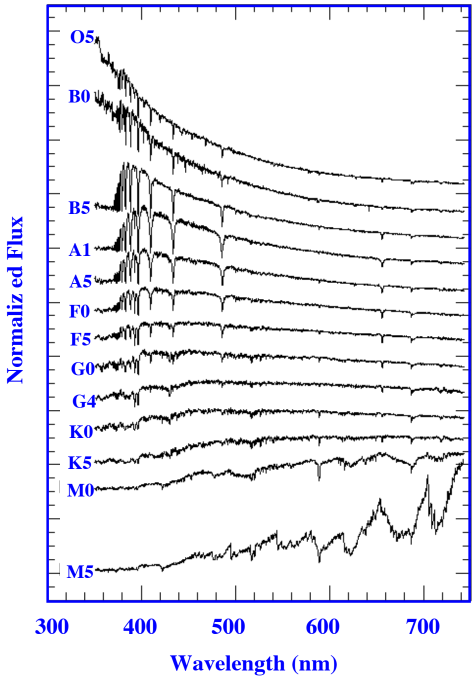
- The luminosity class is essentially based on the ***width of spectral lines***
- narrow lines $\rightarrow$ low surface pressure $\rightarrow$ low surface gravity $\rightarrow$ big star
- supergiants have narrow lines, white dwarfs (the compact remnants of low-/intermediate-mass stars) very broad lines

L Stars/T Dwarfs

- recent extension of the spectral classification for ***very cool*** ($T_{\rm eff} < 2500\,K$) objects, mainly ***brown dwarfs*** (?) (low-mass objects many with $M < 0.08\,M_\odot$ which are not massive enough for nuclear reactions in the core)

Notes:

Spectra of Dwarf Stars (Luminosity Class V)



Notes:

1.5 STELLAR ATMOSPHERES (ZG: 13-1; CO: 9.1, 9.4)

- ***Continuum spectrum:*** defines ***effective temperature*** (T_{eff}) and ***photospheric radius*** (R_{ph}) through
  $L_{bol} = 4\pi\, R_{ph}^2\, \sigma\, T_{eff}^4$
- ***absorption lines*** in the spectrum are caused by cooler material above the photosphere
- ***emission lines*** are caused by hotter material above the photosphere
- ***spectral lines*** arise from transitions between the bound states of atoms/ions/molecules in the star's atmosphere
- spectral lines contain a wealth of information about
  - ▷ the ***temperature*** in regions where the lines are produced → spectral type
  - ▷ the ***chemical composition*** → ***nucleosynthesis*** in stars
  - ▷ ***pressure*** → ***surface gravity*** → ***luminosity class***
  - ▷ ***stellar rotation:*** in rapidly rotating stars, spectral lines are Doppler broadened by rotation
  - ▷ ***orbital velocities*** (due to periodic Doppler shifts) in binaries

Notes:

1.6 STELLAR MASSES (ZG: 12-2/3; CO: 7.2, 7.3)

Only one direct method of mass determination: study dynamics of binary systems. By *Kepler's third law:*

$$\boxed{(M_1 + M_2)/M_\odot = a^3/P^2}$$

a = semi-major axis of apparent orbit in astronomical units; P = period in years.

a) *Visual binary stars:*

- ▷ Sum of masses from above
- ▷ Ratio of masses if absolute orbits are known

$$M_1/M_2 = a_2/a_1 \qquad a = a_1 + a_2$$

- ▷ Hence M_1 and M_2 but only a few reliable results.

b) *Spectroscopic binary stars:*

- ▷ Observed *radial velocity* yields $v \sin i$ (inclination i of orbit in general unknown). From both velocity curves, we can obtain M_1/M_2 and $M_1 \sin^3 i$ and $M_2 \sin^3 i$ i.e. lower limits to mass (since $\sin i < 1$).
- ▷ For *spectroscopic eclipsing binaries* $i \sim 90^o$; hence determination of M_1 and M_2 possible. About 100 good mass determinations; all main-sequence stars.

- Summary of mass determinations:
  - ▷ Apart from main-sequence stars, reliable masses are known for 3 white dwarfs
    a few giants
  - ▷ *Range of masses:* $0.1 M_\odot < M_s < 200 M_\odot$.

Notes:

1.7 STELLAR RADII (ZG: 12-4/5; 7.3)

In general, stellar angular diameters are too small to be accurately measurable, even for nearby stars of known distance.

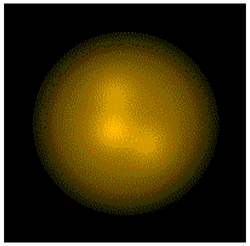
$$R_\odot = 6.96 \times 10^5 \text{ km}$$

- *Interferometric measurements:*
  - a) Michelson stellar interferometer results for 6 stars ($R_s >> R_\odot$)
  - b) Intensity interferometer results for 32 stars (all hot, bright main-sequence stars with $R_s \sim R_\odot$)
- *Eclipsing binaries:*
  - ▷ Measure periodic brightness variations
  - ▷ reliable radii for a few hundred stars.
- *Lunar occultations:*
  - ▷ Measure diffraction pattern as lunar limb occults star
  - ▷ results for about 120 stars.

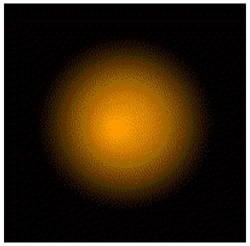
Notes:

Optical Interferometry (WHT, COAST): Betelgeuse

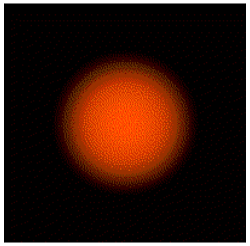
700 nm

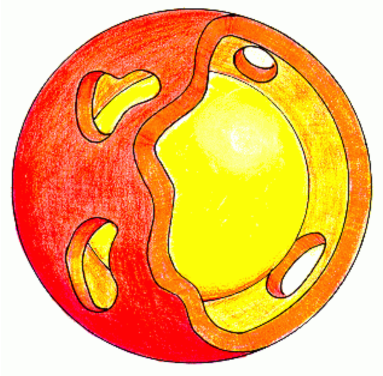


905 nm



1290 nm



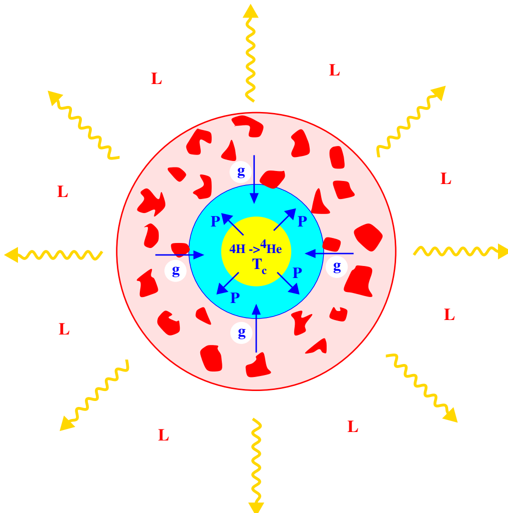


Notes:

- ***Indirect methods:***
  - ▷ e.g. use of $L_s = 4\pi R_s^2 \sigma T_{eff}^4$ with estimates of L_s and T_{eff}.

- ***Summary of measurements of radii:***
  - ▷ ***Main-sequence stars*** have similar radii to the Sun; R_s increases slowly with surface temperature.
  - ▷ Some stars have much smaller radii $\sim 0.01R_\odot$ (white dwarfs)
  - ▷ Some stars have much larger radii $> 10R_\odot$ (giants and supergiants)
  - ▷ ***Range of radii:*** $0.01R_\odot < R_s < 1000R_\odot$.

Notes:



Notes:

Stellar Structure and Stellar Evolution

- ***physical laws*** **that determine the equilibrium structure of a star**
- ***stellar birth*** **in protostellar clouds** $\rightarrow$ ***planet*** **formation in circumstellar discs,** ***binarity, brown dwarfs***
- **stellar evolution driven by successive phases of** ***nuclear burning,*** $\rightarrow$ ***giants, supergiants***
- **final stages of stars:**
  - ▷ ***white dwarfs*** **and** ***planetary nebula ejection*** ($\mathrm{M} \lesssim 8\,\mathrm{M}_\odot$)
  - ▷ ***supernova explosions*** **for massive stars** ($\mathrm{M} \gtrsim 8\,\mathrm{M}_\odot$)**, leaving** ***neutron star (pulsar), black-hole*** **remnants**

Stellar Atmospheres

- ***basic physics*** **that determines the structure of stellar atmospheres,** ***line formation***
- **modelling spectral lines to determine** ***atmospheric properties, chemical composition***

Notes:

Selected Properties of Main-Sequence Stars

Sp	M_V	$B - V$	B.C.	M_{bol}	$\log T_{eff}$ (K)	$\log R$ ($R_\odot$)	$\log M$ ($M_\odot$)
O5	-5.6	-0.32	-4.15	-9.8	4.626	1.17	1.81
O7	-5.2	-0.32	-3.65	-8.8	4.568	1.08	1.59
B0	-4.0	-0.30	-2.95	-7.0	4.498	0.86	1.30
B3	-1.7	-0.20	-1.85	-3.6	4.286	0.61	0.84
B7	-0.2	-0.12	-0.80	-1.0	4.107	0.45	0.53
A0	0.8	+0.00	-0.25	0.7	3.982	0.36	0.35
A5	1.9	+0.14	0.02	1.9	3.924	0.23	0.26
F0	2.8	+0.31	0.02	2.9	3.863	0.15	0.16
F5	3.6	+0.43	-0.02	3.6	3.813	0.11	0.08
G0	4.4	+0.59	-0.05	4.4	3.774	0.03	0.02
G2	4.7	+0.63	-0.07	4.6	3.763	0.01	0.00
G8	5.6	+0.74	-0.13	5.5	3.720	-0.08	-0.04
K0	6.0	+0.82	-0.19	5.8	3.703	-0.11	-0.07
K5	7.3	+1.15	-0.62	6.7	3.643	-0.17	-0.19
M0	8.9	+1.41	-1.17	7.5	3.591	-0.22	-0.26
M5	13.5	+1.61	-2.55	11.0	3.491	-0.72	-0.82

Exercise 1.1: The V magnitudes of two main-sequence stars are both observed to be 7.5, but their blue magnitudes are $B_1 = 7.2$ and $B_2 = 8.65$. (a) What are the colour indices of the two stars. (b) Which star is the bluer and by what factor is it brighter at blue wavelength. (c) Making reasonable assumptions, deduce as many of the physical properties of the stars as possible e.g. temperature, luminosity, distance, mass, radius [assume $A_V = 0$].

Notes:

Notes:

Summary I

Concepts:

- relation between *astronomical observables* (flux, spectrum, parallax, radial velocities) and *physical properties* (luminosity, temperature, radius, mass, composition)
- the *stellar magnitude system* (apparent and absolute magnitudes, bolometric magnitude, bolometric correction, distance modulus), the *UBV system* and *stellar colours*
- the *black-body* spectrum, *effective temperature*
- *spectral classification:* spectral type and luminosity classes and its implications
- measuring masses and radii

Important equations:

- *distance modulus:* $(\mathrm{m}-\mathrm{M})_{\mathrm{V}} = 5\log \mathrm{D}/10\mathrm{pc}$
- *absolute V magnitude:* $\mathrm{M_V} = -2.5\log \mathrm{L}/\mathrm{L}_\odot + 4.72 + \mathrm{B.C.} + \mathrm{A_V}$
- *Salpeter initial mass function (IMF):* $\mathrm{f(M)\,dM} \propto \mathrm{M}^{-2.35}\,\mathrm{dM}$
- *black-body relation:* $\mathrm{L} = 4\pi \mathrm{R_s^2}\,\sigma \mathrm{T_{eff}^4}$
- *Kepler's law:* $\mathrm{a}^3\left(\frac{2\pi}{\mathrm{P}}\right)^2 = \mathrm{G}(\mathrm{M_1}+\mathrm{M_2})$

Notes:

Notes:

2. Correlations between Stellar Properties

2.1 Mass-luminosity relationship (ZG: 12.2; CO: 7.3)

- Most stars obey

$$\boxed{\mathrm{L_s} = \textit{constant} \times \mathrm{M_s^{\nu}} \qquad 3 < \nu < 5}$$

Exercise 2.1: Assuming a Salpeter IMF, show that most of the mass in stars in a galaxy is found in low-mass stars, while most of the stellar light in a galaxy comes from massive stars.

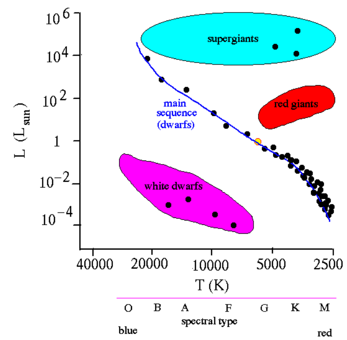
2.2 Hertzsprung–Russell diagram (ZG: 13-3; CO: 8.2) (plot of $\mathrm{L_s}$ vs. $\mathrm{T_{eff}}$): and ***Colour–Magnitude Diagram*** (e.g. plot of V vs. B-V) From diagrams for nearby stars of known distance we deduce:

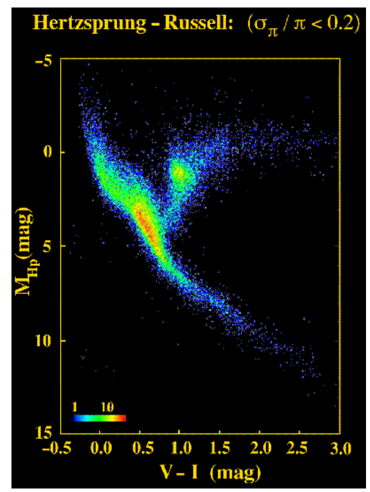
1. About 90% of stars lie on the main sequence (broad band passing diagonally across the diagram)
2. Two groups are very much more luminous than MS stars (giants and supergiants)
3. One group is very much less luminous; these are the white dwarfs with $\mathrm{R_s} << \mathrm{R_\odot}$ but $\mathrm{M_s} \sim \mathrm{M_\odot}$.

log g – log $\mathrm{T_{eff}}$ diagram, determined from atmosphere models (does not require distance)

Notes:

Hertzsprung-Russell (Colour-Magnitude) Diagram





Hipparcos (1989 - 1993)

Notes:

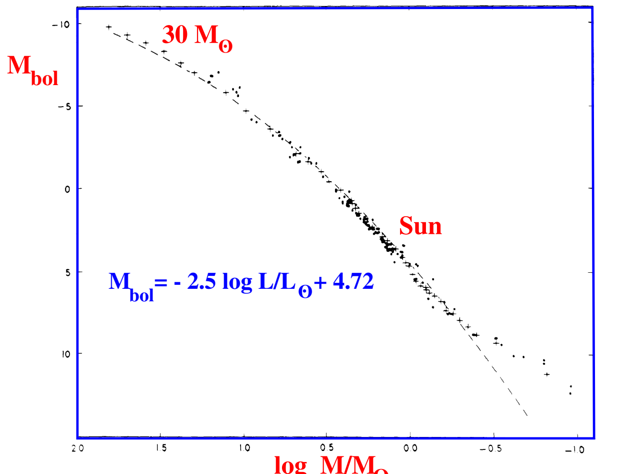


Fig. 2. Mass-luminosity relationship for the luminosity V stars from this paper (plus signs), from Popper (1980) (points) and from Heintze (1973) (broken line).

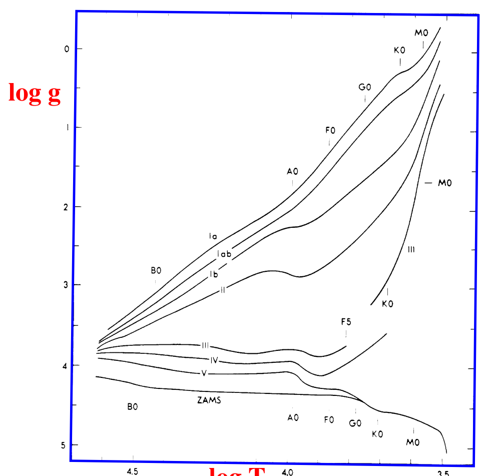


Fig. 3. The diagram log g, log T, for stars of different luminosities plotted from the data of Ta

Notes:

2.3 Cluster H-R Diagrams

(ZG:13-3, 14-2; OG: 13.4)

- ***Galactic or open clusters*** – 10 to 1000 stars, not concentrated towards centre of cluster – found only in *disc of Galaxy*
- ***Globular clusters*** – massive spherical associations containing 10^5 or more stars, *spherically distributed* about centre of Galaxy, many at great distances from plane.
- All stars within a given cluster are effectively *equidistant* from us; we are probably seeing *homogeneous, coeval* groups of stars, and with the *same chemical composition.* We can construct ***H-R diagrams*** of apparent brightness against temperature.

Main features of H-R diagrams:

1. *Globular clusters*
   - (a) All have *main-sequence turn-offs* in similar positions and giant branches joining the main sequence at that point.
   - (b) All have *horizontal branches* running from near the top of the giant branch to the main sequence above the turn-off point.
   - (c) In many clusters *RR Lyrae stars* (of variable luminosity) occupy a region of the horizontal branch.
2. *Galactic clusters*
   - (a) Considerable variation in the MS turn-off point; lowest in about the same position as that of globular clusters.
   - (b) Gap between MS and giant branch *(Hertzsprung gap)* in clusters with high turn-off point.

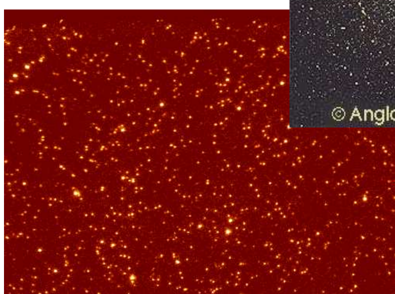
Notes:

STAR CLUSTERS

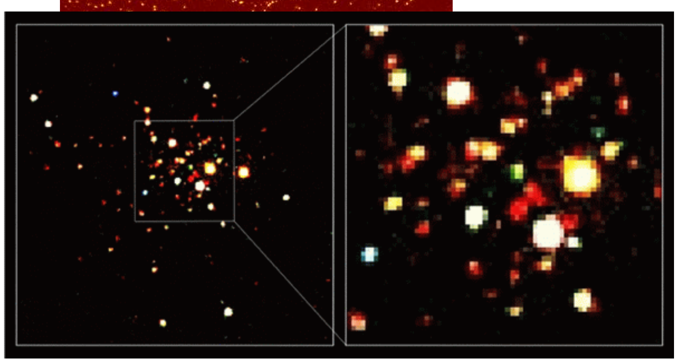
Open Cluster (Pleiades)

Globular Cluster (47 Tuc)

HST



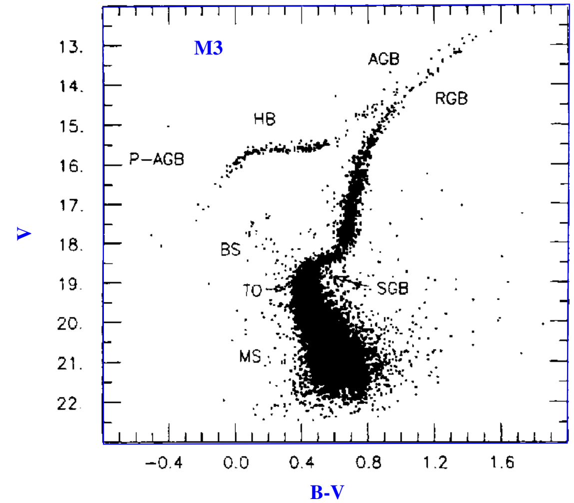
47 Tucanae



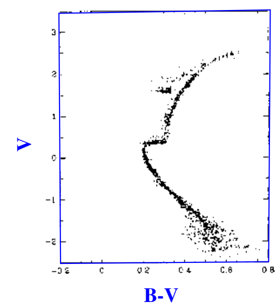
Chandra (X-rays)

Notes:

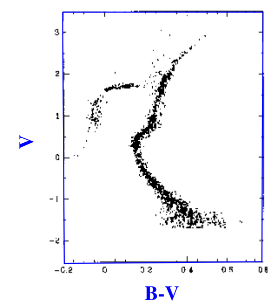
Globular Cluster CM Diagrams



47 Tuc



M15



Notes:

2.4 Chemical Composition of Stars
(ZG: 13-3; CO: 9.4)

- We deduce the *photospheric composition* by studying *spectra:* information often incomplete and of doubtful precision.
- *Solar system* abundances: Reasonable agreement between analysis of solar spectrum and laboratory studies of meteorites (carbonaceous chondrites).
- *Normal stars* (vast majority): Similar composition to Sun and interstellar medium
  Typically: *Hydrogen 90% by number; Helium 10%; other elements (metals)* $\ll 1\,\%$
  (by mass: $X \simeq 0.70$, $Y \simeq 0.28$, $Z \simeq 0.02$)
- *Globular cluster stars: Metal deficient* compared to Sun by factors of $10-1000$,
  Hydrogen and helium normal

Assuming uniform initial composition for the Galaxy, we conclude that about 99% of metals must have been synthesized within stars.

