

## ASTRONOMY 220C

### *Problem Set 3*

*Due February 21, 2019*

#### Short answers

- 1) What is the condition on reaction rates that leads to the “ $\beta$ -limited CNO cycle? Which reaction rates have to be faster than which other rates? At about what temperature does this happen (there is of course a weak density dependence)?
- 2) What reactions are chiefly responsible for creating a neutron excess,  $\eta = 1 - 2Y_e$ , during helium burning? Why is this important? What value of  $\eta$  do you expect to characterize the composition of a Pop I star just prior to carbon ignition? How might this neutron excess vary with the initial metallicity of the star?
- 3) For what sorts of Wolf-Rayet stars are lines of hydrogen seen in the spectrum?
- 4) What is a typical equatorial rotation speed for a massive O or B star? Is the Eddington Sweet time scale for such a star less than or longer than hydrogen burning time scale?
- 5) White dwarfs of Ne and O are thought to be quite rare in nature compared with white dwarfs composed of C and O, yet many classical nova outbursts show compositions consistent with having come from such Ne-rich objects. Why is this possible?
- 5) What is the heaviest element that can be made in great abundance by the  $rp$ -process operating in a Type I x-ray burst on a neutron star?
- 6) What is the balanced power approximation and why does it give a good estimate for the burning temperature for nuclear fuels heavier than helium in a massive star?
- 7) a) How many independent parameters does it take to specify the abundances of all isotopes heavier than silicon in silicon quasiequilibrium and what are they? b) How many does it take to specify the abundances in nuclear statistical equilibrium and what are they? c) What is the most abundant isotope in nuclear statistical equilibrium when  $Y_e = 0.45$  (don’t calculate, just look it up).
- 8) List and identify with a phrase or two, 5 effects that modify the so called “Chandrasekhar Mass” from the oft quoted value of  $1.44 M_\odot$ . Which effects cause increases and which cause decreases and which two are most important within the context of iron cores in massive stars? What is the (cold) Chandrasekhar mass for a white dwarf composed of  $^{56}\text{Fe}$ .
- 9) Where in nature, in what kind of star or explosion, during what burning phase, and by what reactions are each of the following made: a)  $^{12}\text{C}$ , b)  $^{13}\text{C}$ , c)  $^{14}\text{N}$ , d)  $^{15}\text{N}$ , e)  $^{22}\text{Ne}$ , and f) the s-process?

## Longer Problems

1) A neutron star of mass  $1.4 M_{\odot}$  and radius 10 km accretes a mixture of 75% H, 25% He by mass at a rate of  $10^{-9} M_{\odot} \text{ yr}^{-1}$  (nb., year, not second). It is a source of repeated Type I x-ray bursts. A burst happens every 12 hours. Assuming hydrostatic equilibrium and pressure due to non-relativistic electrons ( $P = 1.004 \times 10^{13} (\rho Y_e)^{5/3}$ ; only approximately true), calculate the density at the base of the accreted layer at the time of a flash.

2) a) What reaction is chiefly responsible for producing neutrons for the *s*-process during helium burning in a massive star? Where do most of the neutrons go? b) If the temperature and density histories of helium burning do not change, i.e., at each helium mass fraction the temperature and density are the same, but the metallicity of the star varies, do you expect the steady state neutron flux to vary? Would the number of neutrons captured by each iron nucleus vary greatly? c) Can the complete solar abundances set of *s*-process nuclei be produced in massive stars? If not, why not, and where might the necessary circumstances actually be achieved?

