ASTRONOMY 220C

Problem Set 2
Due February 7, 2019

Short answers

1) What is the spin and parity of the ground state of deuterium (^{2}H)? Why is it not 0^+?

2) For which cases will the S-factor be approximately a constant in the “Gamow energy window” (i.e., $E_o \pm \Delta/2$ for incident charged particles or kT for neutrons): a) direct capture, b) tail of a broad resonance, c) two narrow resonances, d) hundreds of broad resonances within the Gamow window?

3) The reaction ^{16}O + ^{16}O is very important in oxygen burning to be studied later. Given that oxygen burning occurs at 1.8×10^9 K, calculate the power of temperature to which the energy generation will be sensitive, i.e., $\epsilon_{\rm nuc} \propto T^n$, what is n? ^{16}O has 8 protons.

4) As hydrogen burns in a massive star does the convective core shrink with time or grow with time. Similarly for helium burning, does it grow or shrink with time [for single stars]?

5) What is the Eddington limit in erg s^{-1} for a star of mass M solar masses whose atmosphere is pure helium? Evaluate for a mass of 4 solar masses and assume electron scattering opacity.

6) What is the shortest lifetime for any star to burn hydrogen (once the hydrogen has ignited)?

7) For solar metallicity stars, assuming current estimates of mass loss (inaccurate as they certainly are), what it the greatest mass any single star is likely to have at the time it becomes a supernova? An estimate good to a few solar masses will suffice. What does that mass correspond to on the main sequence?

8) Sketch the flows of matter for Eddington Sweet circulation in a rigidly rotating star. How might they be different in a star that is highly differentially rotating?

9) What is semiconvection? If the efficiency of semiconvection in a star is increased does one obtain more red supergiants (vs. blue) or fewer? That is, will a star spend more of its helium burning lifetime or less as a red supergiant if one has a lot of semiconvection?

10) What are the chief effects of rotation on the evolution of a massive star (M greater than 10 $M_\odot$ say)? [I am interested in qualitative effects like the helium core mass and surface abundances. A short description will suffice]. Why is Eddington-Sweet circulation more important in massive stars than in the sun (short answer, no numbers needed)?

Longer Problems

1) Calculate, or look up on the web the Q-value for the weak interaction ^{56}Fe(e$^-$,ν)^{56}Mn. It will be a negative number signaling energy must be supplied to make the reaction occur. For a fully ionized plasma in which the electrons are totally degenerate and relativistic at what density will this reaction begin to occur appreciably? (hint: the Fermi energy of a relativistic degenerate electron gas is 0.515 MeV $(\rho_6 Y_e)^{1/3}$ where ρ_6 is the density in units of 10^6 g cm^{-3}, Clayton 2-39) For purposes of this calculation, assume that $Y_e = 0.50$, that the temperature is zero and iron nucleus is in its ground state only. Speculate briefly (no calculation) upon the effect of a large finite temperature on your answer.

2) What is the Gamow energy in MeV for the reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ at a temperature $T_9 =$ 0.2 GK? If the S-factor for that reaction at that energy is 170 keV, what would the reaction rate $\lambda = N_A < \sigma v >$ be at that temperature? If the density is 10^3 g cm^{-3}, and the helium mass fraction, $X_\alpha = 0.5$, what is the lifetime of carbon against this reaction? (Hint: Lifetime $= [\rho Y_\alpha \lambda_{\alpha,\gamma}(^{12}C)]^{-1}$).

3) The binding energies of ^{4}He, ^{12}C, and ^{16}O are 28.296, 92.163, and 127.621 MeV respectively. Calculate how much energy would be released, in erg/gm, if pure helium burned to a final mixture of 1/2 C and 1/2 O by mass.

4) **Optional:** This problem requires compiling and running a code from the class website.
a) Download and compile the fortran program cno.f from the class website under programs. Even if you don't know fortran, you should be able to do this if you have a working knowledge of any computer language and access to a fortran compiler, e.g. g77 or gfortran. If you have trouble with this question talk with me.
Also download the file fort.4 which is the input data for the program. It is set up to run with an initial composition of 70% H, 28% He and 1% C and O, all by mass fraction, at a constant temperature of 3×10^7 K and density 100 g cm^{-3}. Look at the data.
b) Run the program. The output will be in the files fort.7 and fort.9. The first file contains the time evolution of the abundances (the elapsed time is in column 1). The second file gives the nuclear energy generation in the last column. What is the difference (in physics included) in enuc and snuc, the last two columns in fort.9? Examine the program to see.
c) Plot as a function of log time from 1 y to 1 million years the log of the mass fractions of H, ^{12}C, ^{13}C, ^{14}N, and ^{16}O. Use IDL or any plot routine you may be familiar with to make the plot. Make a paper copy and include with this assignment.
d) Throughout the latter half of hydrogen burning, the abundance of ^{12}C is nearly a constant. Why?
This program shows an example of a "nuclear reaction network". You may notice that it inverts a 14x14 matrix every time step. The matrix ("a") is built in subroutine burn and inverted in subroutine leqs. There is a matrix because the differential equations describing the abundances have been forward differenced in time. The solution is therefore "implicit" not "explicit". We will talk about this in class. Such an approach provides an important degree of stability when taking large time steps in situations where complex flows are tightly coupled.