THIS IS THE PRIMARY EVIDENCE FOR NUCLEOSYNTHESIS DURING STELLAR EVOLUTION.
Notes:

2.5 STELLAR POPULATIONS (ZG: 14-3; CO: 13.4)

Population I: metallicity: $Z \sim 0.02$ (i.e. solar), old and young stars, mainly in the Galactic disc, open clusters

Population II: metallicity: $Z \sim 0.1 - 0.001\,Z_\odot$, old, high-velocity stars in the Galactic halo, globular clusters

Population III: hypothetical population of zero-metallicity stars (first generation of stars?), possibly with very different properties (massive, leading to relatively massive black holes?), may not exist as a major separate population (HE0107-5240, a low-mass star with $Z \sim 10^{-7}$: the first pop III star discovered?)

Stars with peculiar surface composition

- Most stars seem to retain their initial surface composition as the centre evolves. A small number show anomalies, which can occur through:
  1) *mixing* of central material to the surface
  2) large scale *mass loss* of outer layers exposing interior (e.g. helium stars)
  3) *mass transfer* in a binary (e.g. barium stars)
  4) pollution with *supernova* material from a binary companion (e.g. Nova Sco)

Sub-stellar objects

- *Brown Dwarfs:* star-like bodies with *masses too low* to create the central temperature required *to ignite fusion reactions* (i.e. $M \lesssim 0.08\,M_\odot$ from theory).
- *Planets: self-gravitating* objects formed in *disks* around stars (rocky planets [e.g. Earth], giant gas planets [e.g. Jupiter])

Notes:

Summary II

Concepts:

- How does one determine ***mass-luminosity relations***?
- The importance of the ***Hertzsprung-Russell*** and ***Colour-Magnitude diagram***
- Basic properties of ***open and globular clusters***
- The ***chemical composition*** of stars (metallicity)
- The different ***stellar populations***
- Difference between ***stars, brown dwarfs and planets***

Notes:

Notes:

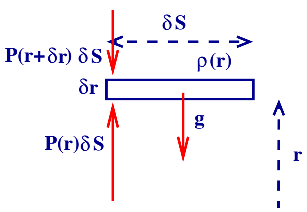
3. THE PHYSICAL STATE OF THE STELLAR INTERIOR

Fundamental assumptions:

- Although *stars evolve*, their properties change so *slowly* that at any time it is a good approximation to neglect the rate of change of these properties.
- Stars are *spherical* and symmetrical about their centres; all physical quantities depend just on r, the distance from the centre:

3.1 The Equation of hydrostatic equilibrium (ZG: 16-1; CO: 10.1)

> *Fundamental principle 1: stars are self-gravitating bodies in dynamical equilibrium*
> *→ balance of gravity and internal pressure forces*



Consider a small volume element at a distance r from the centre, cross section δS, length δr.

$$(P_{r+\delta r} - P_r)\,\delta S + GM_r/r^2\,(\rho_r\,\delta S\,\delta r) = 0$$

$$\boxed{\frac{dP_r}{dr} = -\frac{GM_r\rho_r}{r^2} \qquad (1)}$$

Equation of distribution of mass:

$$M_{r+\delta r} - M_r = (dM_r/dr)\,\delta r = 4\pi r^2 \rho_r\,\delta r$$

$$\boxed{\frac{dM_r}{dr} = 4\pi r^2 \rho_r \qquad (2)}$$

Notes:

Exercise: *3.1* Use dimensional analysis to estimate the *central pressure* and *central temperature* of a star.
– consider a point at $r = R_s/2$

$$dP_r/dr \sim -P_c/R_s \qquad \rho_r \sim \bar{\rho} = 3M_s/(4\pi R_s^3)$$

$$M_r \sim M_s/2 \qquad P_c \sim (3/8\pi)(GM_s^2/R_s^4)$$

$$(P_c)_\odot \sim 5\times 10^{14}\,\mathrm{N\,m^{-2}} \quad \text{or} \quad 5\times 10^{9}\,\mathrm{atm}$$

Estimate of central temperature:
Assume stellar material obeys the ideal gas equation

$$P_r = \frac{\rho_r}{\mu m_H} k T_r$$

(μ = mean molecular weight in proton masses; $\mu \sim 1/2$ for fully ionized hydrogen) and using equation (1) to obtain

$$kT_c \simeq \frac{GM_s\,\mu m_H}{R_s}$$

$$\boxed{(T_c)_\odot \sim 2\times 10^{7}\,\mathrm{K} \quad \bar{\rho}_\odot \sim 1.4\times 10^{3}\,\mathrm{kg\,m^{-3}} \quad (\text{c.f. } (T_s)_\odot \sim 5800\ \mathrm{K})}$$

- Although the Sun has a *mean density similar* to that of *water*, the high temperature requires that it should be *gaseous throughout.*
- the average *kinetic energy* of the particles is *higher* than the *binding energy of atomic hydrogen* so the material will be *highly ionized,* i.e is a *plasma.*

Notes:

3.2 The Dynamical timescale (ZG: P5-4; CO: 10.4): $\mathbf{t_D}$

- ***Time for star to collapse*** **completely if pressure forces were negligible** $(\delta M\ddot{r} = -\delta M\, g)$

$$(\rho\ \delta S \delta r)\, \ddot{r} = -(GM_r/r^2)\, (\rho\ \delta S \delta r)$$

- **Inward displacement of element after time t is given by**

$$s = (1/2)\, gt^2 = (1/2)\, (GM_r/r^2)\, t^2$$

- **For estimate of** t_{dyn}**, put** $s \sim R_s, r \sim R_s, M_r \sim M_s$**; hence**

$$t_{dyn} \sim (2R_s^3/GM_s)^{1/2} \sim \{3/(2\pi G\bar{\rho})\}^{1/2}$$

$$(t_{dyn})_\odot \sim 2300\ \mathrm{s} \sim 40\ \mathrm{mins}$$

Stars adjust very quickly to maintain a balance between pressure and gravitational forces.
General rule of thumb: $t_{dyn} \simeq 1/\sqrt{4G\bar{\rho}}$
Notes:

Notes:

3.3 The virial theorem (ZG: P5-2; CO: 2.4)

$$dP_r/dr = -GM_r\rho_r/r^2$$

$$4\pi r^3 dP_r = -(GM_r/r)4\pi r^2\rho_r dr$$

$$4\pi[r^3P_r]_{r=0,P=P_c}^{r=R_s,P=P_s} - 3\int_0^{R_s} P_r\,4\pi r^2 dr = -\int_0^{R_s}(GM_r/r)4\pi r^2\rho_r dr$$

$$\int_0^{R_s} 3P_r\,4\pi r^2 dr = \int_0^{R_s}(GM_r/r)4\pi r^2\rho_r dr$$

Thermal energy/unit volume $u = nfkT/2 = (\rho/\mu m_H)fkT/2$
Ratio of specific heats $\gamma = c_p/c_v = (f+2)/f$ $(f = 3 : \gamma = 5/3)$

$$u = \{1/(\gamma-1)\}(\rho kT/\mu m_H) = P/(\gamma-1)$$

$$\boxed{3(\gamma-1)U + \Omega = 0}$$

U = total *thermal energy;* Ω = total *gravitational energy.*
For a fully ionized, ideal gas $\gamma = 5/3$ **and** $2U + \Omega = 0$
Total energy of star $E = U + \Omega$

$$\boxed{E = -U = \Omega/2}$$

Note: **E is negative and equal to Ω/2 or −U.** ***A decrease in E*** **leads to a decrease in Ω but an** ***increase in*** **U** ***and hence*** **T. A star, with no hidden energy sources, composed of a perfect gas** ***contracts*** **and** ***heats up*** **as it radiates energy.**

> ***Fundamental principle 2: stars have a negative 'heat capacity', they heat up when their total energy decreases***

Notes:

Important implications of the virial theorem:

- **stars become hotter when their total energy decreases ($\rightarrow$ normal stars contract and heat up when there is no nuclear energy source because of energy losses from the surface);**
- ***nuclear burning is self-regulating*** **in non-degenerate cores: e.g. a sudden increase in nuclear burning causes expansion and cooling of the core:** ***negative feedback $\rightarrow$ stable nuclear burning.***

3.4 Sources of stellar energy: **(CO: 10.3)**

> ***Fundamental principle 3: since stars lose energy by radiation, stars supported by thermal pressure require an energy source to avoid collapse***

Provided stellar material always behaves as a perfect gas, thermal energy of star $\sim$ gravitational energy.

- ***total energy*** **available $\sim \mathrm{GM_s^2/2R_s}$**
- ***thermal time-scale*** **(Kelvin-Helmholtz timescale, the timescale on which a star radiates away its thermal energy)):**
  $\mathrm{t_{th} \sim GM_s^2/(2R_sL_s)}$
  $\mathrm{(t_{th})_\odot \sim 0.5 \times 10^{15}}$ **sec** $\sim 1.5 \times 10^7$ **years.**
- **e.g. the** ***Sun*** **radiates $\mathrm{L_\odot \sim 4 \times 10^{26}}$ W, and from geological evidence $\mathrm{L_\odot}$ has not changed significantly over $\mathrm{t \sim 10^9}$ years**

The thermal and gravitational energies of the Sun are not sufficient to cover radiative losses for the total solar lifetime.
Only nuclear energy can account for the observed luminosities and lifetimes of stars
Notes:

- ***Largest*** **possible** ***mass defect*** **available when H is transmuted into Fe: energy released = 0.008 $\times$ total mass. For the Sun $\mathrm{(E_N)_\odot = 0.008\,M_\odot c^2 \sim 10^{45}}$ J**
- ***Nuclear timescale*** $\mathrm{(t_N)_\odot \sim (E_N)_\odot / L_\odot \sim 10^{11}\,yr}$
- ***Energy loss at stellar surface*** **as measured by the stellar luminosity is** ***compensated by*** **energy release from** ***nuclear reactions*** **throughout the stellar interior.**

$$\mathrm{L_s} = \int_0^{\mathrm{R_s}} \varepsilon_\mathrm{r} \rho_\mathrm{r}\, 4\pi \mathrm{r}^2 \mathrm{dr}$$

  ε_r is the nuclear energy released per unit mass per sec and will depend on $\mathrm{T_r}, \rho_\mathrm{r}$ and composition

$$\boxed{\frac{\mathrm{dL_r}}{\mathrm{dr}} = 4\pi \mathrm{r}^2 \rho_\mathrm{r} \varepsilon_\mathrm{r} \qquad (3)}$$

  for any elementary shell.

- **During** ***rapid evolutionary phases,*** **(i.e. $\mathrm{t \ll t_{th}}$)**

$$\boxed{\frac{\mathrm{dL_r}}{\mathrm{dr}} = 4\pi \mathrm{r}^2 \rho_\mathrm{r} \left(\varepsilon_\mathrm{r} - \mathrm{T}\frac{\mathrm{dS}}{\mathrm{dt}}\right) \qquad (3a),}$$

  where $-\mathrm{TdS/dt}$ is called a ***gravitational energy*** **term.**

SUMMARY III: STELLAR TIMESCALES

- ***dynamical timescale:*** $\mathrm{t_{dyn}} \simeq \dfrac{1}{\sqrt{4\mathrm{G}\rho}} \sim 30\,\mathrm{min}\,\left(\rho/1000\,\mathrm{kg\,m^{-3}}\right)^{-1/2}$
- ***thermal timescale*** **(Kelvin-Helmholtz):** $\mathrm{t_{KH}} \simeq \dfrac{\mathrm{GM^2}}{\mathrm{2RL}} \sim 1.5 \times 10^7\,\mathrm{yr}\,(\mathrm{M/M_\odot})^2\,(\mathrm{R/R_\odot})^{-1}\,(\mathrm{L/L_\odot})^{-1}$
- ***nuclear timescale:*** $\mathrm{t_{nuc}} \simeq \underbrace{\mathrm{M_c/M}}_{\text{core mass}}\ \underbrace{\eta}_{\text{efficiency}}\ (\mathrm{Mc^2})/L \sim 10^{10}\,\mathrm{yr}\,(\mathrm{M/M_\odot})^{-3}$

Notes:

3.5 Energy transport (ZG: P5-10, 16-1, CO: 10.4)

The size of the *energy flux* is determined by the mechanism that provides the *energy transport: conduction, convection or radiation.* For all these mechanisms the *temperature gradient determines the flux.*

- ***Conduction* does not contribute seriously to energy transport through the interior**
  - ▷ At high gas density, mean free path for particles << mean free path for photons.
  - ▷ Special case, *degenerate matter* – very effective conduction by electrons.
- The *thermal radiation field* in the interior of a star consists mainly of *X-ray photons in thermal equilibrium with particles.*
- Stellar material is *opaque to X-rays* (bound-free absorption by inner electrons)
- *mean free path for X-rays* in solar interior ~ 1 cm.
- Photons reach the surface by a *"random walk"* process and as a result of many interactions with matter are degraded from X-ray to optical frequencies.
- After N steps of size l, the distribution has spread to $\simeq \sqrt{N}\, l$. For a photon to "random walk" a distance R_s, requires a *diffusion time* (in steps of size l)

$$t_{diff} = N \times \frac{l}{c} \simeq \frac{R_s^2}{lc}$$

For $l = 1$ cm, $R_s \sim R_\odot \to t_{diff} \sim 5 \times 10^3$ yr.

Notes:

Notes:

Energy transport by radiation:

- **Consider a spherical shell of area $A = 4\pi r^2$, at radius r of thickness dr.**
- ***radiation pressure***

$$P_{\rm rad} = \frac{1}{3} a T^4 \qquad (i)$$

(=momentum flux)

- ***rate of deposition of momentum* in region $r \to r + dr$**

$$-\frac{dP_{\rm rad}}{dr}\, dr\, 4\pi r^2 \qquad (ii)$$

- **define *opacity* κ [m^2/kg], so that fractional intensity loss in a beam of radiation is given by**

$$\frac{dI}{I} = -\kappa\rho\, dx,$$

where ρ is the mass density and

$$\tau \equiv \int \kappa\rho dx$$

is called *optical depth* (note: $I = I_0 \exp(-\tau)$)

  - ▷ **$1/\kappa\rho$: mean free path**
  - ▷ **$\tau \gg 1$: optically thick**
  - ▷ **$\tau \ll 1$: optically thin**

- **rate of momentum absorption in shell $L(r)/c\, \kappa\rho\, dr$. Equating this with equation (ii) and using (i):**

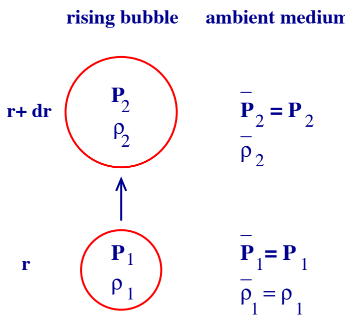
$$\boxed{L_r = -4\pi r^2 \, \frac{4ac}{3\kappa\rho} T^3 \frac{dT}{dr} \qquad (4a)}$$

Notes:

Notes:

Energy transport by convection:

- *Convection* occurs in liquids and gases when the temperature gradient exceeds some typical value.
- *Criterion for stability against convection (Schwarzschild criterion)*



▹ consider a *bubble* with initial ρ_1, P_1 *rising* by an amount dr, where the ambient pressure and density are given by $\overline{\rho(r)}, \overline{P(r)}$.

▹ the *bubble expands adiabatically*, i.e $P_2 = P_1 \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$ (γ = adiabatic exponent)

▹ assuming the bubble remains in *pressure equilibrium* with the ambient medium, i.e. $P_2 = \overline{P_2} = \overline{P(r+dr)} \simeq P_1 + (dP/dr)\,dr$,

$$\begin{aligned} \rho_2 &= \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} \simeq \rho_1 \left(1 + \frac{1}{P}\frac{dP}{dr}\,dr\right)^{1/\gamma} \\ &\simeq \rho_1 + \frac{\rho}{\gamma P}\frac{dP}{dr}\,dr \end{aligned}$$

▹ *convective stability* if $\rho_2 - \overline{\rho_2} > 0$ (bubble will sink back)

$$\boxed{\frac{\rho}{\gamma P}\frac{dP}{dr} - \frac{d\rho}{dr} > 0}$$

Notes:

- For a perfect gas (negligible radiation pressure)

$$P = \rho k T/(\mu m_H)$$

- Provided μ does not vary with position (no changes in ionization or dissociation)

$$-[1-(1/\gamma)](T/P)\,dP/dr > -dT/dr \text{ (both negative)}$$

- or *magnitude of adiabatic* dT/dr (l.h.s) > *magnitude of actual* dT/dr (r.h.s).
- Alternatively, $\quad \dfrac{P}{T}\dfrac{dT}{dP} < \dfrac{\gamma-1}{\gamma}$
- There is *no generally accepted theory of convective energy transport* at present. The stability criterion must be checked at every layer within a stellar model: dP/dr from equation (1) and dT/dr from equation (4). The stability criterion can be broken in two ways:

  1. *Large opacities* or *very centrally concentrated nuclear burning* can lead to *high (unstable) temperature gradients* e.g. in stellar cores.
  2. $(\gamma-1)$ can be much smaller than 2/3 for a monatomic gas, e.g. in *hydrogen ionization zones.*

Notes:

Influence of convection

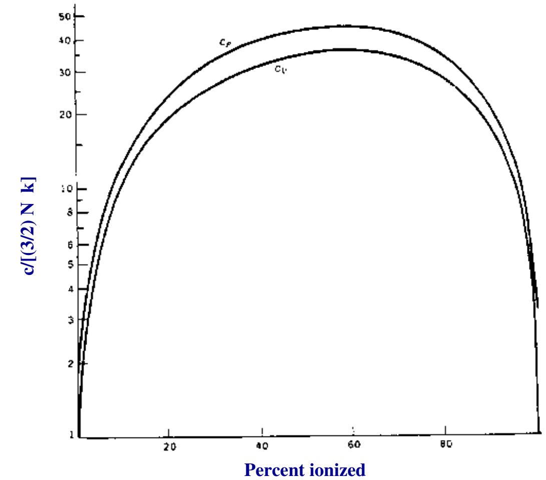
(a) Motions are ***turbulent:*** too slow to disturb hydrostatic equilibrium.
(b) ***Highly efficient energy transport:*** high thermal energy content of particles in stellar interior.
(c) Turbulent ***mixing*** so ***fast*** that ***composition*** of convective region ***homogeneous*** at all times.
(d) Actual dT/dr only exceeds adiabatic dT/dr by very slight amount.

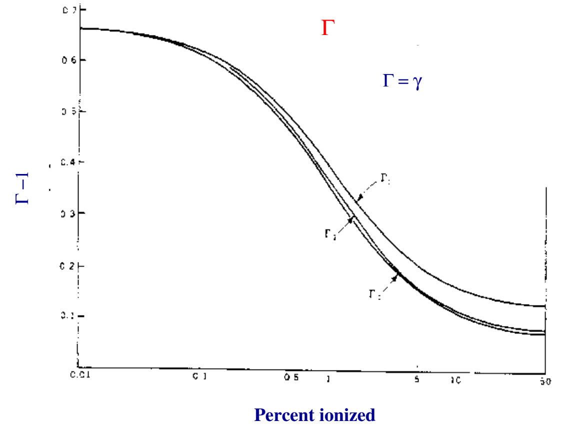
Hence to sufficient accuracy (in convective regions)

$$\boxed{\frac{\mathrm{dT}}{\mathrm{dr}} = \frac{\gamma - 1}{\gamma}\frac{\mathrm{T}}{\mathrm{P}}\frac{\mathrm{dP}}{\mathrm{dr}} \qquad (4\mathrm{b})}$$

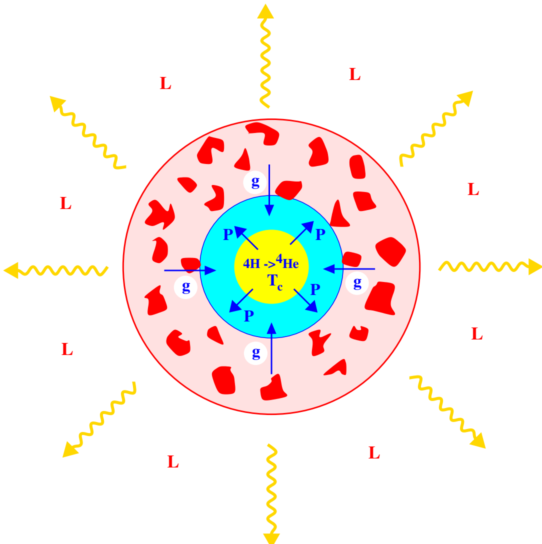
This is not a good approximation close to the surface (in particular for giants) where the density changes rapidly.
Notes:

Specific Heats





Notes:



Notes:

SUMMARY IV: FUNDAMENTAL PRINCIPLES

- Stars are *self-gravitating* bodies in *dynamical equilibrium* → *balance of gravity and internal pressure forces* (hydrostatic equilibrium);
- stars lose energy by *radiation from the surface* → stars supported by thermal pressure require an *energy source* to avoid collapse, e.g. *nuclear energy, gravitational energy* (energy equation);
- the *temperature structure* is largely determined by the mechanisms by which *energy* is *transported* from the core to the surface, *radiation, convection, conduction* (energy transport equation);
- the *central temperature* is determined by the *characteristic temperature* for the appropriate *nuclear fusion reactions* (e.g. H-burning: 10^7 K; He-burning: 10^8 K);
- normal stars have a *negative 'heat capacity'* (virial theorem): they heat up when their total energy decreases (→ normal stars contract and heat up when there is no nuclear energy source);
- *nuclear burning is self-regulating* in non-degenerate cores (virial theorem): e.g. a sudden increase in nuclear burning causes expansion and cooling of the core: *negative feedback* → *stable nuclear burning;*
- the *global structure* of a star is determined by the *simultaneous satisfaction* of these principles → the *local properties* of a star are determined by the *global structure.*
  (Mathematically: it requires the simultaneous solution of a set of coupled, non-linear differential equations with mixed boundary conditions.)