3) You will need to use the website, <https://www.nndc.bnl.gov/nudat2/>, listed under “periodic chart” at the class website. Zoom in on the isotope Te128 ( $Z=52, N=76$ ). Turn “Tooltips” off and try starting with zoom level 2. Click on “half life” on the toolbar. You will find that Tellurium has 8 stable isotopes (black squares). The abundance and 30 KeV cross section of each are:  $^{120}\text{Te}$ , (0.0058, 480 mb),  $^{122}\text{Te}$ , (0.159, 290 mb),  $^{123}\text{Te}$ , (0.058, 820 mb),  $^{124}\text{Te}$ , (0.299, 160 mb)  $^{125}\text{Te}$ , (0.454, 440 mb)  $^{126}\text{Te}$ , (1.22, 80 mb)  $^{128}\text{Te}$ , (2.07, 40 mb), and  $^{130}\text{Te}$ , (2.24, 16 mb). a) From inspection of the periodic chart classify each of these isotopes as “p”, “s-only”, “rs”, or “r-only”. To trace the path of the *s*-process assume that all nuclei with beta-halfives less than 1 year decay before capturing a neutron. b) Calculate the products of  $n$  and  $\sigma$  for each isotope. What is noticeable about these numbers for the isotopes you labeled as “s-only”? c) Can you estimate the the contributions to  $^{126}\text{Te}$  from the *s*-process and the *r*-process separately? Please do so.

4) The nuclear physics of neon burning is clearly very different from either carbon or oxygen burning. Explain why neon burns by a photodisintegration rearrangement reaction rather than by simple fusion (i.e.,  $^{20}\text{Ne} + ^{20}\text{Ne} \rightarrow ^{40}\text{Ca}$ ). Why does neon, with its larger Coulomb barrier ( $Z = 10$ ), burn before oxygen ( $Z = 8$ )?

5) This question involves the use of the 19 isotope approximation network, approx1, that is available on the class website under “programs”. It requires no additional files to run and prints output on the screen though you could change that.

a) Download the Fortran code and compile it. Look at least at the main driver program. The rest of the program sets up a matrix of rate equations and solves it by inversion. Rates are calculated each time step and the composition is updated one time step by subroutine burn which also provides the energy generation and neutrino losses. The program is currently set up to start at a temperature of  $6 \times 10^8$  K and a density of  $2 \times 10^5 \text{ g cm}^{-3}$ , precarbon burning

conditions, with a composition of 20%  $^{12}\text{C}$  and 80%  $^{16}\text{O}$  by mass fraction. This roughly represents the conditions in a  $15 M_{\odot}$  star. The code strives to approximately maintain the condition nuclear energy generation equals three times the neutrino losses (see lecture 11). When the generation and loss rates are unbalanced, like when a given fuel is exhausted, the temperature rises. The burning conditions for C, Ne, O, and Si burning are approximately recreated.

b) Run the code. The columns give the time until the end (s), the time step (s), the current temperature (GK) and density ( $\text{g cm}^{-3}$ ), the nuclear energy generation rate and neutrino loss rate ( $\text{erg g}^{-1} \text{s}^{-1}$ ), and some of the (19) abundances. The last column is  $^{54}\text{Fe}$ . The rest should be obvious, e.g., 12 =  $^{12}\text{C}$ , 16 =  $^{16}\text{O}$ , ... 36 =  $^{36}\text{Ar}$ , and 40 =  $^{40}\text{Ca}$ . The code also assumes  $\rho \propto T^3$  as we discussed in class. Plot the temperature and density vs remaining time on a log time scale with small values on the right of the plot. Also, probably on a separate page but with the same time range, plot the abundances of C (A=12), oxygen (A=16), neon (A=20), and silicon (A=28).

c) At about what temperatures and densities do carbon, oxygen, neon and silicon burn? (Find where each has its maximum and look at the point where half has burned)? About how much time does each burning phase take?

d) What is the major composition in mass fractions at the end of carbon, neon, and oxygen burning. The composition at the end of silicon burning is iron and  $^{56}\text{Ni}$  but the code does not presently print those abundances. You could change that if you want.

e) In two or three words each, what appear to be the major functions of subroutines i) burn, ii) sleqs, iii) screen, iv) rate and v) sneutrX?