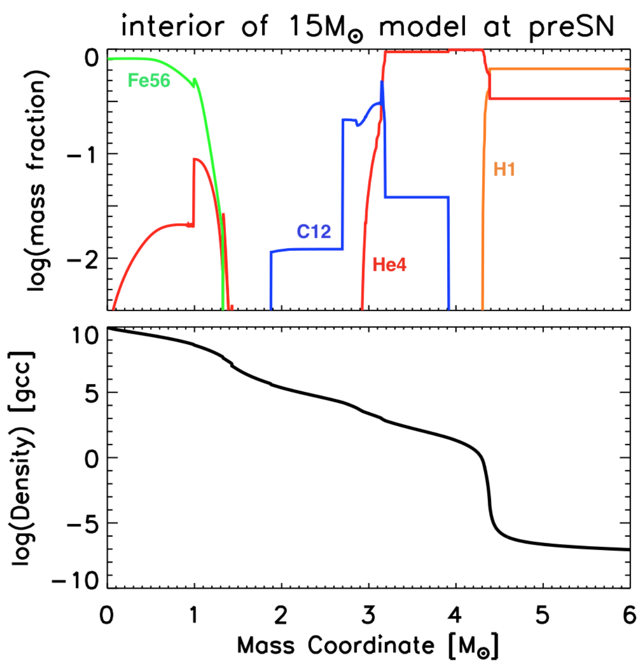
ASTRONOMY 220C

Problem Set 1
Due January 22, 2019

Short answers

1) Four characteristic time scales were given in class; the hydrodynamic time, τ_{HD}, thermal time, τ_T, Kelvin-Helmholtz time, τ_{KH}, and nuclear time, $\tau_{\rm nuc}$. Which time scale or equality of time scales sets the characteristic condition for the following? a) lifetime on the main sequence; b) duration of the intense neutrino emitting phase of neutron star formation; c) peak of a light curve powered by trapped radiation; d) explosive burning of a nuclear fuel in a shock wave; e) interval between one burning phase and the next in a massive star (neglect residual burning in a shell); f) explosive disruption of a star by a nuclear runaway.

2) Attached is a figure showing the composition and density structure of the inner 6 $M_\odot$ of a presupernova 15 $M_\odot$ star. Visually estimate a) the mass of the helium core; b) the mass of the carbon-oxygen core; c) the mass of the iron core. d) From the density structure - which is mimics in the pressure - which cores will evolve more like individual stars?



3) a) What is the hydrodynamic time scale for a white dwarf star with an average density of 10^7 g cm^{-3}? b) The helium core of a 20 $M_\odot$ star has a radius of 2.5×10^{10} cm and a mass of 6.0 $M_\odot$. What is the hydrodynamical time scale for this core? c) About how long would

it take a sound wave (or a weak shock) to cross that core?

4) If a polytrope of constant composition and constant $\beta = P_{ideal}/P_{total}$ contracts and increases its central density by a factor of 64, by what factor will its central temperature increase? Does your answer depend on the polytropic index if β at the center of the star does not change?

5) What are the critical *main sequence* masses of stars that will ignite a) helium burning, b) carbon burning, c) oxygen burning, d) silicon burning? Above what mass are supernovae possible (neglect rotation and assume Pop I abundances)?

6)a) What type of meteorite is used as a reference point for the composition of the primitive solar nebula (three words)? b) From what source would the following abundances or ratios must accurately be extracted (solar photosphere and corona or meteorite) *and why*? i) helium, ii) lithium, iii) oxygen, iv) iron, v) uranium, and vi) ^{16}O/^{17}O?

7) On the class website under "handouts" - Lodders(2009), table - you will find the table of standard solar abundances by mass fractions. At a mass density of 100 g cm^{-3}, what would be the *number density* (per cm^{-3}) of ^{56}Fe?

8) What is the most tightly bound isotope of all (nb., isotope, not element)? What is the most tightly bound isotope that has equal numbers of neutrons and protons? (hint: it may not be stable).

9) Use the "reaction rate threshold" calculator given under "links" on the class website (or enter http://t2.lanl.gov/nis/data/qtool.html) to calculate the Q-value for ^{12}C(p,γ)^{13}N and for ^{56}Fe(n,γ)^{57}Fe and write down your answers. [the EMAX does not matter since these are both exoergic reactions, enter 1.0 or any number greater than zero. But the bigger the number, the more reaction channels will open up, try 100. for fun.]

10) From the "Nuclear Wallet Cards", http://www.nndc.bnl.gov/wallet/wccurrent.html, the atomic mass excesses of ^{56}Ni, ^{56}Co, ^{56}Fe, and ^{56}Mn are -53.906, -56.039, -60.606, and -56.910 MeV respectively. Which nucleus is stable? For the nuclei that could decay by electron capture, what would be the energy release. For the nucleus that could beta decay, what would be the energy release? For electron capture, where would the energy go, i.e., would any be deposited locally?

11) For neutrons, what are the magic numbers corresponding to shell closures for N up to 126 and in which shell does the last neutron added reside? What is a stable nucleus that contains each of these magic numbers? See http://www.nndc.bnl.gov/chart/.

12) Why are the most abundant elements below calcium characterized by equal numbers of

neutrons and protons (Z= N) while the abundant isotopes of heavier nuclei have N ¿ Z?

(Somewhat) Longer Questions

1) What is the ideal gas pressure in dyne cm^{-2} for a fully ionized gas of 20% by mass carbon (^{12}C) and 80% by mass oxygen (^{16}O) at a temperature of 7×10^8 K and density 10^5 g cm^{-3}?

2) The density of nuclear matter is 2.9×10^{14} g cm^{-3}. Consider this matter to consist of 50% each, neutrons and protons so that the density of each is one-half this value. Assume the nucleon gas is non-relativistic and completely degenerate. What is the peak Fermi energy of the nucleons [see notes]? Assume that the average Fermi energy of a nucleon is 3/5 of this [see notes]. Multiply this latter number by 12 to get the approximate total Fermi energy in a ^{12}C nucleus. Express your answers in MeV (1 MeV = 1.602 $\times 10^{-6}$ erg). Compare your result to the total Coulomb energy, for a sphere of constant charge density $\sim \frac{3}{5}\frac{Z^2e^2}{R}$ $R = 1.12A^{1/3}$ fm and e^2 = 1.44 MeV fm.

3) Write down the shell model energy states ($1s^2_{1/2}$ ...) for ^{17}O. What would be the spin and parity of the ground state of this nucleus? To what resonances in ^{18}F could the reaction $^{17}O(p,\gamma)^{18}F$ couple if the proton had angular momentum l = 1?

4) Recent observations of supernova progenitors for about a dozen events (Smartt ARAA, 2009) suggest an upper bound for the stars that become Type II supernovae of about 20 solar masses. What happens above 20 $M_\odot$ is uncertain, but black hole formation with no bright accompanying supernova is possible. On the other hand, recent theoretical modeling suggests that the minimum mass above which supernova occur may be as high as 9 $M_\odot$. Using these limits and assuming a Salpeter IMF (Γ = -1.35), estimate the mass of the numerical average supernova.