Notes:

4 THE EQUATIONS OF STELLAR STRUCTURE

In the absence of convection:

$$\boxed{\begin{aligned}
\frac{dP_r}{dr} &= \frac{-GM_r\rho_r}{r^2} && (1)\\
\frac{dM_r}{dr} &= 4\pi r^2\rho_r && (2)\\
\frac{dL_r}{dr} &= 4\pi r^2\rho_r\left(\varepsilon_r - T\frac{dS}{dt}\right) && (3)\\
\frac{dT_r}{dr} &= \frac{-3\kappa_r L_r\rho_r}{16\pi acr^2T_r^3} && (4a)
\end{aligned}}$$

4.1 The Mathematical Problem (Supplementary) (GZ: 16-2; CO: 10.5)

- $P_r, \kappa_r, \varepsilon_r$ are functions of ρ, T, chemical composition
- Basic physics can provide expressions for these.
- In total, there are *four, coupled, non-linear, partial differential equations (+ three physical relations) for seven unknowns:* $P, \rho, T, M, L, \kappa, \varepsilon$ as functions of r.
- These completely determine the structure of a star of given composition subject to boundary conditions.
- In general, only numerical solutions can be obtained (i.e. computer).
- *Four (mixed) boundary conditions* needed:
  - ▷ at centre: $M_r = 0$ and $L_r = 0$ at $r = 0$ (exact)
  - ▷ at surface: $L_s = 4\pi R_s^2\sigma T_{eff}^4$ (blackbody relation)
    (surface = photosphere, where $\tau \simeq 1$)
    $P = (2/3)\,g/\kappa$ (atmosphere model)

  (sometimes: $P(R_s) = 0$ [rough], but *not* $T(R_s) = 0$)

Notes:

4.1.1 Uniqueness of solution: the Vogt Russell "Theorem" (CO: 10.5)

> *"For a given chemical composition, only a single equilibrium configuration exists for each mass; thus the internal structure of the star is fixed."*

- This "theorem" has not been proven and is not even rigorously true; there are known exceptions

4.1.2 The equilibrium solution and stellar evolution:

- If there is *no bulk motions* in the interior of a star (i.e. no convection), *changes of chemical composition* are *localised* in *regions of nuclear burning* The structure equations (1) to (4) can be supplemented by equations of the type:

$$\partial/\partial t\,(\text{composition})_M = f(\rho, T, \text{composition})$$

- Knowing the composition as a function of M at a time t_0 we can solve (1) to (4) for the structure at t_0. Then

$$\begin{aligned}(\text{composition})_{M,t_0+\delta t} &= (\text{composition})_{M,t_0} + \\ &\quad \partial/\partial t\,(\text{composition})_M\,\delta t\end{aligned}$$

- *Calculate modified structure for new composition and repeat* to discover how star evolves (not valid if stellar properties change so rapidly that time dependent terms in (1) to (4) cannot be ignored).

4.1.3 Convective regions: (GZ: 16-1; CO: 10.4)

- Equations (1) to (3) unchanged.
- for efficient convection (neutral buoyancy):

$$\frac{P}{T}\frac{dT}{dP} = \frac{\gamma - 1}{\gamma} \qquad (4b)$$

- L_{rad} is calculated from equation (4) once the above have been solved.

Notes:

4.2 THE EQUATION OF STATE

4.2.1 Perfect gas: (GZ: 16-1: CO: 10.2)

$$P = NkT = \frac{\rho}{\mu m_H} kT$$

N is the number density of particles; μ is the mean particle mass in units of m_H. Define:

X = mass fraction of hydrogen (Sun: 0.70)

Y = mass fraction of helium (Sun: 0.28)

Z = mass fraction of heavier elements (metals) (Sun: 0.02)

- $X + Y + Z = 1$
- If the material is assumed to be *fully ionized:*

Element	No. of atoms	No. of electrons
Hydrogen	$X\rho/m_H$	$X\rho/m_H$
Helium	$Y\rho/4m_H$	$2Y\rho/4m_H$
Metals	$[Z\rho/(Am_H)]$	$(1/2)AZ\rho/(Am_H)$

- A represents the average atomic weight of heavier elements; each metal atom contributes $\sim A/2$ electrons.
- Total number density of particles:

$$N = (2X + 3Y/4 + Z/2)\,\rho/m_H$$

$\triangleright$ $(1/\mu) = 2\,X + 3/4\,Y + 1/2\,Z$

- This is a good approximation to μ *except in cool, outer regions.*

Notes:

- When Z is negligible: $Y = 1 - X$; $\mu = 4/(3 + 5X)$
- Inclusion of *radiation pressure* in P:

$$P = \rho kT/(\mu m_H) + aT^4/3.$$

(important for massive stars)

4.2.2 Degenerate gas: (GZ: 17-1; CO: 15.3)

- First deviation from perfect gas law in stellar interior occurs when electrons become degenerate.
- The *number density of electrons* in phase space is *limited by the Pauli exclusion principle.*

$$n_e\, dp_x dp_y dp_z\, dxdydz \le (2/h^3)\, dp_x dp_y dp_z\, dxdydz$$

- In a *completely degenerate gas* all cells for momenta smaller than a threshold momentum p_0 are completely filled (Fermi momentum).
- The number density of electrons within a sphere of radius p_0 in momentum space is (at $T = 0$):

$$N_e = \int_0^{p_0} (2/h^3)\, 4\pi p^2 dp = (2/h^3)(4\pi/3)p_0^3$$

- From *kinetic theory*

$$P_e = (1/3) \int_0^\infty p\, v(p) n(p) dp$$

(a) Non-relativistic complete degeneracy:

$$v(p) = p/m_e \qquad \text{for all } p$$

$$P_e = (1/3) \int_0^{p_0} (p^2/m_e)(2/h^3)\, 4\pi p^2\, dp$$

$$= \{8\pi/(15 m_e h^3)\} p_0^5 = \{h^2/(20 m_e)\}(3/\pi)^{2/3}\, N_e^{5/3}.$$

Notes:

(b) Relativistic complete degeneracy:

$$\mathbf{v(p) \sim c}$$

$$\mathbf{P_e = (1/3) \int_0^{p_0} pc(2/h^3)\, 4\pi p^2\, dp}$$

$$\mathbf{= (8\pi c/3h^3) p_0^4/4 = (2\pi c/3h^3)\, p_0^4}$$

$$\mathbf{= (hc/8)(3/\pi)^{1/3}\, N_e^{4/3}.}$$

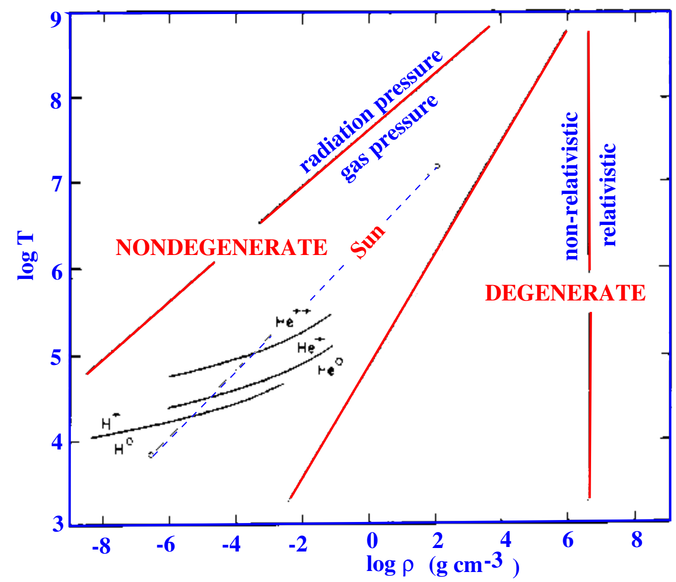
- **Also** $\mathbf{N_e = (X + Y/2 + Z/2)\, \rho/m_H = (1/2)(1 + X)\, \rho/m_H}$.
- **For intermediate regions use the full relativistic expression for v(p).**
- **For ions we may continue to use the non-degenerate equation:**
- $\mathbf{P_{ions} = (1/\mu_{ions})(\rho k T/m_H)}$ **where** $\mathbf{(1/\mu_{ions}) = X + Y/4}$.

Conditions where degeneracy is important:

(a) ***Non-relativistic*** **– interiors of** ***white dwarfs***; ***degenerate cores of red giants.***

(b) ***Relativistic*** **- very high densities only; interiors of** ***white dwarfs.***

Notes:



Temperature-density diagram for the equation of state (Schwarzschild 1958)

Notes:

4.3 THE OPACITY (GZ: 10-2; CO: 9.2)

The rate at which energy flows by radiative transfer is determined by the opacity *(cross section per unit mass [m^2/kg])*

$$dT/dr = -3\kappa L\rho/(16\pi acr^2T^3) \qquad (4)$$

In degenerate stars a similar equation applies with the opacity representing resistance to energy transfer by electron conduction.

Sources of stellar opacity:

1. *bound-bound absorption* (negligible in interiors)

2. *bound-free absorption*

3. *free-free absorption*

4. *scattering by free electrons*

- usually use a mean opacity averaged over frequency, *Rosseland mean opacity* (see textbooks).

Approximate analytical forms for opacity:

High temperature: $\kappa = \kappa_1 = 0.020\,\mathrm{m^2\,kg^{-1}}\,(1+X)$

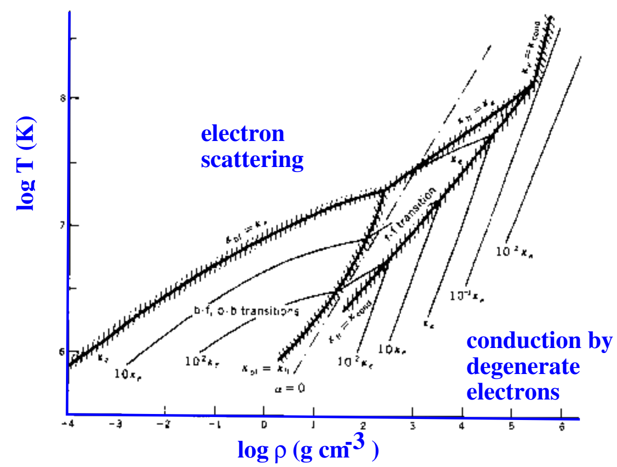
Intermediate temperature: $\kappa = \kappa_2\rho T^{-3.5}$ *(Kramer's law)*

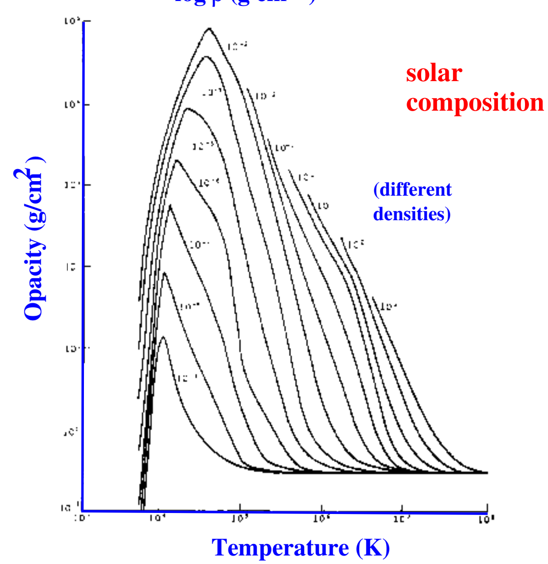
Low temperature: $\kappa = \kappa_3\rho^{1/2}T^4$

- $\kappa_1, \kappa_2, \kappa_3$ are constant for stars of given chemical composition but all depend on composition.

Notes:

Opacities





Notes:

5. NUCLEAR REACTIONS
(ZG: P5-7 to P5-9, P5-12, 16-1D; CO: 10.3)

- ***Binding energy*** **of nucleus with Z protons and N neutrons is:**

$$Q(Z,N) = \underbrace{[ZM_p + NM_n - M(Z,N)]}_{\text{mass defect}}\, c^2.$$

- ***Energy release:***

$4\,H \rightarrow {}^4He$ $\quad 6.3 \times 10^{14}\,J\,kg^{-1} = 0.007\,c^2 \;(\varepsilon = 0.007)$

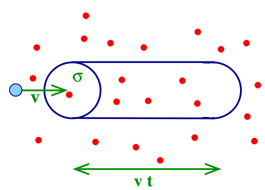
$56\,H \rightarrow {}^{56}Fe$ $\quad 7.6 \times 10^{14}\,J\,kg^{-1} = 0.0084\,c^2 \;(\varepsilon = 0.0084)$

- ***H burning*** **already releases most of the available nuclear binding energy.**

5.1 Nuclear reaction rates: (ZG: P5-7)

$$\boxed{1} + \boxed{2} \rightarrow \boxed{1,2} + \text{Energy}$$

Reaction rate **is proportional to:**



1. ***number density*** n_1 **of particles 1**
2. ***number density*** n_2 **of particles 2**
3. ***frequency of collisions*** **depends on** ***relative velocity*** **v of colliding particles** $r_{1+2} = n_1 n_2\, \langle \sigma(v) v \rangle$
4. ***probability*** $P_p(v)$ ***for penetrating Coulomb barrier*** **(Gamow factor)**

$$P_p(v) \propto \exp[-(4\pi^2 Z_1 Z_2 e^2/hv)]$$

Notes:

Nuclear Binding Energy

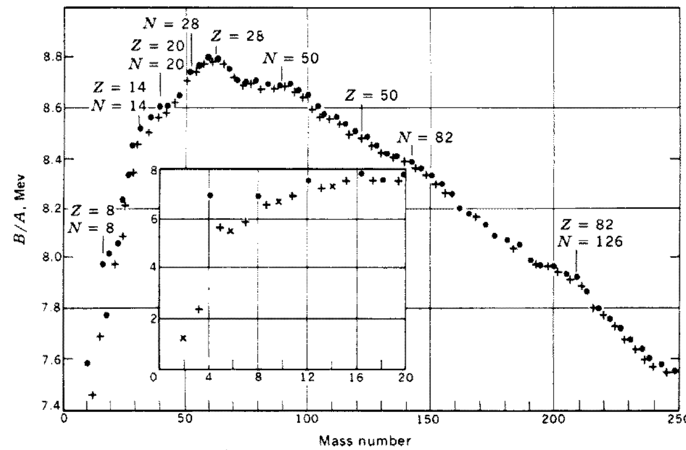
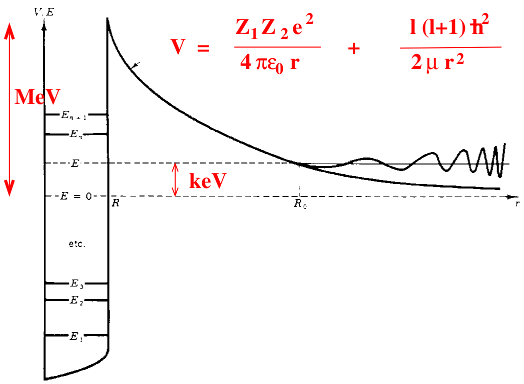


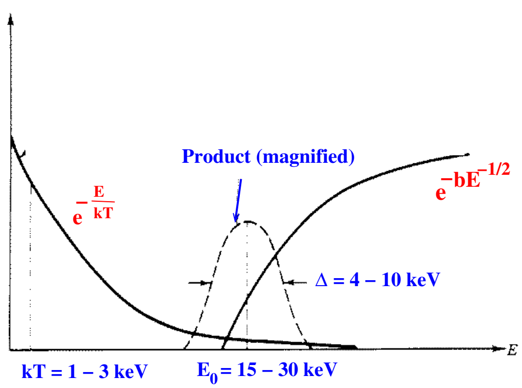
Fig. 7-1 The binding energy per nucleon of the most stable isobar of atomic weight *A*. The solid circles represent nuclei having an even number of protons and an even number of neutrons, whereas the crosses represent odd-*A* nuclei. (*M. A. Preston, "Physics of the Nucleus," Addison-Wesley Publishing Company, Inc., Reading, Mass.,* 1962.)

Notes:

Coulomb Barrier



Reaction Rate



Notes:

5. define *cross-section factor* S(E)*:* $\sigma = [S(E)/E]\,P_p(E)$
   - ▷ depends on the details of the nuclear interactions
   - ▷ insensitive to particle energy or velocity (non-resonant case)
   - ▷ S(E) is typically a slowly varying function
   - ▷ evaluation requires *laboratory* data except in p-p case (theoretical cross section)
6. *particle velocity distribution* (Maxwellian).

$$D(T,v) \propto (v^2/T^{3/2})\exp[-(m_H A' v^2/2kT)]$$

where $A' = A_1A_2(A_1+A_2)^{-1}$ is the reduced mass.

The *overall reaction rate* per unit volume is:

$$\boxed{R_{12} = \int_0^\infty n_1 n_2\, v[S(E)/E\,P_p(v)]\,D(T,v)\,dv}$$

- Setting $n_1 = (\rho/m_1)\,X_1,\quad n_2 = (\rho/m_2)\,X_2$ and

$$\tau = 3E_0/kT = 3\{2\pi^4 e^4 m_H Z_1^2 Z_2^2 A'/(h^2 kT)\}^{1/3}$$

$$R_{12} = B\rho^2\,(X_1X_2/A_1A_2)\tau^2\exp(-\tau)/(A'Z_1Z_2)$$

where B is a constant depending on the details of the nuclear interaction (from the S(E) factor)

- ▷ Low temperature: τ is large; exponential term leads to small reaction rate.
- ▷ Increasing temperature: reaction rate increases rapidly through exponential term.
- ▷ High temperature: τ^2 starts to dominate and *rate falls again.*
  (In practice, we are mainly concerned with temperatures at which there is a rising trend in the reaction rate.)

Notes:

(1) Reaction rate decreases as Z_1 and Z_2 increase. Hence, ***at low temperatures,*** reactions involving ***low*** Z ***nuclei*** are favoured.

(2) Reaction rates need only be significant over times $\sim 10^9$ years.

5.2 HYDROGEN BURNING

5.2.1 PPI chain: (ZG: P5-7, 16-1D)

1)	${}^1\mathrm{H} + {}^1\mathrm{H} \;\rightarrow\; {}^2\mathrm{D} + \mathrm{e}^+ + \nu$	$+\,1.44\,\mathrm{MeV}$
2)	${}^2\mathrm{D} + {}^1\mathrm{H} \;\rightarrow\; {}^3\mathrm{He} + \gamma$	$+\,5.49\,\mathrm{MeV}$
3)	${}^3\mathrm{He} + {}^3\mathrm{He} \;\rightarrow\; {}^4\mathrm{He} + {}^1\mathrm{H} + {}^1\mathrm{H}$	$+\,12.85\,\mathrm{MeV}$

- for each conversion of ${}^4\mathrm{H} \rightarrow {}^4\,\mathrm{He}$, reactions (1) and (2) have to occur twice, reaction (3) once
- the ***neutrino*** in (1) carries away ***0.26 MeV*** leaving ***26.2 MeV*** to contribute to the luminosity
- reaction (1) is a ***weak interaction*** $\rightarrow$ ***bottleneck*** of the reaction chain
- ***Typical reaction times*** for $\mathrm{T} = 3 \times 10^7\,\mathrm{K}$ are
  (1) $14 \times 10^9\,\mathrm{yr}$
  (2) 6 s
  (3) $10^6\,\mathrm{yr}$
  - ▷ (these depend also on ρ, X_1 and X_2).
  - ▷ ***Deuterium*** is burned up very rapidly.

Notes:

If ^{4}***He is sufficiently abundant,*** two further chains can occur:

PPII chain:

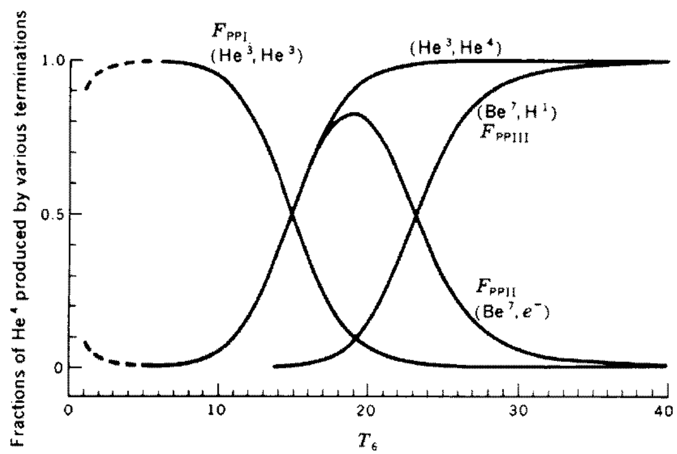
3a)	${}^3\mathrm{He} + {}^4\mathrm{He} \;\rightarrow\; {}^7\mathrm{Be} + \gamma$	$+\,1.59\,\mathrm{MeV}$
4a)	${}^7\mathrm{Be} + \mathrm{e}^- \;\rightarrow\; {}^7\mathrm{Li} + \nu$	$+\,0.86\,\mathrm{MeV}$
5a)	${}^7\mathrm{Li} + {}^1\mathrm{H} \;\rightarrow\; {}^4\mathrm{He} + {}^4\mathrm{He}$	$+\,17.35\,\mathrm{MeV}$

PPIII chain:

4b)	${}^7\mathrm{Be} + {}^1\mathrm{H} \;\rightarrow\; {}^8\mathrm{B} + \gamma$	$+\,0.14\,\mathrm{MeV}$
5b)	${}^8\mathrm{B} \;\rightarrow\; {}^8\mathrm{Be} + \mathrm{e}^+ + \nu$	
6b)	${}^8\mathrm{Be} \;\rightarrow\; {}^4\mathrm{He} + {}^4\mathrm{He}$	$+\,18.07\,\mathrm{MeV}$

- In both ***PPII and PPIII,*** a ^{4}***He atom*** acts as a ***catalyst*** to the conversion of ${}^3\mathrm{He} + {}^1\mathrm{H} \;\rightarrow\; {}^4\mathrm{He} + \nu$.
- $\mathrm{E}_{\mathrm{total}}$ is the same in each case but the energy carried away by the neutrino is different.
- All three PP chains ***operate simultaneously*** in a H burning star containing significant ^{4}He: details of the cycle depend on density, temperature and composition.

Notes:



Notes:

5.2.2 THE CNO CYCLE (ZG: P5-9; 16-1D) (T $< 10^8$ K)

- ***Carbon, nitrogen and oxygen*** serve as ***catalysts*** for the ***conversion of H to He***

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

- The ***seed nuclei*** are believed to be predominantly ^{12}C and ^{16}O: these are the main ***products of He burning,*** a later stage of nucleosynthesis.
- ***cycle timescale:*** is determined by the ***slowest reaction*** ($^{14}\mathrm{N} + ^{1}\mathrm{H}$)
- ***Approach to equilibrium*** in the CNO cycle is determined by the second slowest reaction ($^{12}\mathrm{C} + ^{1}\mathrm{H}$)
- in equilibrium $\lambda_{^{12}\mathrm{C}}\,{}^{12}\mathrm{C} = \lambda_{^{13}\mathrm{C}}\,{}^{13}\mathrm{C} = \lambda_{^{14}\mathrm{N}}\,{}^{14}\mathrm{N} = \lambda_{^{15}\mathrm{N}}\,{}^{15}\mathrm{N}$ (where λ_* are reaction rates and ^{13}C, etc. number densities)
- most of the ***CNO seed elements*** are converted into ^{14}N
- ***Observational*** evidence for CNO cycle:
  1. In ***some red giants*** $^{13}\mathrm{C}/^{12}\mathrm{C} \sim 1/5$
     (terrestrial ratio $\sim 1/90$)
  2. ***Some stars*** with ***extremely nitrogen-rich*** compositions have been discovered

Notes:

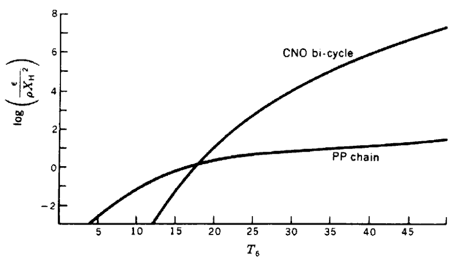
5.3 Energy generation from H burning (CO: 10.3)

- **Using experimental or extrapolated reaction rates, it is possible to calculate $\varepsilon(T)$ for the various chains.**

$$\varepsilon_{PP} \propto \rho X_H^2 \qquad \varepsilon_{CNO} \propto \rho X_H X_{CNO}$$

- **Energy generation occurs by *PP chain* at $T \sim 5 \times 10^6$ K.**
- ***High-mass stars* have higher T_c (CNO cycle dominant) than low-mass stars (pp chain)**
- ***Analytical fits to the energy generation rate:***

$$\varepsilon_{PP} \simeq \varepsilon_1 X_H^2 \rho T^4; \qquad \varepsilon_{CNO} \simeq \varepsilon_2 X_H X_{CNO} \rho T^{20}.$$



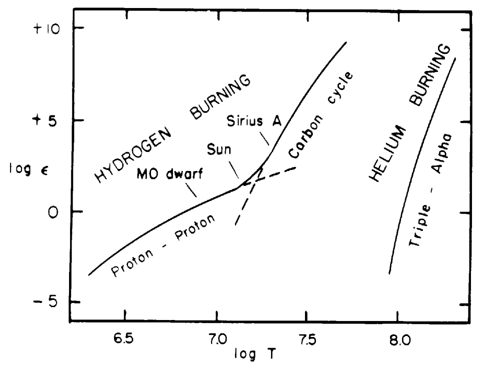


Fig. 10.1. Nuclear energy generation as a function of temperature (with $\rho X^2 = 100$ and $X_{CN} = 0.005X$ for the proton-proton reaction and the carbon cycle, but $\rho^2 Y^3 = 10^8$ for the triple-alpha process).

Notes:

5.4 Other Reactions Involving Light Elements *(Supplementary)*

- **Both the *PP chain* and the *CNO cycle* involve *weak interactions.* First reaction of PP chain involves two steps**

$$^1H + {}^1H \rightarrow {}^2He \begin{cases} \rightarrow {}^2D + e^+ + \nu \\ \rightarrow {}^1H + {}^1H \end{cases}$$

- **In the *CNO cycle, high nuclear charges slow the reaction rate.* D, Li, Be and B burn at lower temperatures than H, because all can burn without β-decays and with $Z < 6$.**

$$\begin{aligned}
{}^2D + {}^1H &\rightarrow {}^3He + \gamma \\
{}^6Li + {}^1H &\rightarrow {}^4He + {}^3He \\
{}^7Li + {}^1H &\rightarrow {}^8Be + \gamma \rightarrow {}^4He + {}^4He + \gamma \\
{}^9Be + {}^1H &\rightarrow {}^6Li + {}^4He \\
{}^{10}B + {}^1H &\rightarrow {}^7Be + {}^4He \\
{}^{11}B + {}^1H &\rightarrow {}^4He + {}^4He + {}^4He + \gamma
\end{aligned}$$

- **^{7}Be is destroyed as in the PP chain**
- **These elements always have *low abundances* and play no major role for nuclear burning**
- **they take place at $T \sim 10^6 - 10^7$ K**
- **they *are largely destroyed,* including in the surface layers, because convection occurs during *pre-main-sequence contraction.***

Notes:

5.5 HELIUM BURNING (ZG: P5-12; 16-1D)

- When ***H is exhausted*** in central regions, further ***gravitational contraction*** will occur leading to a ***rise in*** T_c, (provided material remains perfect gas)
- ***Problem with He burning: no stable nuclei at*** $A = 8$***;*** no chains of light particle reactions bridging gap between ^{4}He and ^{12}C (next most abundant nucleus).
  - ▷ Yet ^{12}C and ^{16}O are equivalent to 3 and 4 α-particles.
  - ▷ Perhaps many body interactions might be involved? These would only occur fast enough if ***resonant.***
  - ▷ ***Triple α reaction:*** $^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$
  - ▷ Ground state of ^{8}Be has $\gamma = 2.5\,\mathrm{eV}$ $\rightarrow \tau = 2.6 \times 10^{-16}\,\mathrm{s}$
  - ▷ Time for two α's to scatter off each other: $t_{\mathrm{scatt}} \sim 2d/v \sim 2 \times 10^{-15}/2 \times 10^5 \sim 10^{-20}$ sec
  - ▷ A small concentration of ^{8}Be builds up in ^{4}He gas until rate of break-up = rate of formation.
  - ▷ At $T = 10^8$ K and $\rho = 10^8\,\mathrm{kg\,m^{-3}}$, $n(^8\mathrm{Be})/n(^4\mathrm{He}) \sim 10^{-9}$.
  - ▷ This is sufficient to allow: $^8\mathrm{Be} + {}^4\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \gamma$
- The overall reaction rate would still not be fast enough unless this reaction were ***also resonant at stellar temperatures.***
  - ▷ An s-wave resonance requires ^{12}C to have a 0^+ state with energy $E_0 + 2\Delta E_0$ where $E_0 = 146(T \times 10^{-8})^{2/3}\,\mathrm{keV}$ and $2\Delta E_0 = 164(T \times 10^{-8})^{5/6}\,\mathrm{keV}$.
  - ▷ Such an excited state is found to lie at a resonance energy $E_{\mathrm{res}} = 278\,\mathrm{keV}$ above the combined mass of $^8\mathrm{Be} + {}^4\mathrm{He}$.

Notes:

  - ▷ Best available estimates of partial widths are: $\gamma_\alpha \simeq \gamma = 8.3$ eV; $\gamma_\gamma = (2.8 \pm 0.5)10^{-3}$ eV.
  - ▷ Thus resonant state breaks up ***almost every time.***
  - ▷ ***Equilibrium concentration of ^{12}C*** and the energy generation rate can be calculated.
  - ▷ ***At*** $T \sim 10^8$ ***K*** $\quad \varepsilon_{3\alpha} \simeq \varepsilon_3 X_{\mathrm{He}}^3 \rho^2 T^{30}$.
- ***energy generation in He core*** strongly concentrated towards regions of highest T
- other important ***He-burning reactions:***

$$
\begin{aligned}
{}^{12}\mathrm{C} + \alpha &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{13}\mathrm{C} + \alpha &\rightarrow {}^{16}\mathrm{O} + \mathrm{n} \\
{}^{14}\mathrm{N} + \alpha &\rightarrow {}^{18}\mathrm{O} + e^+ + \nu \\
{}^{16}\mathrm{O} + \alpha &\rightarrow {}^{20}\mathrm{Ne} + \gamma \\
{}^{18}\mathrm{O} + \alpha &\rightarrow {}^{22}\mathrm{Ne} + \gamma \\
{}^{20}\mathrm{Ne} + \alpha &\rightarrow {}^{24}\mathrm{Mg} + \gamma
\end{aligned}
$$

  in some phases of stellar evolution and outside the core, these can be the dominant He-burning reactions
- in a stellar core supported by ***electron degeneracy,*** the onset of He burning is believed to be accompanied by an explosive reaction – ***THE HELIUM FLASH***
- once He is used up in the central regions, further contraction and heating may occur, leading to additional nuclear reactions e.g. ***carbon burning***
- by the time that H and He have been burnt most of the possible energy release from fusion reactions has occurred

Notes:

6.1 THE STRUCTURE OF MAIN-SEQUENCE STARS (ZG: 16.2; CO 10.6, 13.1)

- main-sequence phase: *hydrogen core burning* phase
  - ▷ *zero-age main sequence (ZAMS): homogeneous* composition

Scaling relations for main-sequence stars

- use *dimensional analysis* to derive scaling relations (relations of the form $L \propto M^{\gamma}$)
- replace differential equations by characteristic quantities (e.g. $dP/dr \sim P/R$, $\rho \sim M/R^3$)
- *hydrostatic equilibrium* $\rightarrow P \sim \frac{GM^2}{R^4}$ (1)
- *radiative transfer* $\rightarrow L \propto \frac{R^4 T^4}{\kappa M}$ (2)
- to derive *luminosity–mass relationship,* specify *equation of state* and *opacity law*

(1) *massive stars:* ideal-gas law, electron scattering opacity, i.e.

▷ $P = \frac{\rho}{\mu m_H} kT \sim \frac{kT}{\mu m_H}\left(\frac{M}{R^3}\right)$ and $\kappa \simeq \kappa_{Th} = \text{constant}$

$$\Rightarrow \frac{kT}{\mu m_H} \sim \frac{GM}{R} \quad (3)$$

▷ substituting (3) into (2): $L \propto \frac{\mu^4 M^3}{\kappa_{Th}}$

Notes:

Notes:

(2) ***low-mass stars:*** **ideal-gas law, Kramer's opacity law, i.e.** $\kappa \propto \rho T^{-3.5}$

$$\Rightarrow L \propto \frac{\mu^{7.5} M^{5.5}}{R^{0.5}}$$

- ***mass–radius relationship***
  - ▷ **central temperature determined by characteristic nuclear-burning temperature (hydrogen fusion:** $T_c \sim 10^7\,K$**; helium fusion:** $T_c \sim 10^8\,K$**)**
  - ▷ **from (3)** $\Rightarrow R \propto M$ **(in reality** $R \propto M^{0.6-0.8}$**)**

(3) ***very massive stars:*** **radiation pressure, electron scattering opacity, i.e.**

▷ $P = \frac{1}{3} a T^4 \rightarrow T \sim \frac{M^{1/2}}{R} \Rightarrow L \propto M$

- **power-law index in mass–luminosity relationship decreases from** ~ 5 ***(low-mass)*** **to 3** ***(massive)*** **and 1** ***(very massive)***
- **near a solar mass:** $L \simeq L_\odot \left(\frac{M}{M_\odot}\right)^4$
- ***main-sequence lifetime:*** $T_{MS} \propto M/L$

  typically: $T_{MS} = 10^{10}\,yr \left(\frac{M}{M_\odot}\right)^{-3}$

- ***pressure*** **is inverse proportional to the** ***mean molecular weight*** μ
  - ▷ **higher** μ **(fewer particles) implies higher temperature to produce the same pressure, but** T_c ***is fixed*** **(hydrogen burning** ***(thermostat):*** $T_c \sim 10^7\,K$**)**
  - ▷ **during H-burning** μ **increases from** ~ 0.62 **to** ~ 1.34

  $\rightarrow$ ***radius increases*** **by a factor of** ~ 2 **(equation [3])**

Notes:

Notes:

- *opacity* at low temperatures depends strongly on *metallicity* (for bound-free opacity: $\kappa \propto Z$)
  - ▷ *low-metallicity stars* are much *more luminous* at a given mass and have proportionately shorter lifetimes
  - ▷ mass-radius relationship only weakly dependent on metallicity
  - → low-metallicity stars are *much hotter*
  - ▷ *subdwarfs:* low-metallicity main-sequence stars lying just below the main sequence

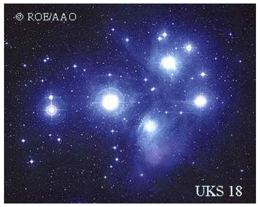
General properties of homogeneous stars:

	Upper main sequence ($M_s > 1.5\,M_\odot$)	Lower main sequence ($M_s < 1.5\,M_\odot$)
core	*convective;* well mixed	*radiative*
ϵ	*CNO cycle*	*PP chain*
κ	*electron scattering*	*Kramer's opacity* $\kappa \simeq \kappa_3 \rho T^{-3.5}$
surface	*H fully ionized* energy transport by *radiation*	*H/He neutral* *convection* zone just below surface

N.B. T_c increases with M_s; ρ_c decreases with M_s.

- *Hydrogen-burning limit:* $M_s \simeq 0.08\,M_\odot$
  - ▷ low-mass objects (brown dwarfs) do not burn hydrogen; they are supported by *electron degeneracy*
- maximum mass of stars: $100 - 150\,M_\odot$
- *Giants, supergiants and white dwarfs* cannot be chemically homogeneous stars supported by nuclear burning

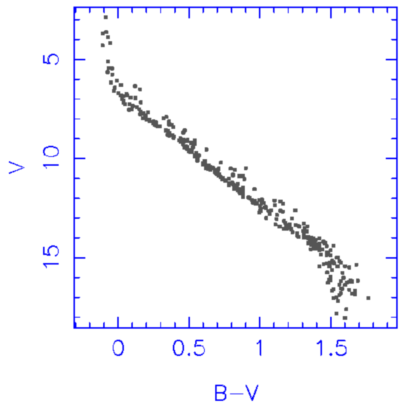
Notes:



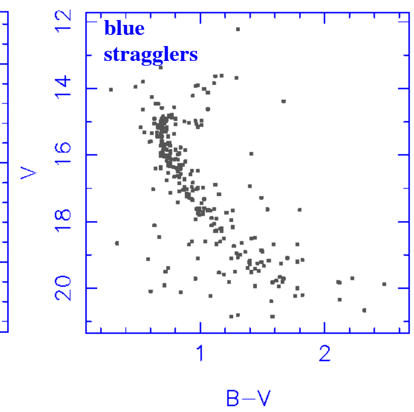
Turnoff Ages in Open Clusters

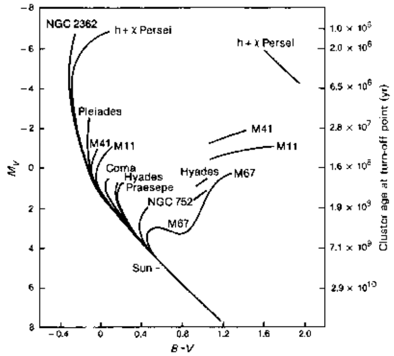
$$T = 10^{10}\,\text{yr}\left(\frac{L_{TO}}{L_\odot}\right)^{-3/4}$$

Pleiades



NGC 188





Notes:

6.2 THE EVOLUTION OF LOW-MASS STARS
($M \lesssim 8\,M_\odot$) (ZG: 16.3; CO: 13.2)

6.2.1 Pre-main-sequence phase

- observationally new-born stars appear as *embedded protostars/T Tauri stars* near the *stellar birthline* where they burn *deuterium* ($T_c \sim 10^6\,K$), often still *accreting* from their birth clouds
- *after deuterium burning* → star *contracts*
  → $T_c \sim (\mu m_H/k)\,(GM/R)$ increases until hydrogen burning starts ($T_c \sim 10^7 K$) → main-sequence phase

6.2.2 Core hydrogen-burning phase

- energy source: *hydrogen burning* (4 H→ ^{4}He)
  → mean molecular weight μ increases in core from 0.6 to 1.3 → R, L and T_c increase (from $T_c \propto \mu\,(GM/R)$)
- lifetime: $T_{MS} \simeq 10^{10}\,\text{yr}\left(\frac{M}{M_\odot}\right)^{-3}$

after hydrogen exhaustion:

- formation of *isothermal core*
- *hydrogen burning in shell* around inert core (shell-burning phase)

→ growth of core until $M_{core}/M \sim 0.1$ *(Schönberg-Chandrasekhar limit)*

- ▷ core becomes too massive to be supported by thermal pressure
- → *core contraction* → energy source: *gravitational energy* → core becomes denser and hotter

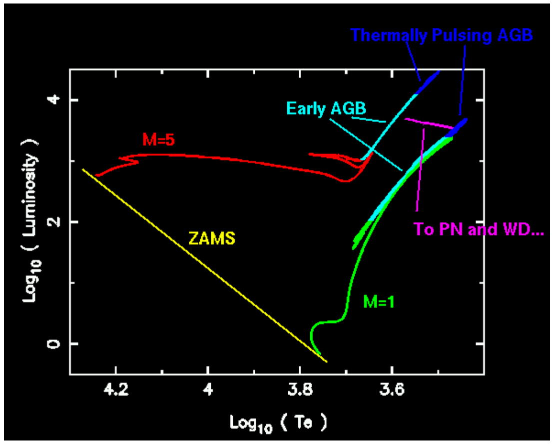
Notes:

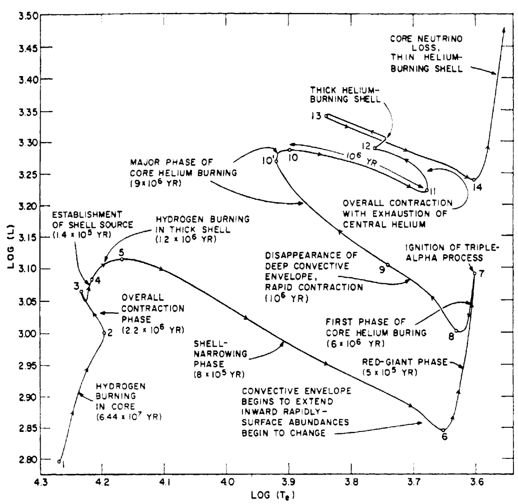
- ▷ contraction stops when the core density becomes high enough that *electron degeneracy pressure* can support the core
  (stars more massive than $\sim 2\,M_\odot$ ignite helium in the core before becoming degenerate)

- while the core contracts and becomes degenerate, the *envelope expands* dramatically
  → star becomes a *red giant*
  - ▷ the transition to the red-giant branch is not well understood (in intuitive terms)
  - ▷ for stars with $M \gtrsim 1.5\,M_\odot$, the transition occurs very fast, i.e. on a thermal timescale of the envelope → few stars observed in transition region *(Hertzsprung gap)*

Notes:

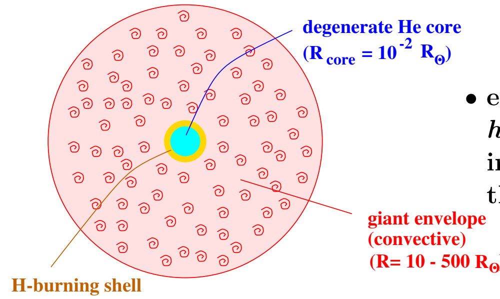
Evolutionary Tracks (1 to 5 $M_\odot$)





Notes:

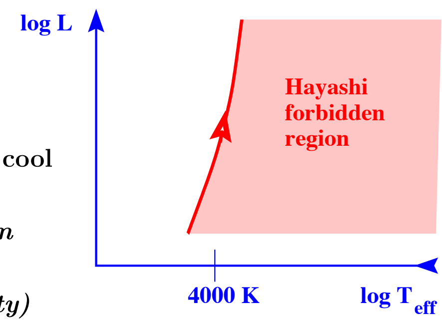
6.2.3 THE RED-GIANT PHASE



- energy source: *hydrogen burning* in thin *shell* above the degenerate core

- core mass grows → temperature in shell increases → luminosity increases → star *ascends red-giant branch*

- *Hayashi track:* vertical track in H-R diagram
  - ▷ no hydrostatic solutions for very cool giants
  - ▷ *Hayashi forbidden region* *(due to H^- opacity)*



- when the core mass reaches $M_c \simeq 0.48\,M_\odot$ → ignition of helium → *helium flash*

Notes:

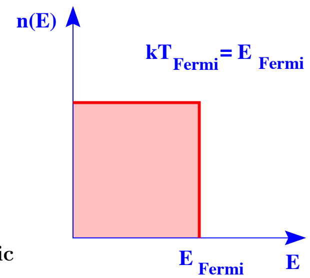
6.2.4 HELIUM FLASH

- *ignition of He under degenerate conditions*
  (for $\mathrm{M} \lesssim 2\,\mathrm{M}_\odot$; core mass $\sim 0.48\,\mathrm{M}_\odot$)
  - ▷ i.e. P is independent of T → *no self-regulation*

    [in normal stars: increase in T → decrease in ρ (expansion) → decrease in T (virial theorem)]
  - ▷ in degenerate case: nuclear burning → increase in T → more nuclear burning → further increase in T

  → *thermonuclear runaway*

- runaway *stops* when matter becomes *non-degenerate* (i.e. $\mathrm{T} \sim \mathrm{T}_{\mathrm{Fermi}}$)
- disruption of star?
  - ▷ energy generated in runaway:
  - ▷ $\Delta\mathrm{E} = \underbrace{\dfrac{\mathrm{M}_{\mathrm{burned}}}{\mu \mathrm{m}_{\mathrm{H}}}}_{\text{number of particles}} \underbrace{\mathrm{kT}_{\mathrm{Fermi}}}_{\text{characteristic energy}}$



→ $\Delta\mathrm{E} \sim 2 \times 10^{42}\,\mathrm{J} \left(\dfrac{\mathrm{M}_{\mathrm{burned}}}{0.1\,\mathrm{M}_\odot}\right) \left(\dfrac{\rho}{10^9\,\mathrm{kg\,m^{-3}}}\right)^{2/3} \qquad (\mu \simeq 2)$

▷ compare $\Delta\mathrm{E}$ to the binding energy of the core
$\mathrm{E}_{\mathrm{bind}} \simeq \mathrm{GM}_{\mathrm{c}}^2/\mathrm{R}_{\mathrm{c}} \sim 10^{43}\,\mathrm{J}$ ($\mathrm{M}_{\mathrm{c}} = 0.5\,\mathrm{M}_\odot$; $\mathrm{R}_{\mathrm{c}} = 10^{-2}\,\mathrm{R}_\odot$)

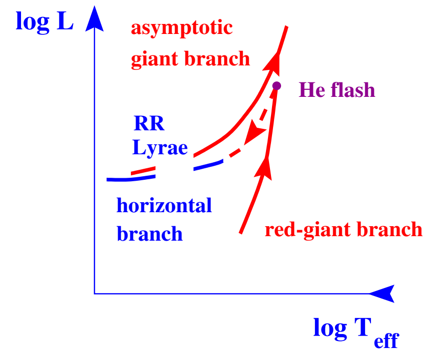
→ expect significant *dynamical expansion,* but no disruption ($\mathrm{t}_{\mathrm{dyn}} \sim$ sec)

→ core expands → *weakening of H shell source* → rapid decrease in luminosity

→ star settles on *horizontal branch*

Notes:

6.2.5 THE HORIZONTAL BRANCH (HB)



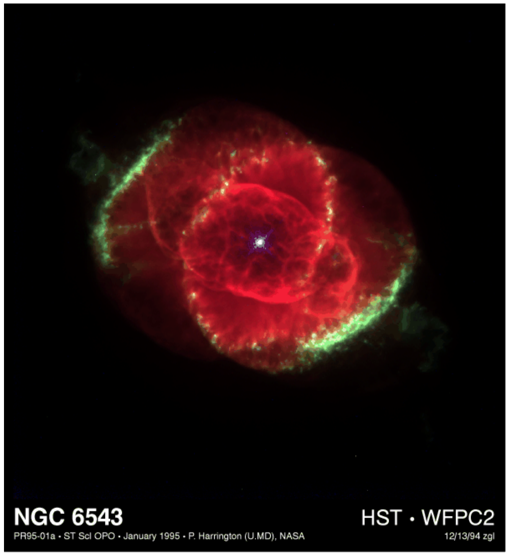
- *He burning* in center: conversion of He to mainly C and O ($^{12}\mathrm{C} + \alpha \rightarrow ^{16}\mathrm{O}$)
- *H burning* in shell (usually the dominant energy source)
- *lifetime:* $\sim 10\,\%$ of main-sequence lifetime
  (lower efficiency of He burning, higher luminosity)
- *RR Lyrae stars* are *pulsationally unstable*
  (L, B − V change with periods $\lesssim 1\,\mathrm{d}$)

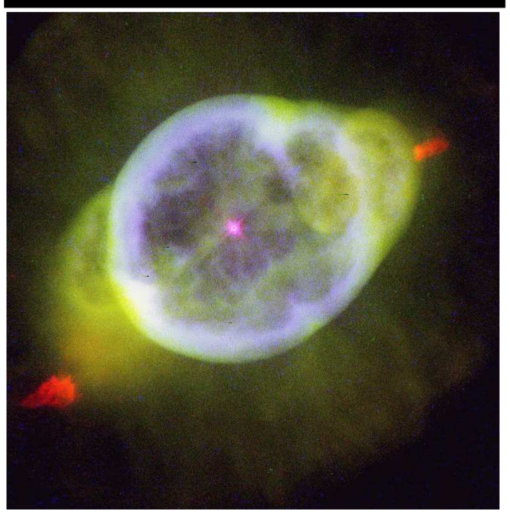
  easy to detect → popular *distance* indicators
- after *exhaustion of central He*

  → core *contraction* (as before) → *degenerate core* → *asymptotic giant branch*

Notes:

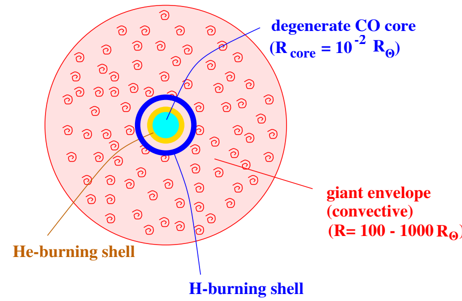
Planetary Nebulae with the HST





Notes:

6.2.6 THE ASYMPTOTIC GIANT BRANCH (AGB)



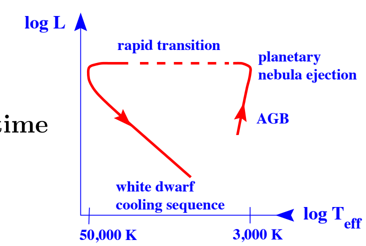
- ***H burning*** and ***He burning*** (in thin shells)
- H/He burning do not occur simultaneous, but alternate → ***thermal pulsations***

- low-/intermediate-mass stars ($\mathrm{M} \lesssim 8\,\mathrm{M}_\odot$) do not experience nuclear burning beyond helium burning
- ***evolution ends*** when the ***envelope*** has been ***lost*** by stellar winds
  - ▷ ***superwind phase:*** very rapid mass loss ($\dot{\mathrm{M}} \sim 10^{-4}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$)
  - ▷ probably because envelope attains ***positive binding energy*** (due to energy reservoir in ionization energy)
  - → envelopes can be dispersed to infinity without requiring energy source
  - ▷ complication: radiative losses

- after ejection:
  hot CO core is exposed
  and ***ionizes*** the ejected shell

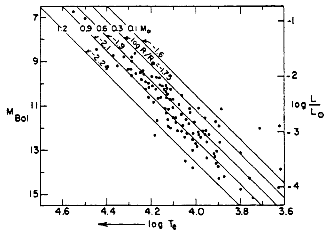
→ ***planetary nebula phase*** (lifetime $\sim 10^4\,\mathrm{yr}$)

- CO core cools, becomes ***degenerate*** → ***white dwarf***



Notes:

6.2.7 WHITE DWARFS (ZG: 17.1; CO: 13.2)



	Mass ($M_\odot$)	Radius ($R_\odot$)
Sirius B	1.053 ± 0.028	0.0074 ± 0.0006
40 Eri B	0.48 ± 0.02	0.0124 ± 0.0005
Stein 2051	0.50 ± 0.05	0.0115 ± 0.0012

- first white dwarf discovered: *Sirius B* (companion of Sirius A)
  - ▷ mass (from orbit): $M \sim 1\,M_\odot$
  - ▷ radius (from $L = 4\pi R^2 \sigma T_{eff}^4$) $R \sim 10^{-2}\,R_\odot \sim R_\oplus$
  - $\rightarrow$ $\rho \sim 10^9\,\mathrm{kg\,m^{-3}}$
- Chandrasekhar (Cambridge 1930)
  - ▷ white dwarfs are supported by *electron degeneracy pressure*
  - ▷ white dwarfs have a *maximum mass* of $1.4\,M_\odot$
- most white dwarfs have a *mass* very close to $M \sim 0.6\,M_\odot$: $M_{WD} = 0.58 \pm 0.02\,M_\odot$
- most are made of carbon and oxygen *(CO white dwarfs)*
- some are made of He or O-Ne-Mg

Notes:

Mass-Radius Relations for White Dwarfs

Non-relativistic degeneracy

- $P \sim P_e \propto (\rho/\mu_e m_H)^{5/3} \sim GM^2/R^4$

$$\rightarrow \boxed{R \propto \frac{1}{m_e}(\mu_e m_H)^{5/3} M^{-1/3}}$$

- note the *negative exponent*
  - $\rightarrow$ R decreases with increasing mass
  - $\rightarrow$ ρ increases with M

Relativistic degeneracy (when $p_{Fe} \sim m_e c$)

- $P \sim P_e \propto (\rho/\mu_e m_H)^{4/3} \sim GM^2/R^4$

$\rightarrow$ M *independent of* R

$\rightarrow$ existence of a *maximum mass*

Notes:

THE CHANDRASEKHAR MASS

- consider a star of radius R containing N Fermions (electrons or neutrons) of mass m_f
- the mass per Fermion is $\mu_f m_H$ (μ_f = mean molecular weight per Fermion) $\rightarrow$ number density $n \sim N/R^3$ $\rightarrow$ volume/Fermion $1/n$
- ***Heisenberg uncertainty principle***
  $[\Delta x\, \Delta p \sim \hbar]^3$ $\rightarrow$ typical momentum: $p \sim \hbar\, n^{1/3}$

$\rightarrow$ ***Fermi energy*** of relativistic particle ($E = pc$)

$$E_f \sim \hbar\, n^{1/3}\, c \sim \frac{\hbar c\, N^{1/3}}{R}$$

- ***gravitational energy*** per Fermion

$$E_g \sim -\frac{GM(\mu_f m_H)}{R}, \text{ where } M = N\, \mu_f m_H$$

$\rightarrow$ total energy (per particle)

$$E = E_f + E_g = \frac{\hbar c N^{1/3}}{R} - \frac{GN(\mu_f m_H)^2}{R}$$

- stable configuration has minimum of total energy
- if $E < 0$, E can be decreased without bound by decreasing R $\rightarrow$ no equilibrium $\rightarrow$ *gravitational collapse*
- maximum N, if $E = 0$

$$\rightarrow \quad N_{max} \sim \left(\frac{\hbar c}{G(\mu_f m_H)^2}\right)^{3/2} \sim 2 \times 10^{57}$$

$$M_{max} \sim N_{max}\, (\mu_e m_H) \sim 1.5\, M_\odot$$

Chandrasekhar mass for white dwarfs

$$\boxed{M_{Ch} = 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot}$$

Notes:

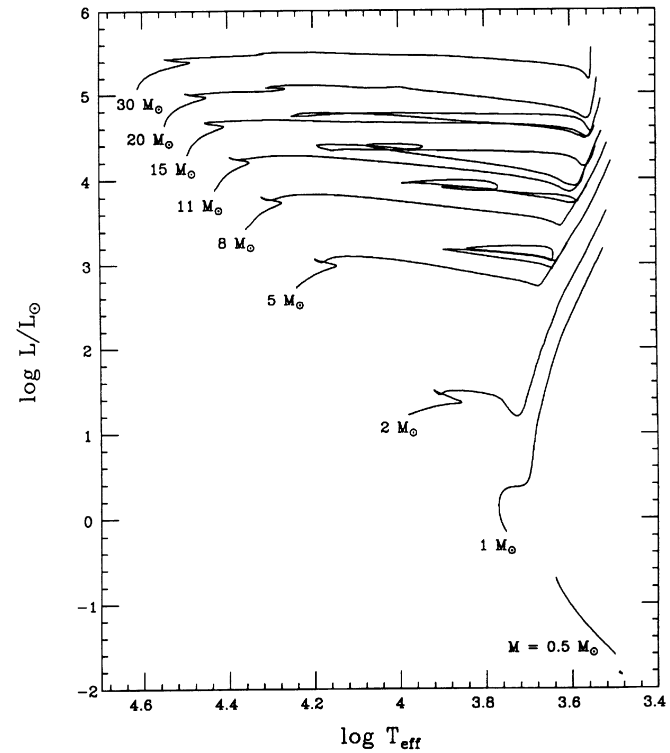
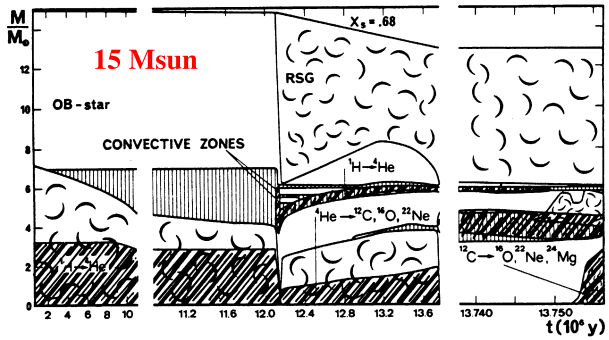


Figure B.1: Composite H-R diagram presenting the evolutionary tracks for stars between $0.5\,M_\odot$ and $30\,M_\odot$. The calculations assume an initially solar composition ($Y = 0.28$, $Z = 0.02$) and a mixing length parameter $\alpha = 1.5$.

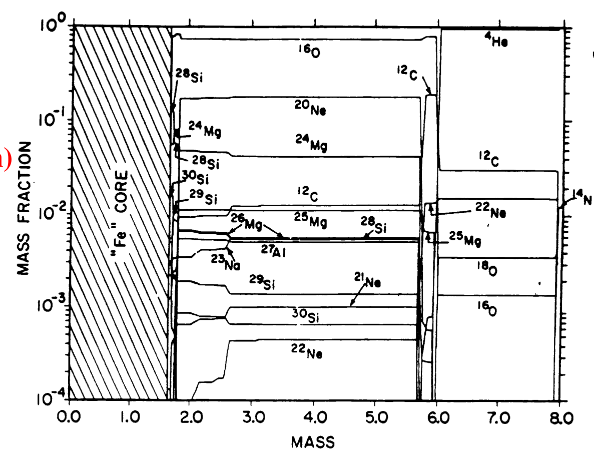
Notes:

Evolution of Massive Stars



Maeder (1987)

Helium Core (8 Msun)



Itoh and Nomoto (1987)

Notes:

6.3 EVOLUTION OF MASSIVE STARS ($M \gtrsim 13\,M_\odot$)
(CO: 13.3)

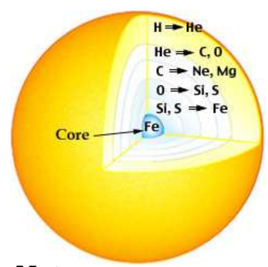
- massive stars continue to burn nuclear fuel beyond hydrogen and helium burning and ultimately form an *iron core*
- alternation of nuclear *burning and contraction* phases
  - ▷ *carbon burning* ($T \sim 6 \times 10^8$ K)

$$
\begin{aligned}
{}^{12}C + {}^{12}C &\rightarrow {}^{20}Ne + {}^{4}He \\
&\rightarrow {}^{23}Na + {}^{1}H \\
&\rightarrow {}^{23}Mg + n
\end{aligned}
$$

  - ▷ *oxygen burning* ($T \sim 10^9$ K)

$$
\begin{aligned}
{}^{16}O + {}^{16}O &\rightarrow {}^{28}Si + {}^{4}He \\
&\rightarrow {}^{31}P + {}^{1}H \\
&\rightarrow {}^{31}S + n \\
&\rightarrow {}^{30}S + 2\,{}^{1}H \\
&\rightarrow {}^{24}Mg + {}^{4}He + {}^{4}He
\end{aligned}
$$

  - ▷ *silicon burning:* photodisintegration of complex nuclei, hundreds of reactions → *iron*



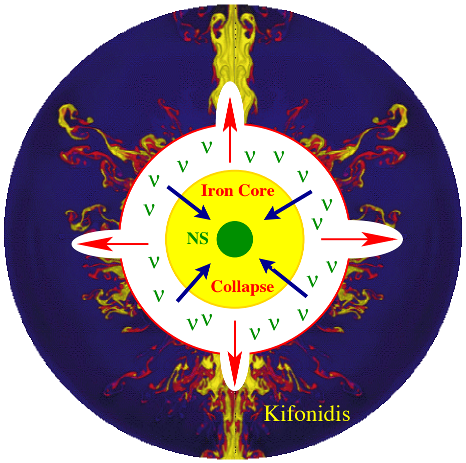
- ▷ form *iron core*
- ▷ *iron* is the most tightly *bound nucleus* → no further energy from nuclear fusion
- ▷ iron core surrounded by *onion-like shell structure*

Notes:

6.4.1 EXPLOSION MECHANISMS (ZG: 18-5B/C/D)

- *two* main, *completely different* mechanisms

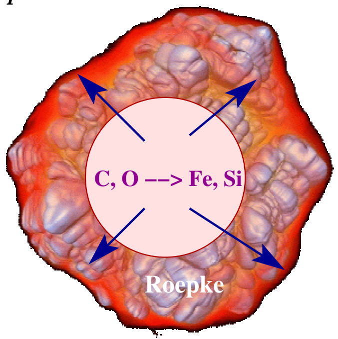
Core-Collapse Supernovae



- triggered after the exhaustion of nuclear fuel in the core of a massive star, if the *iron core mass > Chandrasekhar mass*
- energy source is *gravitational energy* from the collapsing core ($\sim 10\,\%$ of neutron star rest mass $\sim 3 \times 10^{46}$ J)
- most of the energy comes out in *neutrinos* (SN 1987A!)
  - ▷ *unsolved problem:* how is some of the neutrino energy *deposited* ($\sim 1\,\%$, 10^{44} J) in the envelope to *eject* the envelope and produce the supernova?
- leaves *compact remnant* (neutron star/black hole)

Notes:

Thermonuclear Explosions



- occurs in *accreting* carbon/oxygen *white dwarf* when it reaches the *Chandrasekhar mass*
  - → *carbon ignited* under *degenerate* conditions; nuclear burning raises T, but not P
  - → *thermonuclear runaway*
  - → incineration and *complete destruction* of the star
- energy source is *nuclear energy* (10^{44} J)
- *no compact remnant* expected
- main producer of *iron*
- *standard candle* (Hubble constant, acceleration of Universe?)

but: *progenitor* evolution not understood

- ▷ *single-degenerate channel:* accretion from non-degenerate companion
- ▷ *double-degenerate channel:* merger of two CO white dwarfs

Notes:

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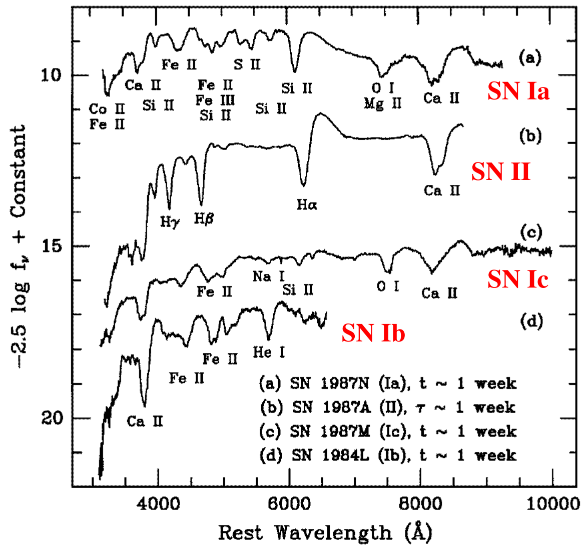
Supernova Classification



Notes:

6.4.2 SUPERNOVA CLASSIFICATION

observational:

- ***Type I: no hydrogen*** lines in spectrum
- ***Type II: hydrogen*** lines in spectrum

theoretical:

- ***thermonuclear explosion*** of degenerate core
- ***core collapse*** → neutron star/black hole

relation no longer 1 to 1 → confusion

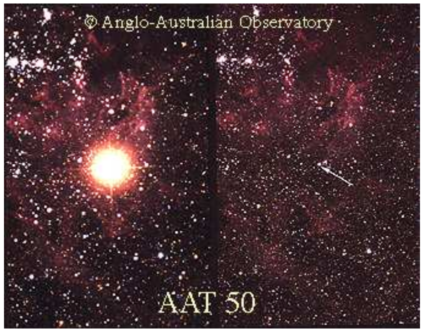
- ***Type Ia*** (Si lines): thermonuclear explosion of white dwarf
- ***Type Ib/Ic*** (no Si; He or no He): core collapse of He star
- ***Type II-P:*** "classical" core collapse of a massive star with hydrogen envelope
- ***Type II-L:*** supernova with linear lightcurve (thermonuclear explosion of intermediate-mass star? probably not!)

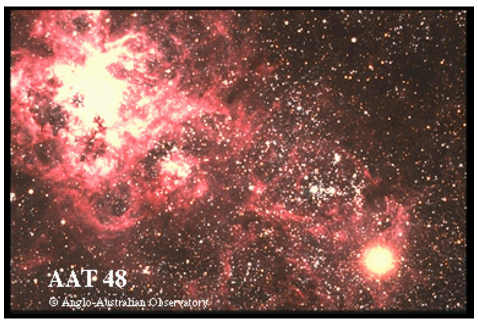
complications

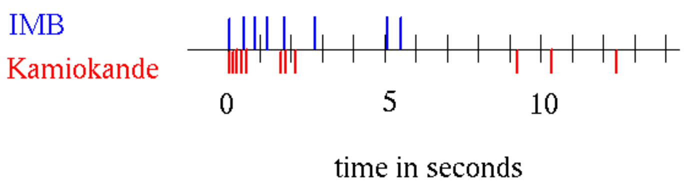
- special supernovae like ***SN 1987A***
- ***Type IIb:*** supernovae that change type, ***SN 1993J*** (Type II → Type Ib)
- some supernova ***"types"*** (e.g., IIn) occur for both explosion types (***"phenomenon"***, not type; also see SNe Ic)
- new types: thermonuclear explosion of He star (Type Iab?)

Notes:

SN 1987A (LMC)







Neutrino Signal

Notes:

6.4.3 SN 1987A (ZG: 18-5)

- SN 1987A in the Large Magellanic Cloud (satellite galaxy of the Milky Way) was the first naked-eye supernova since Kepler's supernova in 1604
- long-awaited, but highly unusual, ***anomalous*** supernova
  - ▷ progenitor ***blue supergiant*** instead of red supergiant
  - ▷ ***complex presupernova nebula***
  - ▷ ***chemical anomalies:*** envelope mixed with part of the helium core

Confirmation of core collapse

- neutrinos ($\overline{\nu}_{\rm e} + {\rm p} \rightarrow {\rm n} + {\rm e}^{+}$), detected with Kamiokande and IMB detectors
  - ▷ confirmation: supernova triggered by core collapse
  - ▷ formation of compact object (neutron star)
  - ▷ energy in neutrinos ($\sim 3 \times 10^{46}\,$J) consistent with the ***binding energy*** of a neutron star

Notes:

SUMMARY III(B): IMPORTANT STELLAR TIMESCALES

- *dynamical timescale:* $t_{dyn} \simeq \frac{1}{\sqrt{4G\rho}}$
$$\sim 30\,\text{min}\,\left(\rho/1000\,\text{kg}\,\text{m}^{-3}\right)^{-1/2}$$
- *thermal timescale* (Kelvin-Helmholtz): $t_{KH} \simeq \frac{GM^2}{2RL}$
$$\sim 1.5\times 10^{7}\,\text{yr}\,(M/M_\odot)^2\,(R/R_\odot)^{-1}\,(L/L_\odot)^{-1}$$
- *nuclear timescale:* $t_{nuc} \simeq \underbrace{M_c/M}_{\text{core mass}}\;\underbrace{\eta}_{\text{efficiency}}\;(Mc^2)/L$
$$\sim 10^{10}\,\text{yr}\,(M/M_\odot)^{-3}$$

Example	t_{dyn}	t_{KH}	t_{nuc}
main-sequence stars			
a) $M = 0.1\,M_\odot$, $L = 10^{-3}\,L_\odot$, $R = 0.15\,R_\odot$	4 min	10^9 yr	10^{12} yr
b) $M = 1\,M_\odot$, $L = 1\,L_\odot$, $R = 1\,R_\odot$	30 min	15×10^6 yr	10^{10} yr
c) $M = 30\,M_\odot$, $L = 2\times10^5\,L_\odot$, $R = 20\,R_\odot$	400 min	3×10^3 yr	2×10^6 yr
red giant ($M = 1\,M_\odot$, $L = 10^3\,L_\odot$, $R = 200\,R_\odot$)	50 d	75 yr	
white dwarf ($M = 1\,M_\odot$, $L = 5\times10^{-3}\,L_\odot$, $R = 2.6\times10^{-3}\,R_\odot$)	7 s	10^{11} yr	
neutron star ($M = 1.4\,M_\odot$, $L = 0.2\,L_\odot$, $R = 10$ km, $T_{eff} = 10^6$ K)	0.1 ms	10^{13} yr	

Notes:

SUMMARY V: THE END STATES OF STARS

Three (main) possibilities

- the star develops a *degenerate core* and nuclear burning stops (+ envelope loss) → *degenerate dwarf (white dwarf)*
- the star develops a *degenerate core* and *ignites* nuclear fuel *explosively* (e.g. carbon) → *complete disruption in a supernova*
- the star *exhausts* all of its *nuclear fuel* and the core exceeds the *Chandrasekhar mass* → *core collapse, compact remnant (neutron star, black hole)*

Final fate as a function of initial mass (M_0) for $Z = 0.02$

$M_0 \lesssim 0.08\,M_\odot$	*no hydrogen* burning (degeneracy pressure + Coulomb forces)	*planets, brown dwarfs*
$[0.08, 0.48]\,M_\odot$	*hydrogen* burning, *no helium* burning	*degenerate He dwarf*
$[0.48, 8]\,M_\odot$	*hydrogen, helium* burning	*degenerate CO dwarf*
$[8, 13]\,M_\odot$	*complicated* burning sequences, *no iron* core	*neutron star*
$[13, 80]\,M_\odot$	formation of *iron* core, *core collapse*	*neutron star, black hole*
$M_0 \gtrsim 80\,M_\odot$	*pair instability?* complete disruption?	*no remnant*
also (?) $[6, 8]\,M_\odot$	*degenerate carbon ignition* possible (but unlikely), complete disruption	*no remnant*

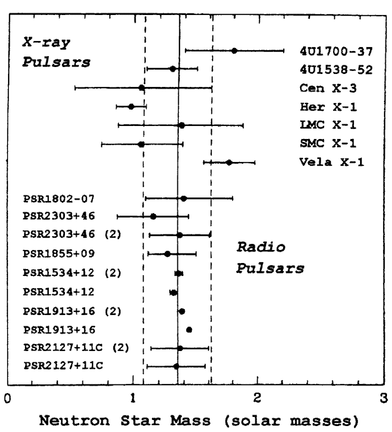
Notes:

6.4.4 NEUTRON STARS (ZG: 17-2; CO: 15.6)

- are the end products of the *collapse* of the cores (mainly Fe) of massive stars (between 8 and $\sim 20\,\mathrm{M}_\odot$)
- in the collapse, all nuclei are dissociated to produce a very compact remnant mainly composed of *neutrons* and some *protons/electrons*

  Note: this dissociation is *endothermic,* using some of the gravitational energy released in the collapse

  ▷ these reactions *undo* all the previous nuclear fusion reactions
- since neutrons are *fermions,* there is a *maximum mass* for a neutron star (similar to the Chandrasekhar mass for white dwarfs), estimated to be between *1.5* – *3* $\mathrm{M}_\odot$
- *typical radii: 10 km* (i.e. density $\sim 10^{18}\,\mathrm{kg\,m^{-3}}$!)



Notes:

6.4.5 SCHWARZSCHILD BLACK HOLES (ZG: 17-3; CO: 16)

- *event horizon:* (after Michell 1784)

  ▷ the *escape velocity* for a particle of mass m from an object of mass M and radius R is $v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$ ($11\,\mathrm{km\,s^{-1}}$ for Earth, $600\,\mathrm{km\,s^{-1}}$ for Sun)

  ▷ assume *photons* have *mass:* $m \propto E$ (Newton's corpuscular theory of light)

  ▷ photons travel with the *speed of light c*

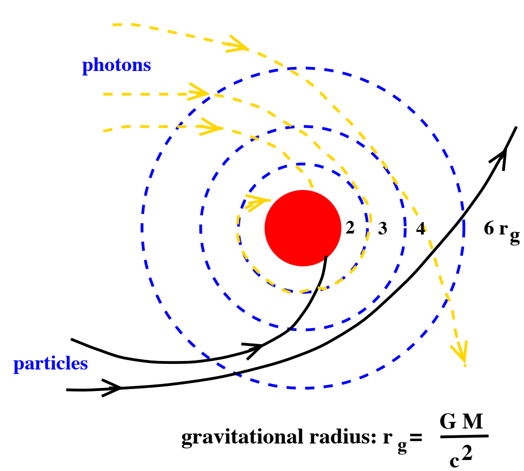
  → photons cannot escape, if $v_{\mathrm{esc}} > c$

  → $\boxed{R < R_s \equiv \frac{2GM}{c^2}}$ *(Schwarzschild radius)*

  ▷ $R_s = 3\,\mathrm{km}\,(M/\,M_\odot)$

  Note: for neutron stars $R_s \simeq 5\,\mathrm{km}$; only a factor of 2 smaller than R_{NS} → GR important

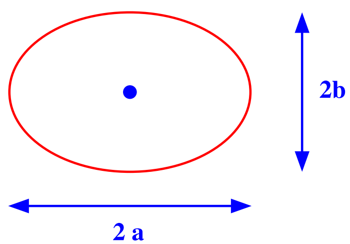
Orbits near Schwarzschild Black Holes



Notes:

7. BINARY STARS (ZG: 12; CO: 7, 17)

- most stars are members of binary systems or multiple systems (triples, quadruples, quintuplets, ...)
- *orbital period* distribution: $P_{orb} = 11\,\mathrm{min}$ to $\sim 10^6\,\mathrm{yr}$
- the majority of binaries are wide and do not interact strongly
- *close binaries* (with $P_{orb} \lesssim 10\,\mathrm{yr}$) can transfer mass $\rightarrow$ *changes structure and subsequent evolution*
- *approximate period distribution:* $f(\log P) \simeq \mathrm{const.}$ (rule of thumb: 10 % of systems in each decade of log P from 10^{-3} to 10^7 yr)
- generally large scatter in distribution of eccentricities $e^2 \equiv 1 - b^2/a^2$, a = semi-major, b = semi-minor axis



- systems with eccentricities $\lesssim 10\,\mathrm{d}$ tend to be circular $\rightarrow$ evidence for *tidal circularization*

Notes:

7.1 Classification

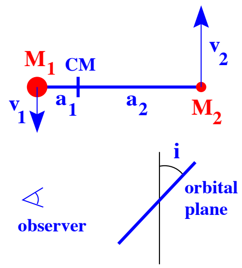
- *visual binaries:* see the periodic wobbling of two stars in the sky (e.g. Sirius A and B); if the motion of only one star is seen: *astrometric binary*
- *spectroscopic binaries:* see the periodic *Doppler shifts* of spectral lines
  - ▷ *single-lined:* only the Doppler shifts of one star detected
  - ▷ *double-lined:* lines of both stars are detected
- *photometric binaries:* periodic variation of fluxes, colours, etc. are observed (caveat: such variations can also be caused by single variable stars: Cepheids, RR Lyrae variables)
- *eclipsing binaries:* one or both stars are eclipsed by the other one $\rightarrow$ inclination of orbital plane $i \simeq 90^\circ$ (most useful for determining basic stellar parameters)

Notes:

Notes:

7.2 THE BINARY MASS FUNCTION
(Supplementary)

- consider a *spectroscopic binary*
- measure the *radial velocity curve* along the line of sight from $\frac{v_r}{c} \simeq \frac{\Delta\lambda}{\lambda}$ (Doppler shift)



▷ $M_1\,a_1 = M_2\,a_2$

▷ $P = \frac{2\pi}{\omega} = 2\pi\,\frac{a_1 \sin i}{v_1 \sin i} = 2\pi\,\frac{a_2 \sin i}{v_2 \sin i}$

▷ *gravitational force*
= *centripetal force*

$$\rightarrow\ \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_1 \sin i)^2}{a_1 \sin^2 i}\,M_1,\quad \frac{GM_1M_2}{(a_1+a_2)^2} = \frac{(v_2 \sin i)^2}{a_2 \sin^2 i}\,M_2$$

substituting $(a_1+a_2)^2 = a_1^2\,(M_1+M_2)^2/M_2^2$, etc.

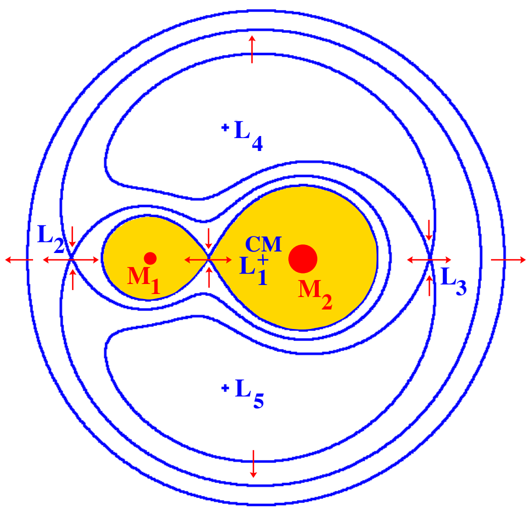
$$\rightarrow\ f_1(M_2) = \frac{M_2^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_1 \sin i)^3}{2\pi G}$$

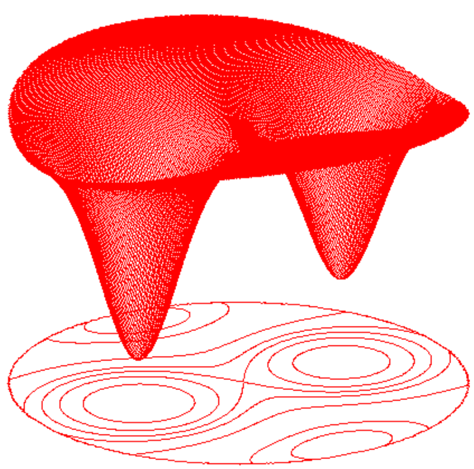
$$f_2(M_1) = \frac{M_1^3 \sin^3 i}{(M_1+M_2)^2} = \frac{P\,(v_2 \sin i)^3}{2\pi G}$$

- f_1, f_2 *mass functions:* relate observables $v_1 \sin i$, $v_2 \sin i$, P to quantities of interest M_1, M_2, $\sin i$
- measurement of f_1 and f_2 (for double-lined spectroscopic binaries only) determines $M_1 \sin^3 i$, $M_2 \sin^3 i$
  - ▷ if i is known (e.g. for visual binaries or eclipsing binaries) $\rightarrow M_1$, M_2
  - ▷ for $M_1 \ll M_2 \quad\rightarrow\quad f_1(M_2) \simeq M_2 \sin^3 i$ (measuring $v_1 \sin i$ for star 1 constrains M_2)
- for *eclipsing binaries* one can also determine the *radii* of both stars (main source of accurate masses and radii of stars [and luminosities if distances are known])

Notes:

The Roche Potential





Notes:

7.3 THE ROCHE POTENTIAL

- ***restricted three-body problem:*** **determine the motion of a test particle in the field of two masses $\mathbf{M_1}$ and $\mathbf{M_2}$ in a circular orbit about each other**
- **equation of motion of the particle in a frame rotating with the binary $\Omega = 2\pi/P$:**

$$\frac{d^2\vec{r}}{dt^2} = -\vec{\nabla}\,U_{\text{eff}} - \underbrace{2\vec{\Omega}\times\vec{v}}_{\text{Coriolis force}},$$

  where the *effective potential* U_{eff} is given by

$$U_{\text{eff}} = -\frac{GM_1}{|\vec{r}-\vec{r}_1|} - \frac{GM_2}{|\vec{r}-\vec{r}_2|} - \underbrace{\frac{1}{2}\Omega^2\,(x^2+y^2)}_{\text{centrifugal term}}$$

- ***Lagrangian points:*** **five stationary points of the Roche potential U_{eff} (i.e. where effective gravity $\vec{\nabla}U_{\text{eff}} = 0$)**
  - ▷ **3 saddle points: L_1, L_2, L_3**
- ***Roche lobe:*** **equipotential surface passing through the *inner Lagrangian point* L_1 ('connects' the gravitational fields of the two stars)**
- **approximate formula for the *effective Roche-lobe radius* (of star 2):**

$$R_L = \frac{0.49}{0.6 + q^{2/3}\ln\left(1+q^{-1/3}\right)}\,A,$$

  where $q = M_1/M_2$ is the mass ratio, A orbital separation.

Notes:

Classification of close binaries

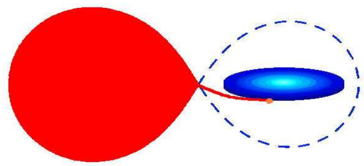
- ***Detached binaries:***
  - ▷ both stars ***underfill their Roche lobes,*** i.e. the photospheres of both stars lie beneath their respective Roche lobes
  - ▷ ***gravitational interactions*** only (e.g. tidal interaction, see Earth-Moon system)
- ***Semidetached binaries:***
  - ▷ one ***star fills its Roche lobe***
  - ▷ the Roche-lobe filling component ***transfers matter*** to the detached component
  - ▷ ***mass-transferring binaries***
- ***Contact binaries:***
  - ▷ ***both stars fill or overfill their Roche lobes***
  - ▷ formation of a common photosphere surrounding both components
  - ▷ ***W Ursae Majoris stars***

Notes:

7.4 BINARY MASS TRANSFER

- ***30 - 50 %*** of all stars experience mass transfer by ***Roche-lobe*** overflow during their lifetimes (generally in late evolutionary phases)

a) (quasi-)***conservative mass transfer***



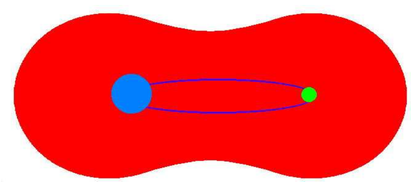
▷ ***mass loss + mass accretion***

▷ the mass loser tends to lose most of its envelope → formation of ***helium stars***

▷ the accretor tends to be ***rejuvenated*** (i.e. behaves like a more massive star with the evolutionary clock reset)

▷ ***orbit*** generally ***widens***

b) ***dynamical mass transfer → common-envelope*** and spiral-in phase (mass loser is usually a red giant)



▷ accreting component also fills its Roche lobe

▷ mass donor (primary) ***engulfs secondary***

▷ ***spiral-in*** of the core of the primary and the secondary immersed in a ***common envelope***

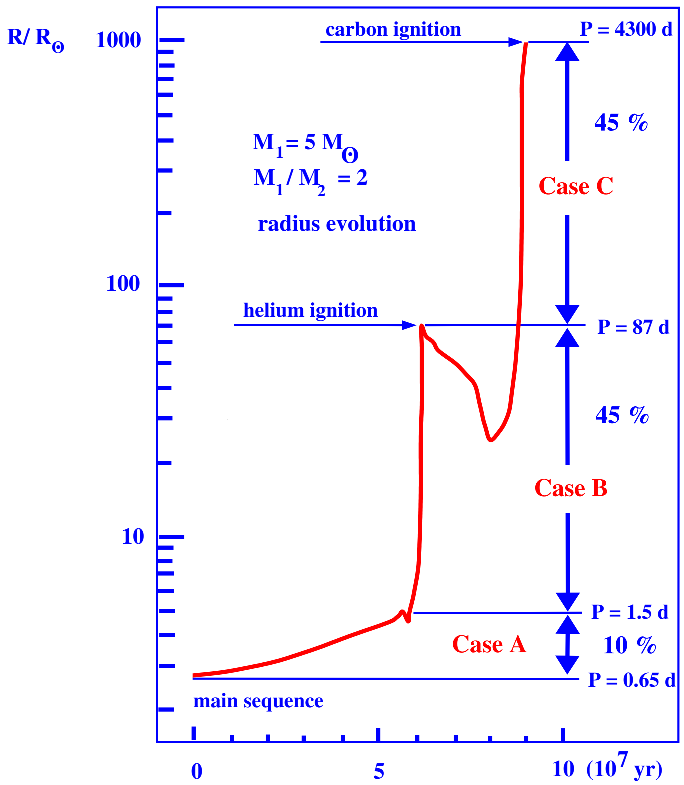
▷ if ***envelope ejected → very close binary*** (compact core + secondary)

▷ otherwise: ***complete merger*** of the binary components → ***formation of a single, rapidly rotating star***

Notes:

Classification of Roche-lobe overflow phases

(Paczynski)



Notes:

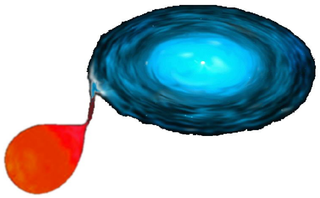
7.5 INTERACTING BINARIES (SELECTION)
(Supplementary)

Algols and the Algol paradox

- *Algol* is an eclipsing binary with orbital period 69 hr, consisting of a B8 dwarf ($M = 3.7\,M_\odot$) and a K0 subgiant ($M = 0.8\,M_\odot$)
- the *eclipse* of the B0 star is *very deep* → *B8 star more luminous* than the more evolved K0 subgiant
- the *less massive star is more evolved*
- inconsistent with stellar evolution → *Algol paradox*
- explanation:
  - ▷ the K star was *initially the more massive* star and evolved more rapidly
  - ▷ *mass transfer* changed the mass ratio
  - ▷ because of the added mass the B stars becomes the more luminous component

Notes:

Interacting binaries containing compact objects (Supplementary)



- short orbital periods (11 min to typically 10s of days) → requires *common-envelope* and *spiral-in* phase

Cataclysmic Variables (CV)

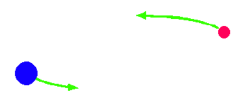
- main-sequence star (usually) transferring mass to a *white dwarf* through an *accretion disk*
- *nova outbursts: thermonuclear explosions* on the surface of the white dwarf
- orbit shrinks because of angular-momentum loss due to *gravitational radiation and magnetic braking*

X-Ray Binaries

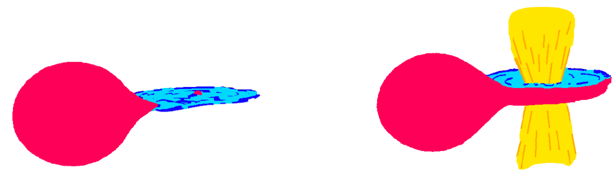
- *compact component: neutron star, black hole*
- *mass donor* can be of low, intermediate or high mass
- very luminous *X-ray sources* (accretion luminosity)
- neutron-star systems: luminosity distribution peaked near the *Eddington limit,* (accretion luminosity for which radiation pressure balances gravity) $\mathrm{L_{Edd}} = \dfrac{4\pi \mathrm{cGM}}{\kappa} \simeq 2\times10^{31}\,\mathrm{W}\left(\dfrac{\mathrm{M}}{1.4\,\mathrm{M_\odot}}\right)$
- accretion of mass and angular momentum → *spin-up* of neutron star → formation of *millisecond pulsar*
- *soft X-ray transients:* best *black-hole* candidates (if $\mathrm{M_X} >$ max. neutron-star mass $\sim 2-3\,\mathrm{M_\odot}$ → likely black hole [but no proof of event horizon yet])

Notes:

Formation of Low-Mass X-Ray Binaries (I)

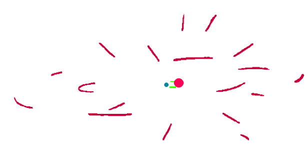


wide binary with large mass ratio



dynamical mass transfer

common-envelope and spiral-in phase



ejection of common envelope and subsequent supernova

Notes:

APPENDICES (Supplementary [=Non-Examinable] Material)

A. Brown Dwarfs

B. Planets

C. The Structure of the Sun and the Solar Neutrino Problem

D. Star Formation

E. Gamma-Ray Bursts

Notes:

A. Brown Dwarfs (Supplementary)

- star-like bodies with ***masses too low*** to create the central temperature required ***to ignite fusion reactions*** (i.e. $M \lesssim 0.08\,M_\odot$ from theory).
- reach maximum temperature by gravitational contraction and then cool steadily becoming undetectable, with surface temperature less than 1000 K, after a few billion years (stars with $T_{\rm eff} < 2000$ and $L \lesssim 5 \times 10^{-4}\,L_\odot$ mainly emit in the infrared).
- Brown dwarfs are prime dark matter candidates (only detectable in the solar neighborhood)
- ***Recent developments*** leading to ***successful searches:***

  (i) Larger optical and IR detectors (CCDs) with large telescopes (8-10 m) (e.g. nearby, young clusters)

  (ii) All-sky IR surveys.

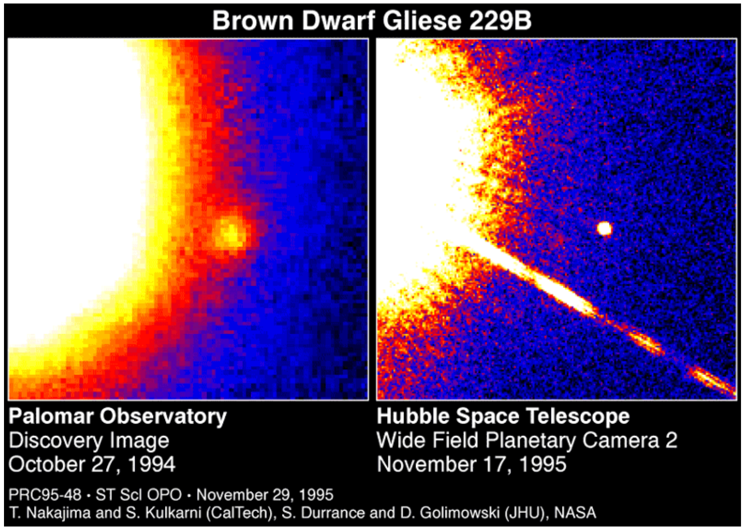
  (iii) Development of powerful IR spectrographs.
- ***Spectral signatures*** of Brown Dwarfs:

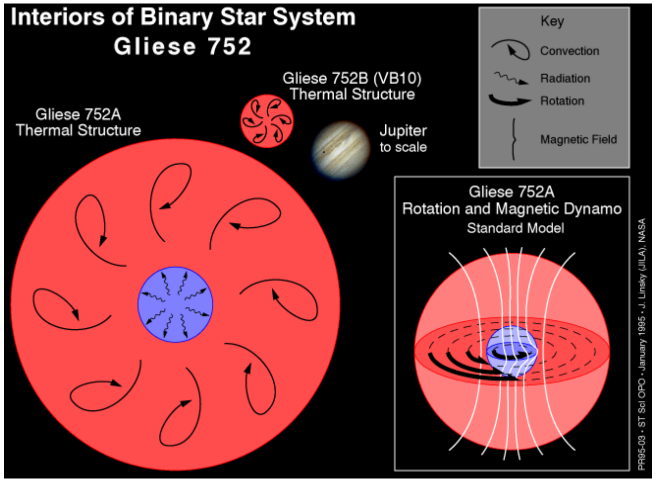
  (i) ***Strong Li lines*** - Brown Dwarfs retain original Li for ever.

  (ii) ***Methane bands*** c.f. Jupiter - dominant when $T_{\rm eff} < 1500$ K.

  (iii) ***L stars*** - bands of FeH, CrH appear instead of TiO, VO ***(M stars);*** also prominent lines of Cs I, Rb I - related to dust formation at $T_{\rm eff} < 2000$ K.
- ***Missing Mass:*** Detections so far indicate that Brown Dwarfs are ***not sufficiently abundant*** to account significantly ***for the missing mass.***

Notes:





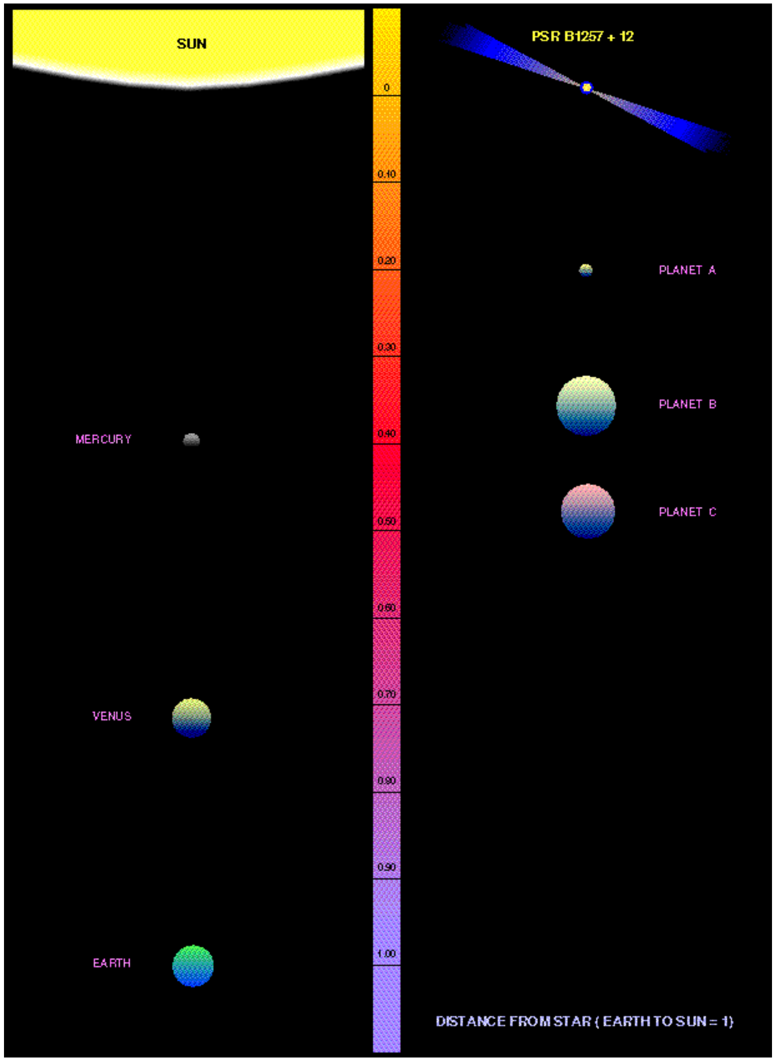
Notes:

B. Extrasolar Planets (Supplementary)

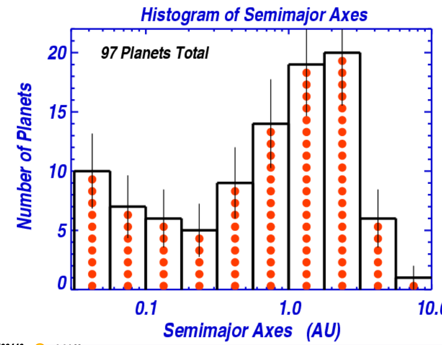
(http://ast.star.rl.ac.uk/darwin/links.html#exoplanets)

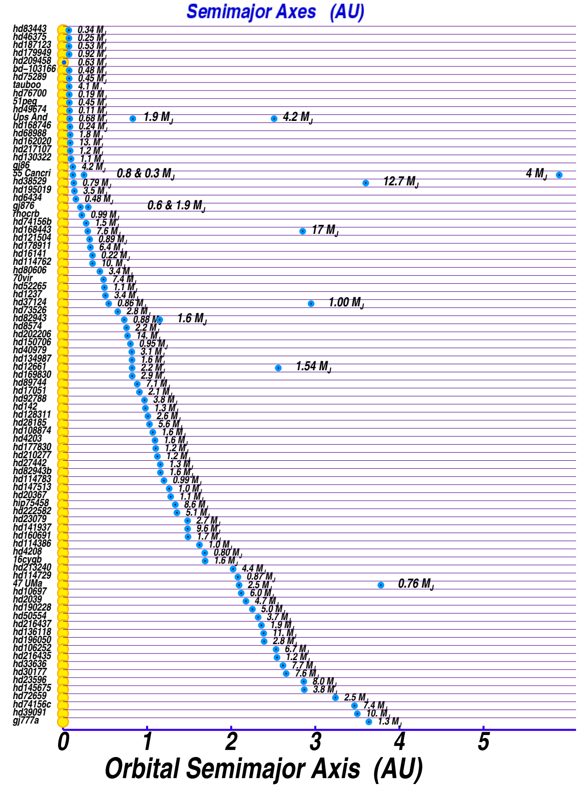
- large numbers of planets have been discovered in the last decade
- first planetary system detected outside the solar system was around a *millisecond pulsar, PSR 1257+12*, a rapidly rotating neutron star, spinning with a period of 6.2 msec (Wolszczan 1992)
  - ▷ 3 planets with masses $> 0.015\,\mathrm{M}_\oplus$, (25 d), $> 3.4\,\mathrm{M}_\oplus$ (66 d), $> 2.8\,\mathrm{M}_\oplus$ (98 d)
  - ▷ detection possible because of extreme timing precision of pulsar (measure effects of tiny reflex motion of pulsar caused by planets)
  - ▷ planets almost certainly formed *after the supernova* that formed the neutron star, out of material that was left over from *disrupted companion star* (?) and formed a disk (similar to planet formation in the solar system?)
- *since 1995* many planets (generally very massive $>> \mathrm{M_{Jup}}$) have been discovered around normal stars

Notes:


SUN
PSR B1257 + 12
0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
PLANET A
PLANET B
PLANET C
MERCURY
VENUS
EARTH
DISTANCE FROM STAR { EARTH TO SUN = 1}

Notes:


Histogram of Semimajor Axes
97 Planets Total
Number of Planets
0
5
10
15
20
0.1
1.0
10.0
Semimajor Axes (AU)

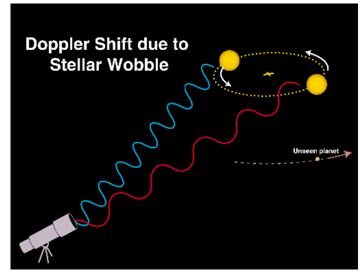

0
1
2
3
4
5
Orbital Semimajor Axis (AU)

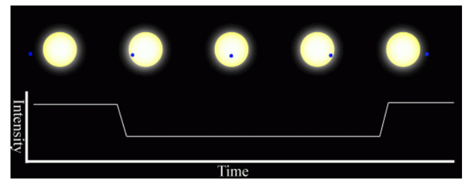
Notes:

Detection Techniques for Extrasolar Planets

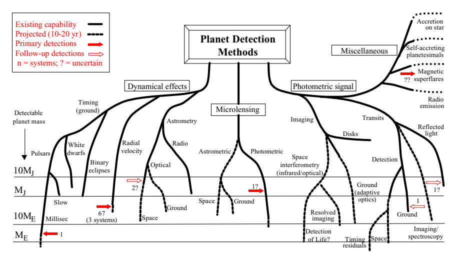
- ***Direct Imaging:*** relies on the fact that planets reflect their parent star's light (April 2005: 2M1207 Brown dwarf with planetary companion)
- ***Photometry – Planetary Transits.*** Photometry can be used to detect a change in the brightness of a star, as in the case when a planet transits (passes in from of) its parent star.
- ***Astrometry:*** by detecting the wobbling motion of a star in the sky due to the motion of the planet
- ***Radial velocity:*** Measure the periodic variation of the velocity of the central star (from the Doppler shifts of spectral lines) caused by the orbiting planets
- Present methods favour detection of massive (gaseous) planets (super-Jupiters) close the central star (→ large radial velocity variations); they are probably completely *unrepresentative* of the majority of planetary systems (which are ubiquitous).

Notes:





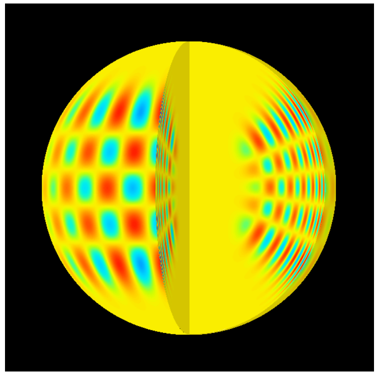
Planet Detection Methods
Michael Perryman, April 2001



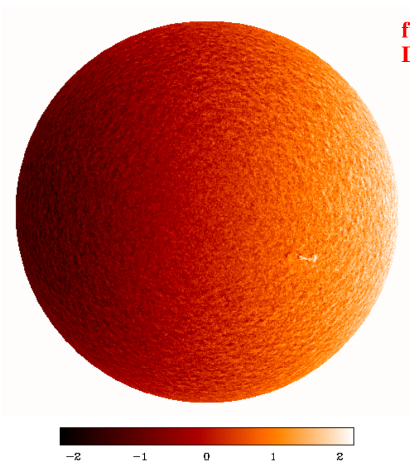
Notes:

HELIOSEISMOLOGY (I)

acoustic mode
in the Sun
(p mode
n=14, l -20)



full-disk
Dopplergram



velocity (km/s)

Notes:

C. STRUCTURE OF THE SUN (ZG: 10, CO: 11)

- The *Sun* is the only star for which we can *measure* internal properties → test of stellar structure theory
- *Composition* (heavy elements) from *meteorites*
- *Density, internal rotation* from *helioseismology*
- *Central conditions* from *neutrinos*

HELIOSEISMOLOGY

- The Sun acts as a *resonant cavity, oscillating* in millions of (acoustic, gravity) modes (like a bell)

→ can be used to reconstruct the internal density structure (like earthquakes on Earth)

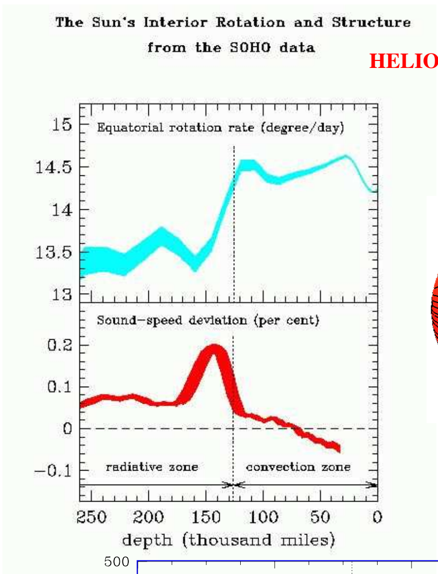
- oscillation modes are *excited* by *convective eddies*
- *periods* of typical modes: *1.5 min to 20 min*
- *velocity amplitudes:* $\sim 0.1\,m/s$
- need to measure *Doppler shifts* in spectral lines relative to their width to an accuracy of $1{:}10^6$
  - ▹ possible with good spectrometers and long integration times (to average out noise)

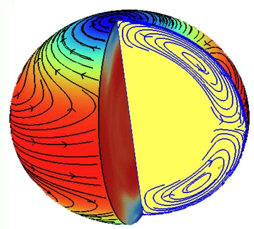
Results

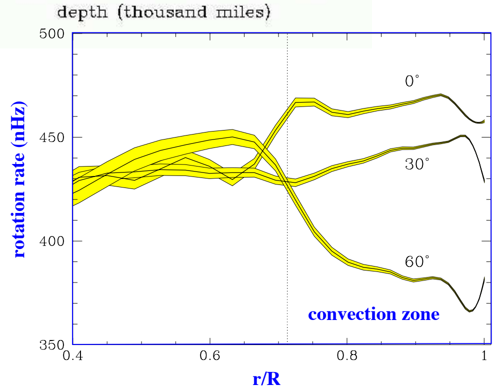
- *density* structure, *sound speed*
- depth of outer *convective zone:* $\sim 0.28\,\mathrm{R}_\odot$
- *rotation* in the core is *slow* (almost like a solid-body) → the core must have been spun-down with the envelope (efficient core–envelope coupling)

Notes:

HELIOSEISMOLOGY (II)







Notes:

SOLAR NEUTRINOS (ZG: 5-11, 16-1D, CO: 11.1)

- ***Neutrinos,*** generated in solar core, *escape* from the Sun and carry away $2-6\,\%$ of the energy released in H-burning reactions
- they can be observed in *underground experiments* $\rightarrow$ *direct probe of the solar core*
- neutrino-emitting reactions (in the pp chains)

$$\begin{aligned} {}^1\mathrm{H} + {}^1\mathrm{H} &\rightarrow {}^2\mathrm{D} + \mathrm{e}^+ + \nu \qquad \mathrm{E}_\nu^{\max} = 0.42\,\mathrm{Mev} \\ {}^7\mathrm{Be} + \mathrm{e}^- &\rightarrow {}^7\mathrm{Li} + \nu \qquad \mathrm{E}_\nu^{\max} = 0.86\,\mathrm{Mev} \\ {}^8\mathrm{B} &\rightarrow {}^8\mathrm{Be} + \mathrm{e}^+ + \nu \qquad \mathrm{E}_\nu^{\max} = 14.0\,\mathrm{Mev} \end{aligned}$$

- The *Davis experiment* (starting around 1970) has shown that the neutrino flux is about a factor of 3 lower than predicted $\rightarrow$ *the solar neutrino problem*

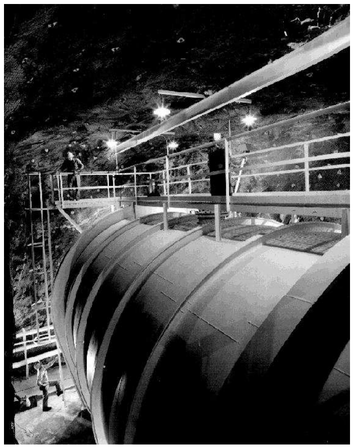
The Homestake experiment (Davis)

- *neutrino detector:* underground tank filled with 600 tons of Chlorine ($\mathrm{C_2\,Cl_4}$: dry-cleaning fluid)
- some neutrinos react with Cl

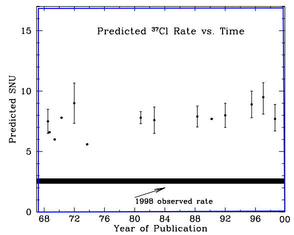
  $\nu_\mathrm{e} + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + \mathrm{e}^- - 0.81\,\mathrm{Mev}$
- rate of absorption $\sim 3 \times 10^{-35}\,\mathrm{s}^{-1}$ per ${}^{37}\mathrm{Cl}$ atom
- every 2 months *each ^{37}Ar atom is filtered out* of the tank (expected number: 54; observed number: 17)
- *caveats*
  - ▷ difficult experiment, only a tiny number of the neutrinos can be detected
  - ▷ the experiment is only sensitive to the most energetic neutrinos in the ^{8}B reaction (only minor reaction in the Sun)

Notes:

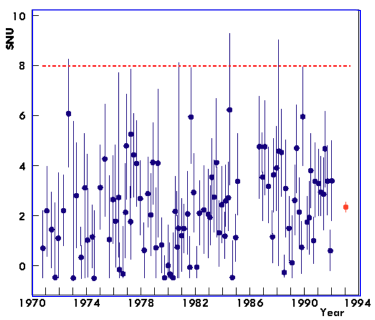
The Davis Neutrino Experiment



Model Predictions



Homestake Mine
(with Cl tank)



Results

Notes:

Proposed Solutions to the Solar Neutrino Problem

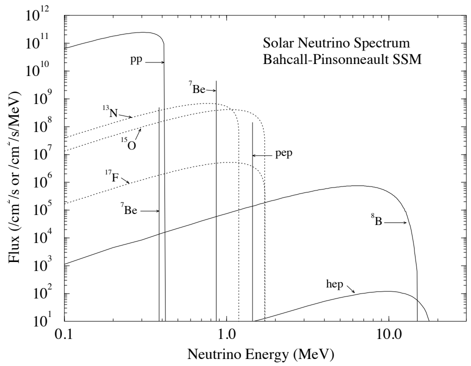
- dozens of solutions have been proposed

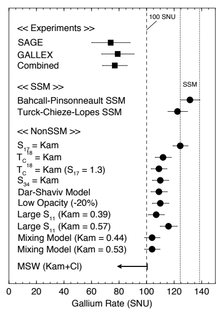
1) *Astrophysical solutions*
   - ▷ require a *reduction in central temperature* of about 5 % (standard model: 15.6×10^6 K)
   - ▷ can be achieved if the solar core is mixed (due to convection, rotational mixing, etc.)
   - ▷ if there are *no nuclear reactions in the centre* (inert core: e.g. central black hole, iron core, degenerate core)
   - ▷ if there are *additional energy transport* mechanisms (e.g. by WIMPS = weakly interacting particles)
   - ▷ most of these astrophysical solutions also change the density structure in the Sun → can now be *ruled out by helioseismology*

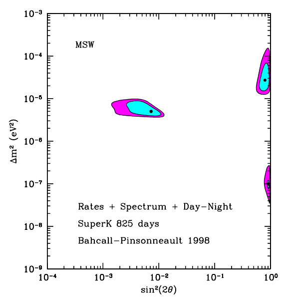
2) *Nuclear physics*
   - ▷ errors in *nuclear cross sections* (cross sections sometimes need to be revised by factors up to ~ 100)
   - ▷ improved experiments have *confirmed the nuclear cross sections* for the key nuclear reactions

Notes:

Solar Neutrinos







Notes:

3) ***Particle physics***

- ▷ all neutrinos generated in the Sun are ***electron neutrinos***
- ▷ if neutrinos have a ***small mass*** (actually mass differences), neutrinos may change type on their path between the centre of the Sun and Earth: ***neutrino oscillations,*** i.e. change from electron neutrino to μ or τ neutrinos, and then cannot be detected by the Davis experiment
- ▷ ***vacuum oscillations:*** occur in vacuum
- ▷ ***matter oscillations (MSW [Mikheyev-Smirnov--Wolfenstein] effect):*** occur only in matter (i.e. as neutrinos pass through the Sun)

RECENT EXPERIMENTS

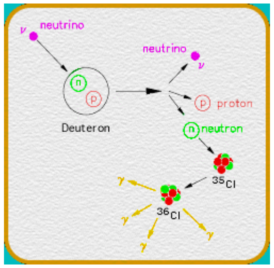
- resolution of the neutrino puzzle requires more sensitive detectors that can also detect neutrinos from the main pp-reaction

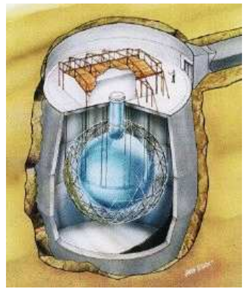
1) ***The Kamiokande experiment***
(also super-Kamiokande)

- ▷ uses 3000 tons of ultra-pure water (680 tons active medium) for
$\nu + \mathrm{e}^- \rightarrow \nu + \mathrm{e}^-$ (inelastic scattering)
- ▷ about six times more likely for ν_e than ν_μ and ν_τ
- ▷ ***observed flux:*** half the predicted flux (energy dependence of neutrino interactions?)

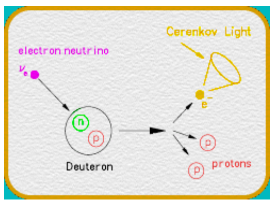
Notes:

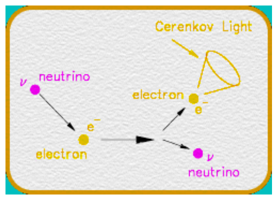
The Sudbury Neutrino Observatory

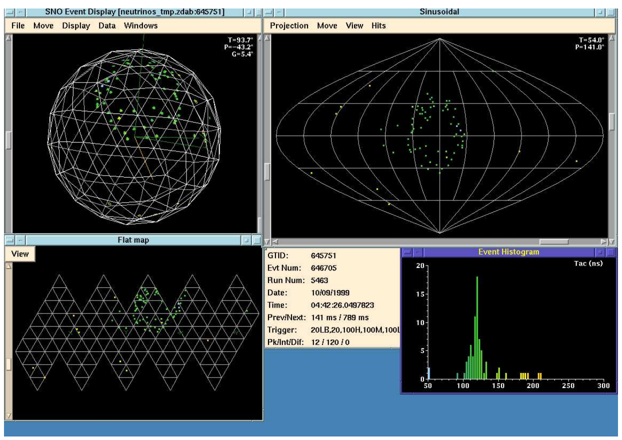




1000 tons of heavy water







Notes:

2) *The Gallium experiments* (GALLEX, SAGE)

- ▷ uses Gallium to measure low-energy *pp neutrinos* directly
  $\nu_e + ^{71}\mathrm{Ga} \rightarrow ^{71}\mathrm{Ge} + e^- - 0.23\,\mathrm{Mev}$
- ▷ *results:* about 80 ± 10 SNU vs. predicted 132 ± 7 SNU (1 SNU: 10^{-36} interactions per target atom/s)

3) *The Sudbury Neutrino Observatory (SNO)*

- ▷ located in a deep mine (2070 m underground)
- ▷ 1000 tons of pure, *heavy water* (D_2O)
- ▷ in acrylic plastic vessel with 9456 light sensors/photomultiplier tubes
- ▷ detect *Cerenkov radiation* of electrons and photons from weak interactions and neutrino-electron scattering
- ▷ *results (June 2001): confirmation of neutrino oscillations (MSW effect)?*

• *2005: Solar Models in a Crisis?*

- ▷ *new abundance* determinations (C and O) have led to a significant reduction in the solar metallicity
- → solar models no longer fit helioseismology constraints
- ▷ no clear solution so far

Notes:

Star Formation (I)

Orion Nebula

Notes:

D. STAR FORMATION (ZG: 15.3; CO: 12)

Star-Forming Regions

a) ***Massive stars***

- born in ***OB associations*** in warm molecular clouds
- produce ***brilliant HII regions***
- shape their environment
  - ▷ photoionization
  - ▷ stellar winds
  - ▷ supernovae

  → induce further (low-mass) star formation?

b) ***Low-mass stars***

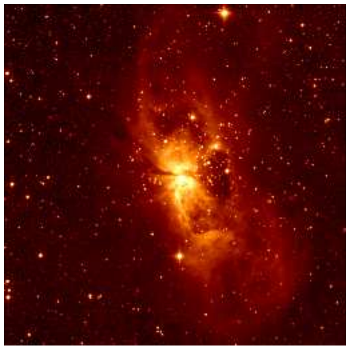
- born in ***cold, dark molecular clouds*** ($T \simeq 10\,K$)
- Bok globules
- near massive stars?
- ***recent:*** most low-mass stars appear to be born in ***cluster-like environments***
- ***but:*** most low-mass stars are not found in clusters → embedded ***clusters do not survive***

  Relationship between massive and low-mass star formation?

  - ▷ massive stars trigger low-mass star formation?
  - ▷ massive stars terminate low-mass star formation?

Notes:

Star Formation (II)



massive star +
cluster of low-mas stars

S 106

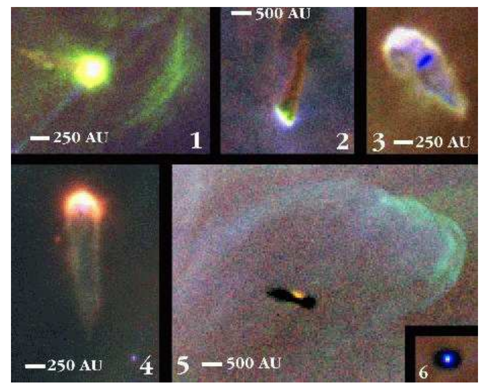
Bok globules

Notes:

Star Formation (III)

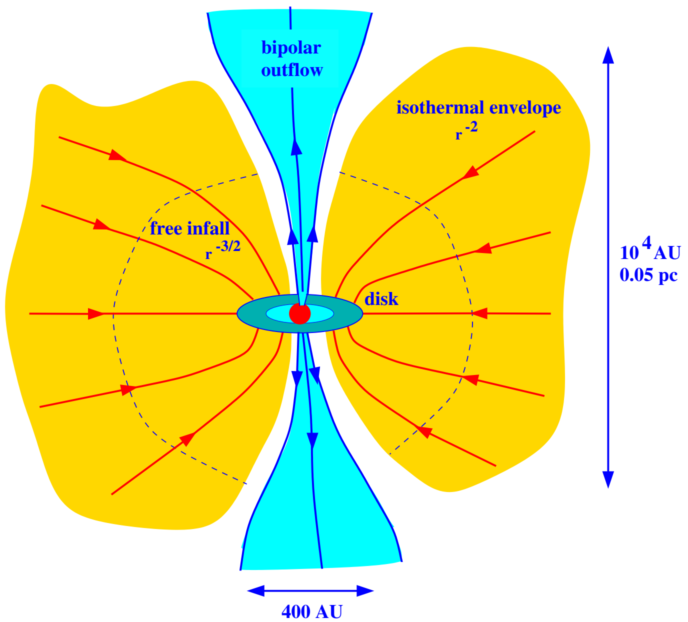
The Trapezium Cluster (IR)

HST



Dusty Disks in Orion (seen as dark silhouettes)

Notes:



Protostar Structure

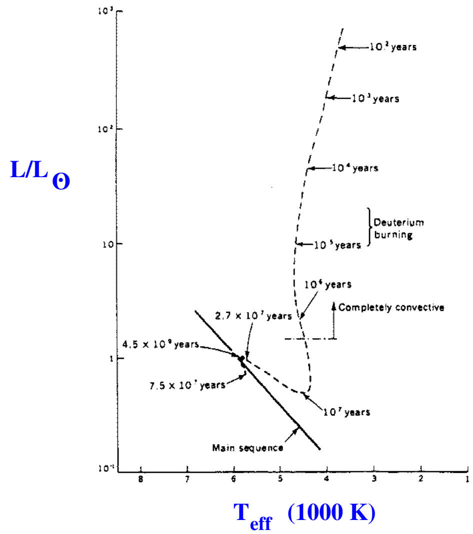
Notes:

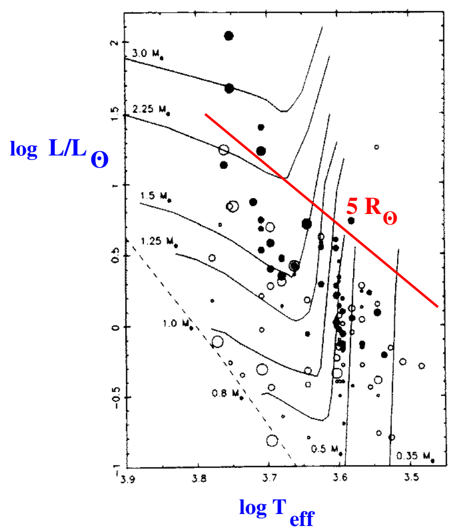
Stellar Collapse (Low-mass)

- cool, ***molecular cores*** (H_2) collapse when their mass exceeds the ***Jeans Mass***
  - ▷ no thermal pressure support if $P_c = \rho/(\mu m_H)kT < GM^2/(4\pi R^4)$
  - ▷ or $M > M_J \simeq 6\,M_\odot \left(\frac{T}{10\,K}\right)^{3/2} \left(\frac{n_{H_2}}{10^{10}\,m^{-3}}\right)^{-1/2}$
- collapse triggered:
  - ▷ by ***loss of magnetic support***
  - ▷ by ***collision*** with other cores
  - ▷ by ***compression*** caused by nearby supernovae
- ***inside-out isothermal collapse*** (i.e. efficient radiation of energy) from $\sim 10^6\,R_\odot$ to $\sim 5\,R_\odot$
- ***timescale:*** $t_{dyn} \sim 1/\sqrt{4\,G\rho} \sim 10^5 - 10^6$ yr
- collapse ***stops*** when material becomes ***optically thick*** and can no longer remain isothermal ***(protostar)***
- ***the angular-momentum problem***
  - ▷ each molecular core has a small amount of angular momentum (due to the velocity shear caused by the Galactic rotation)
  - ▷ characteristic $\Delta v/\Delta R \sim 0.3$km/s/ly
  - → characteristic, specific angular momentum $j \sim (\Delta v/\Delta R\, R_{cloud})\, R_{cloud} \sim 3 \times 10^{16}\, m^2\, s^{-1}$
  - ▷ cores cannot collapse directly
  - → formation of an ***accretion disk***

Notes:

Pre-Main-Sequence Evolution





Notes:

- ▷ characteristic disk size from angular-momentum conservation $\mathrm{j = rv_\perp = rv_{Kepler}} = \sqrt{\mathrm{GMr}}$

  → $\mathrm{r_{min} = j^2/GM \sim 10^4\,R_\odot \simeq 50AU}$

- *Solution:* Formation of *binary systems and planetary systems* which store the angular momentum (Jupiter: 99 % of angular momentum in solar system)

  → *most stars should have planetary systems and/or stellar companions*

  → stars are initially *rotating rapidly* (spin-down for stars like the Sun by magnetic braking)

- inflow/outflow: $\sim 1/3$ of material accreted is ejected from the accreting protostar → bipolar jets

Pre-main-sequence evolution

- Old picture: stars are born with *large radii* ($\sim 100\,\mathrm{R_\odot}$) and slowly contract to the main sequence

  ▷ energy source: *gravitational energy*

  ▷ contraction stops when the central temperature reaches $10^7\,\mathrm{K}$ and H-burning starts (main sequence)

  ▷ note: D already burns at $\mathrm{T_c \sim 10^6\,K}$ → temporarily halts contraction

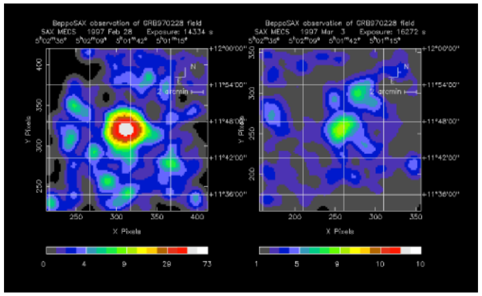
- *Modern picture:* stars are born with *small radii* ($\sim 5\,\mathrm{R_\odot}$) and small masses

  → first appearance in the H-R diagram on the *stellar birthline* (where accretion timescale is comparable to Kelvin-Helmholtz timescale: $\mathrm{t_{\dot M} \equiv M/\dot M \sim t_{KH} = GM^2/(2RL)}$)

  ▷ continued accretion as *embedded protostars/T Tauri stars* until the mass is exhausted or accretion stops because of dynamical interactions with other cores/stars

Notes:

Gamma-Ray Bursts



Beppo-Sax X-ray detection

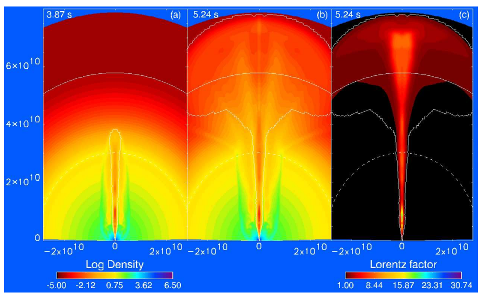


Fig. 1.— Contour maps of the logarithm of the rest–mass density after 3.87 s and 5.24 s (left two panels), and of the Lorentz factor (right panel) after 5.24 s. X and Y axis measure distance in centimeters. Dashed and solid arcs mark the stellar surface and the outer edge of the exponential atmosphere, respectively. The other solid line encloses matter whose radial velocity $> 0.3c$, and whose specific internal energy density $> 5\times10^{19}\,\mathrm{erg\,g^{-1}}$.

Collapsar Model for GRBs

Notes:

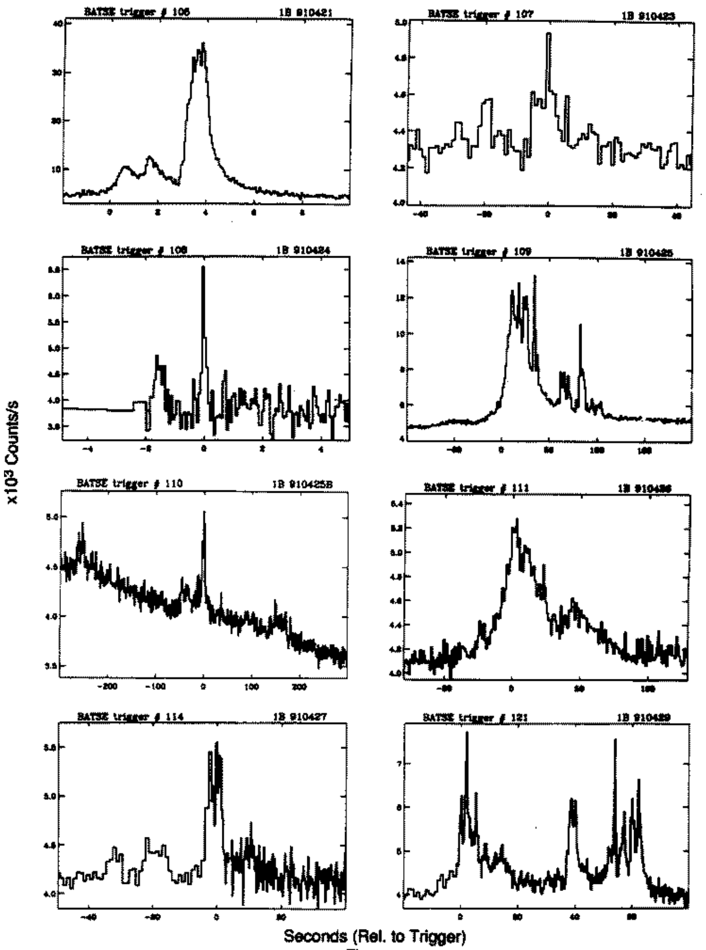
E. GAMMA-RAY BURSTS (ZG: 16-6; CO: 25.4)

- *discovered* by U.S. *spy satellites* (1967; secret till 1973)
- have remained one of the biggest mysteries in astronomy until 1998 (*isotropic* sky distribution; location: solar system, Galactic halo, distant Universe?)
- *discovery of afterglows* in 1998 (X-ray, optical, etc.) with *redshifted absorption lines* has resolved the puzzle of the location of GRBs $\rightarrow$ *GRBs are the some of the most energetic events in the Universe*
- *duration:* 10^{-3} to $10^{3}\,$s (large variety of burst shapes)
- *bimodal* distribution of *durations:* 0.3 s (short-hard), 20 s (long-soft) (different classes/viewing angles?)
- *highly relativistic outflows (fireballs):* ($\gamma \gtrsim 100$,) possibly highly *collimated/beamed*
- GRBs are produced far from the source ($10^{11} - 10^{12}\,$m): interaction of outflow with surrounding medium (external or internal shocks) $\rightarrow$ *fireball model*
- *relativistic energy* $\sim 10^{46} - 10^{47}\,\mathrm{J}\,\epsilon^{-1}\,f_\Omega$ (ϵ: efficiency, f_Ω: beaming factor; typical energy $10^{45}\,$J?)
- *event rate/Galaxy:* $\sim 10^{-7}\,\mathrm{yr}^{-1}\,(3\times10^{45}\,\mathrm{J}/\epsilon\,\mathrm{E})$

Popular Models

- *merging compact objects* (two NS's, BH+NS) $\rightarrow$ can explain short-duration bursts
- *hypernova* (very energetic supernova associated with formation of a rapidly rotating *black hole*) $\rightarrow$ *jet penetrates stellar envelope* $\rightarrow$ GRB along jet axis (large beaming)

Notes:



Notes:

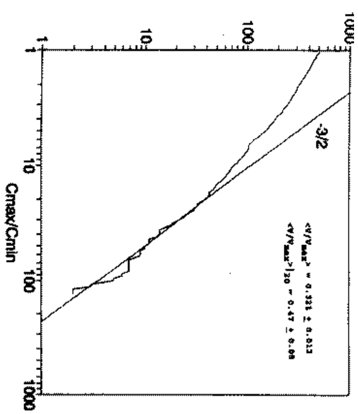


Figure 4. Intensity distribution for BATSE bursts. The measure of intensity is the maximum count rate divided by the threshold count rate.

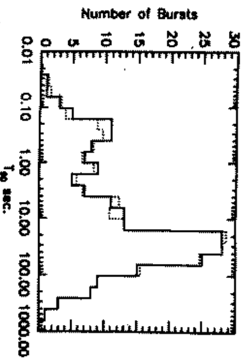


Figure 7. The duration distribution for 222 BATSE bursts, as measured by T_{90}. The solid histogram represents the raw data; the dashed histogram represents the data convolved with measurement errors.

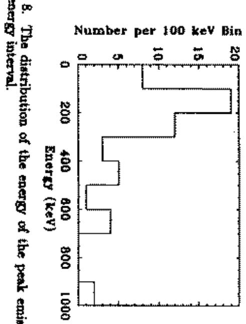


Figure 8. The distribution of the energy of the peak emission per logarithmic energy interval.

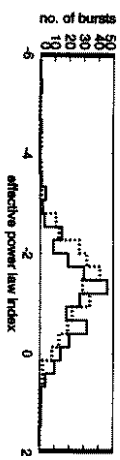


Figure 9. The distribution of burst spectral indices in the 50 to 300 keV energy band. The solid line represents the distribution for the peak rate spectr and the dotted line represents the distribution for the total fluence spectrum

Meegan et al. (1994)

Notes: