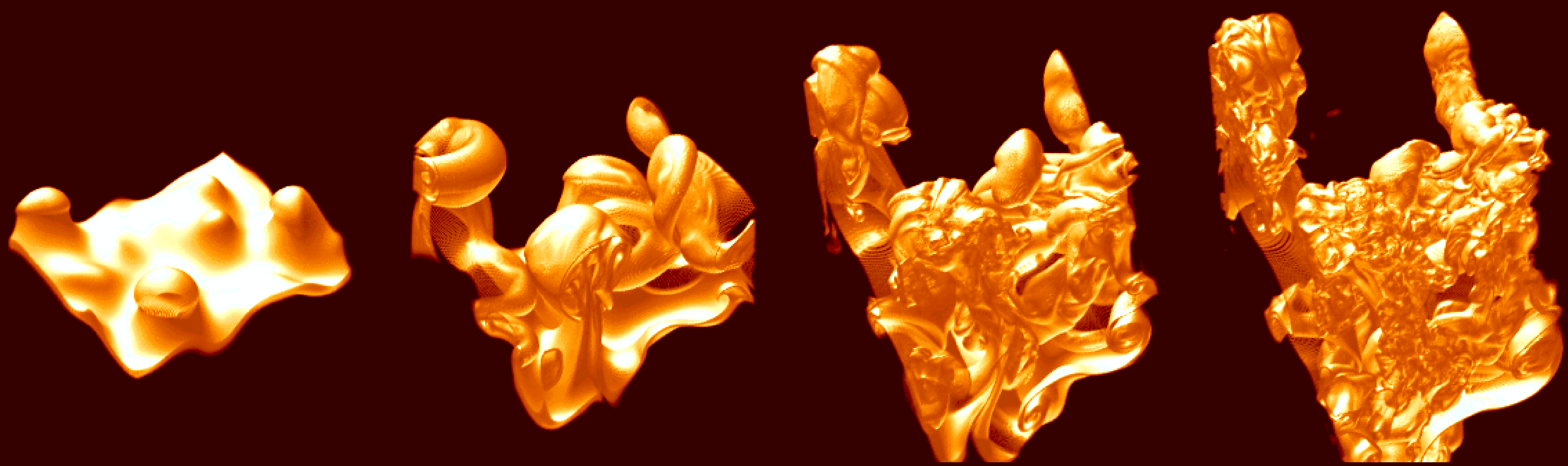
Simulations of Thermonuclear Flames in Type Ia Supernovae



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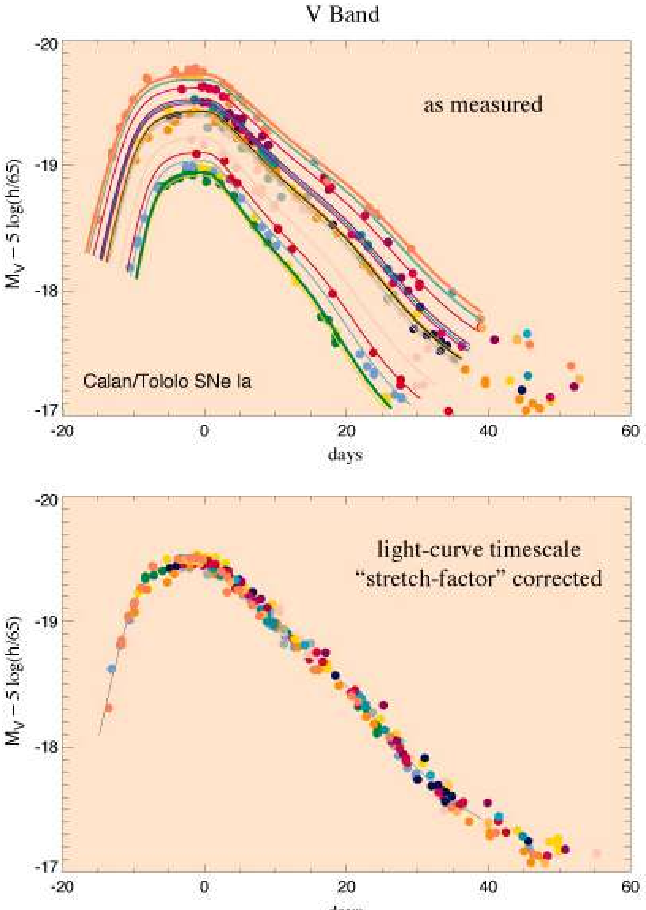
Type Ia Supernovae

- Bright as host galaxy, L ~10^{43} erg s^{-1}
- Large amounts of ^{56}Ni produced
  - Radioactivity powers the lightcurve

SN 1994D (High-Z SN Search team)



Phillips (1993), Perlmutter et al. (1997)

- Lightcurve is robust
  - Variations can be corrected for via a single parameter function.
- Thermonuclear explosion of C/O white dwarf.
  - Must begin as a deflagration
  - Considerable acceleration required

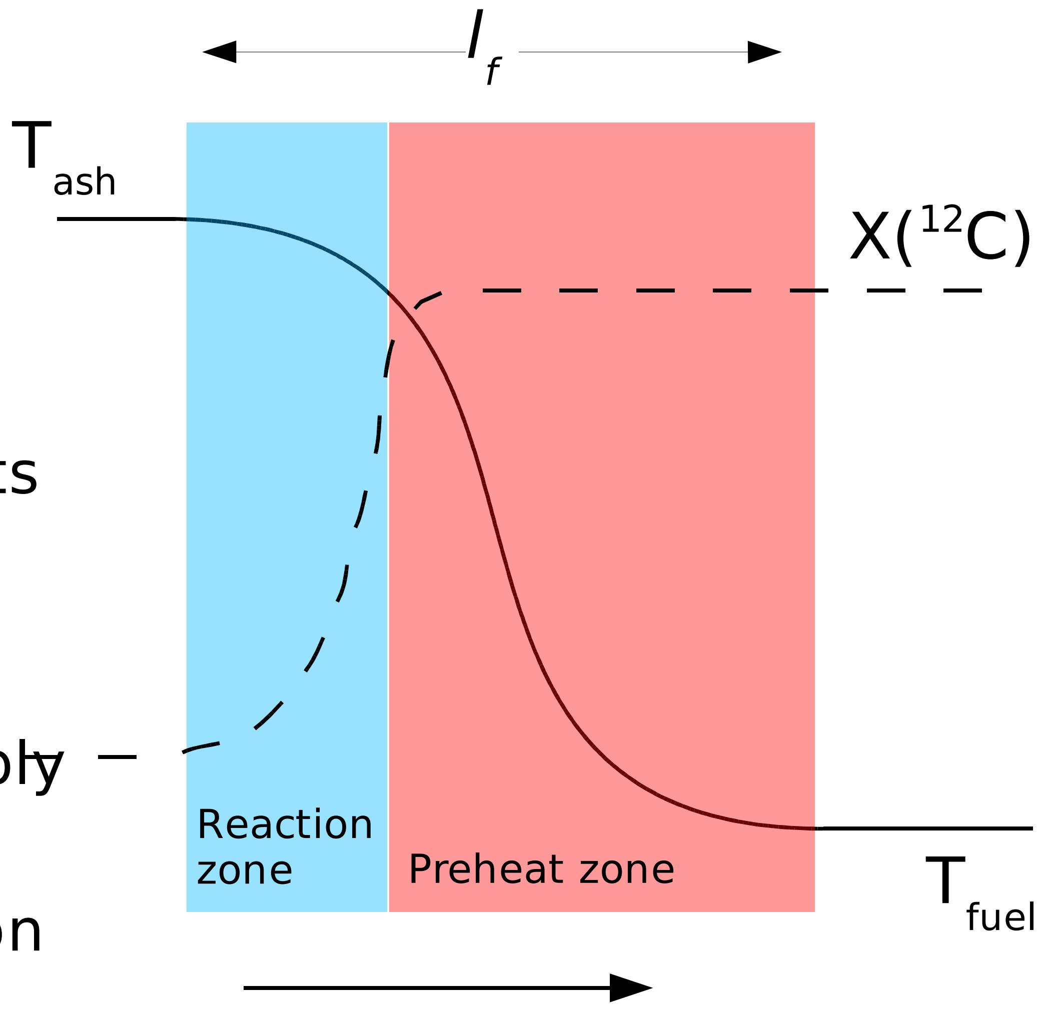
Explosion Requirements

- Flame must accelerate to ~ 1/3 c_s.
- Must produce intermediate mass elements (Si, S, Ar, Ca).
- Produces ~ 0.6 $M_{\odot}$ ^{56}Ni.
- How does the flame accelerate?
  - Flame instabilities (Landau-Darrieus, Rayleigh-Taylor)
  - Interaction with turbulence.

Increase surface area ⇒ increase flame speed.

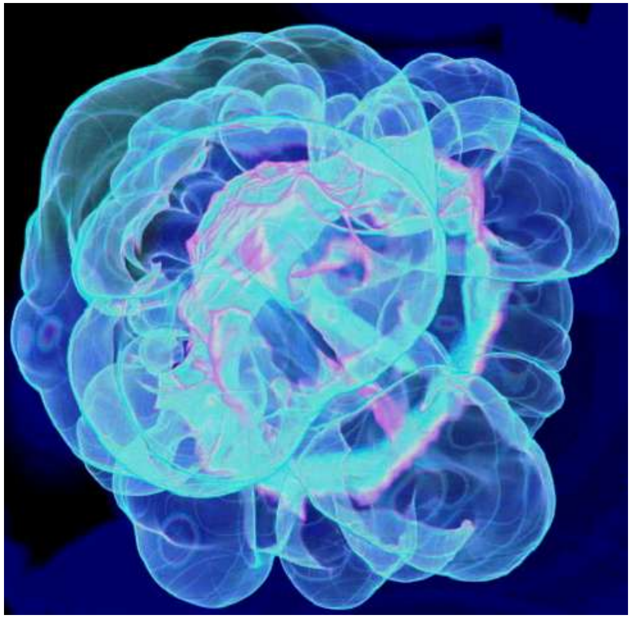
Flames

- Begins as a deflagration
  - Subsonic burning front
    - Pressure is constant
    - Density drops in the ash region.
  - Thermal diffusion transports the heat
- Laminar speed too slow
  - Must accelerate considerably at low densities.
  - May transition to detonation

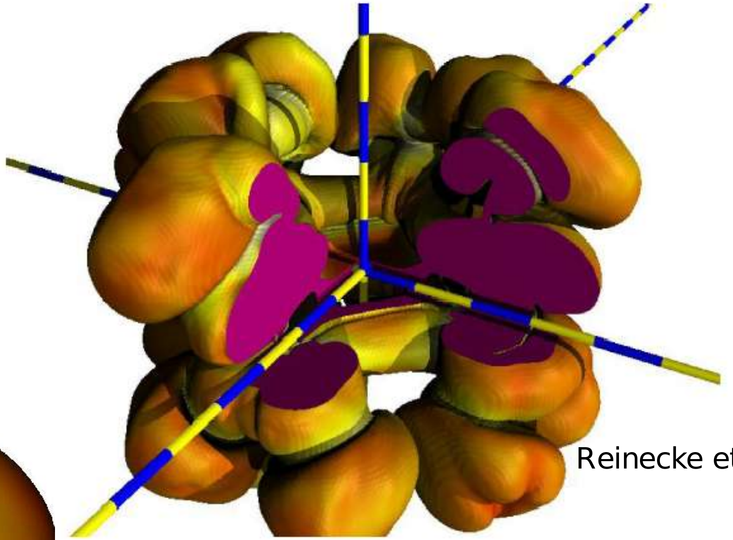


Large Scale Simulations

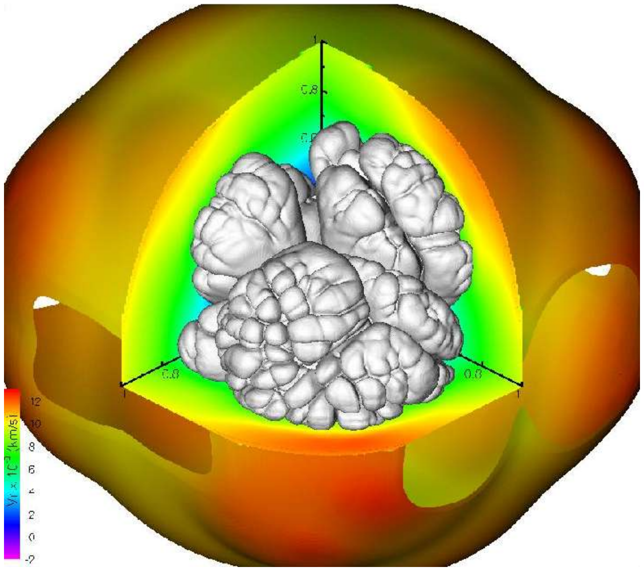
- Instabilities are the dominant acceleration mechanism.
- Pure deflagrations can unbind the star.



Calder et al. (2004)



Reinecke et al. (2003)

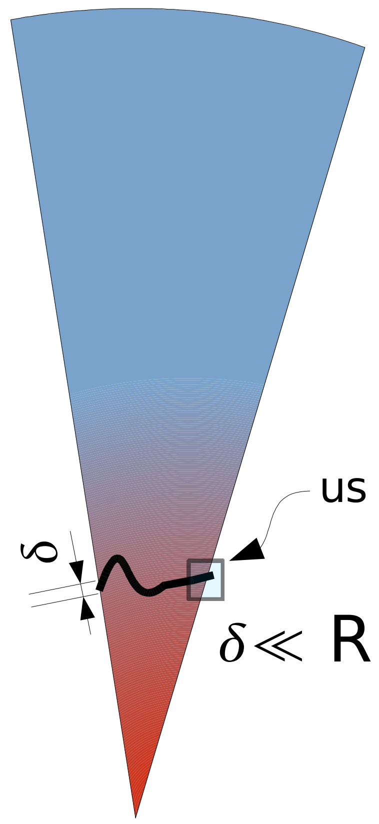


Gamezo et al. (2003)

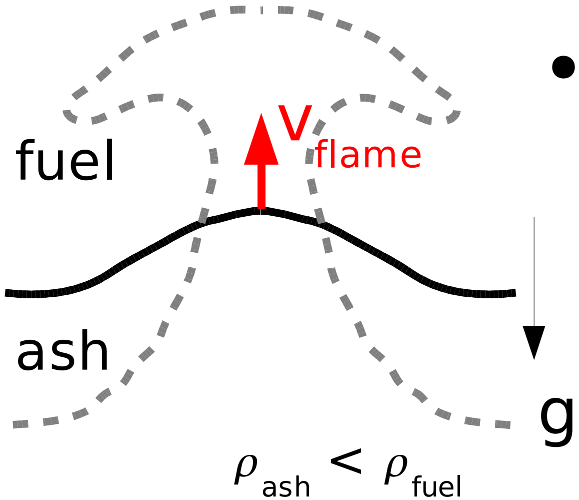
- Some flame model is required.
  - Stellar scale ~ 10^8 cm
  - Flame width ~ 10^{-5} – 10 cm

Bottom-Up Approach

- Simulations cannot resolve the star and the flame.
- We resolve the thermal structure of the flame and work up to large scales
  - Parameter free.
  - Resolved calculations can be used to validate flame models.
- Look for scaling relations that will act as subgrid models.



Reactive Rayleigh-Taylor Instability



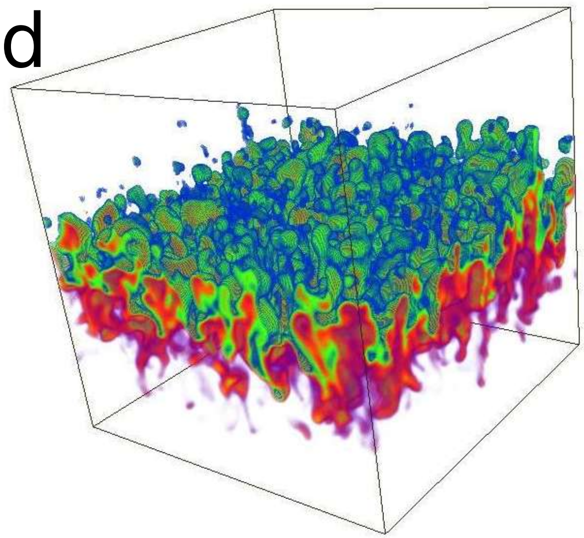
- Rayleigh-Taylor
  - Buoyancy driven instability.
  - Large amounts of surface area generated.

- Sharp-Wheeler model predicts mixed region growth:

$$h = \alpha A g t^2$$

- Reactions set a small scale cutoff to the growth of the instability:

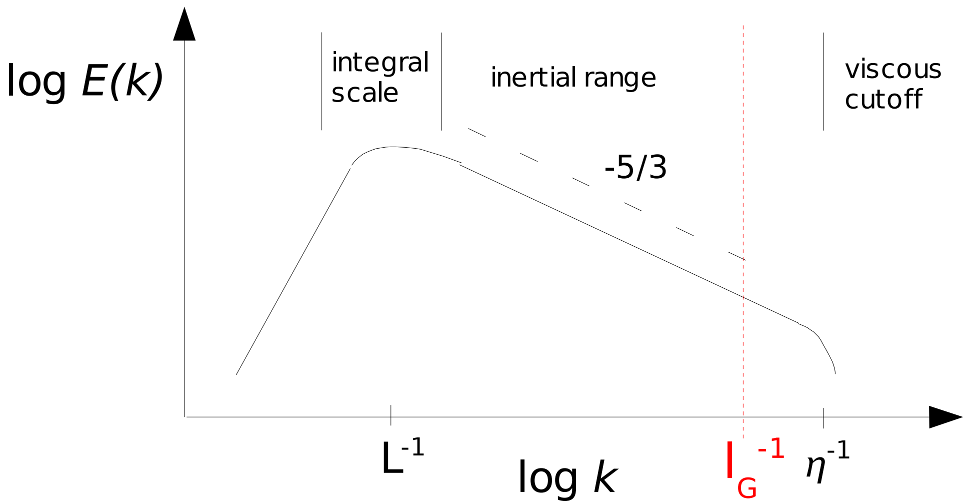
$$\lambda_{\mathrm{fp}} = 4\pi \frac{v_{\mathrm{laminar}}^2}{g_{\mathrm{eff}}}$$



Calder et al. (2002)

Turbulence

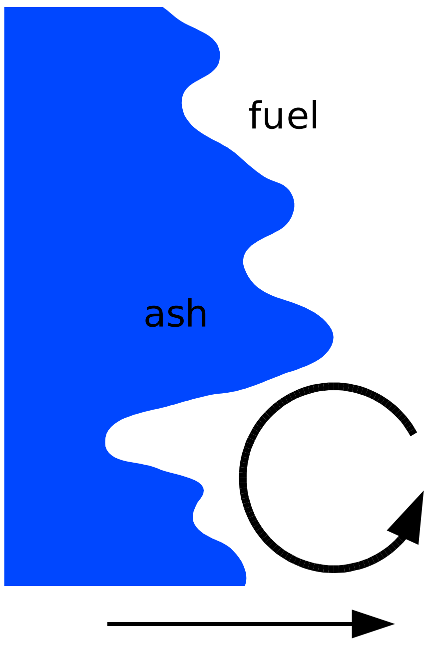
- Kinetic energy cascade over a range of length scales
  - Integral scale, L: bulk of kinetic energy exists
  - Kolmogorov scale, η: inertial and viscous effects balance
  - Gibson scale, l_G: eddy turns over before burning away.



adapted from Peters (2000)

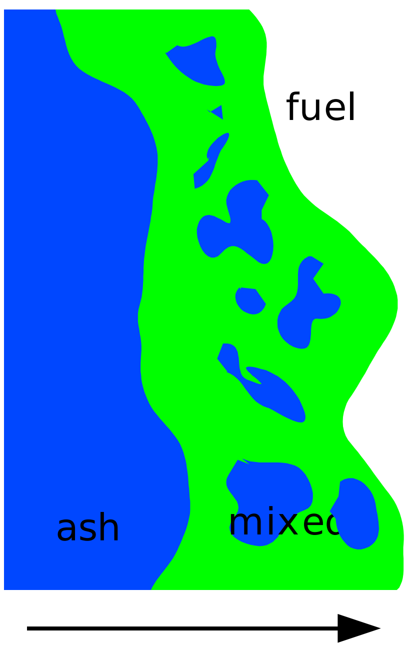
- Size of l_G in comparison to flame width determines the flame regime.

Transition to Distributed Burning



- Flame begins as flamelet
  - Flame is a continuous surface
  - Turbulence serves solely to wrinkle the flame, increasing the area

- Transition to distributed burning regime is proposed at 10^7 g cm^{-3}
  - Mixed region of fuel + ash develops
  - May be possible to quench the flame
  - Possible transition to detonation



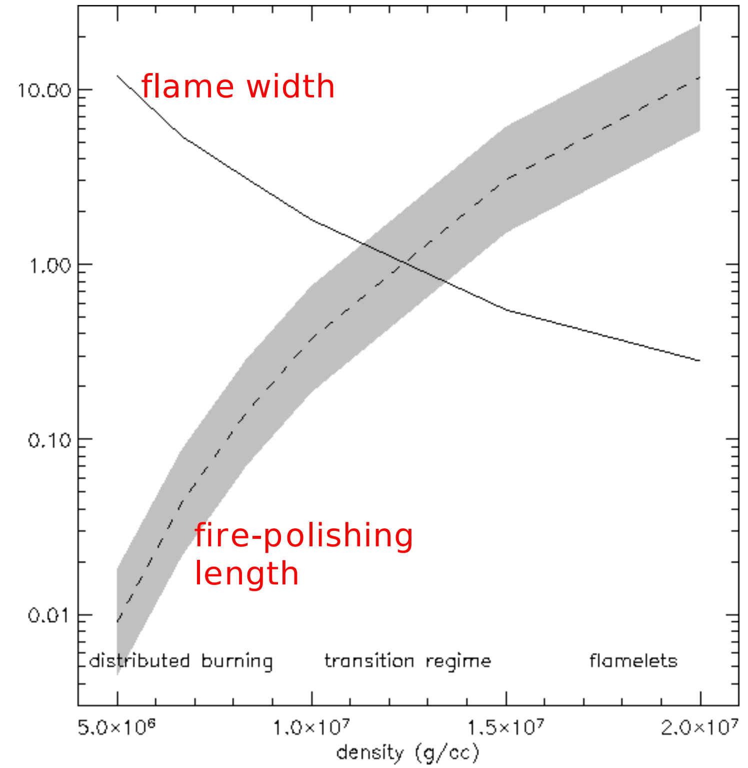
Low Density Flame Properties

ρ ($\mathrm{g\ cm^{-3}}$)	$\Delta\rho/\rho$	v_{laminar} ($\mathrm{cm\ s^{-1}}$)	l_f [a] (cm)	λ_{fp} [b] (cm)	M
6.67×10^6	0.529	1.04×10^3	5.6	0.026	3.25×10^{-6}
10^7	0.482	2.97×10^3	1.9	0.23	8.49×10^{-6}
1.5×10^7	0.436	7.84×10^3	0.54	1.8	2.06×10^{-5}

- Laminar flames are M $\ll$ 1
- Around 10^7 g cm^{-3} pass through the region where

$$\lambda_{\mathrm{fp}} = l_f$$

  - Transition to distributed regime expected here (Niemeyer and Woosley 1997)
  - We need to resolve both scales



Low Mach Number Hydrodynamics

(Bell et al. 2004 JCP 195, 677)

- Low Mach number formulation projects out the compressible components.
  - Pressure decomposed into thermodynamic and dynamic components.

$$p(x,t) = p_0(t) + Mp_1(t) + M^2\pi(x,t)$$

  - Elliptic constraint provided by thermodynamics.

$$0 \equiv \frac{Dp}{Dt} = \frac{\partial p}{\partial \rho}\frac{D\rho}{Dt} + \frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt}$$

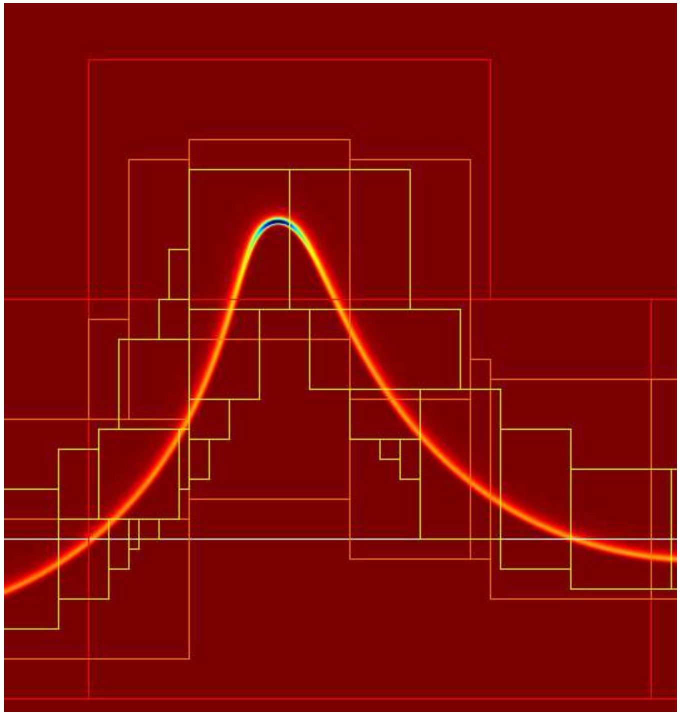
$$\nabla \cdot U = \frac{1}{\rho \frac{\partial p}{\partial \rho}} \left(\frac{\partial p}{\partial T}\frac{DT}{Dt} + \sum_k \frac{\partial p}{\partial X_k}\frac{DX_k}{Dt} \right)$$

  - Advection/Projection/Reaction formulation solves system.
  - Timestep limited by |v| and not |v| + c.

Simulation Method

(Bell et al. 2004 JCP 195, 677)

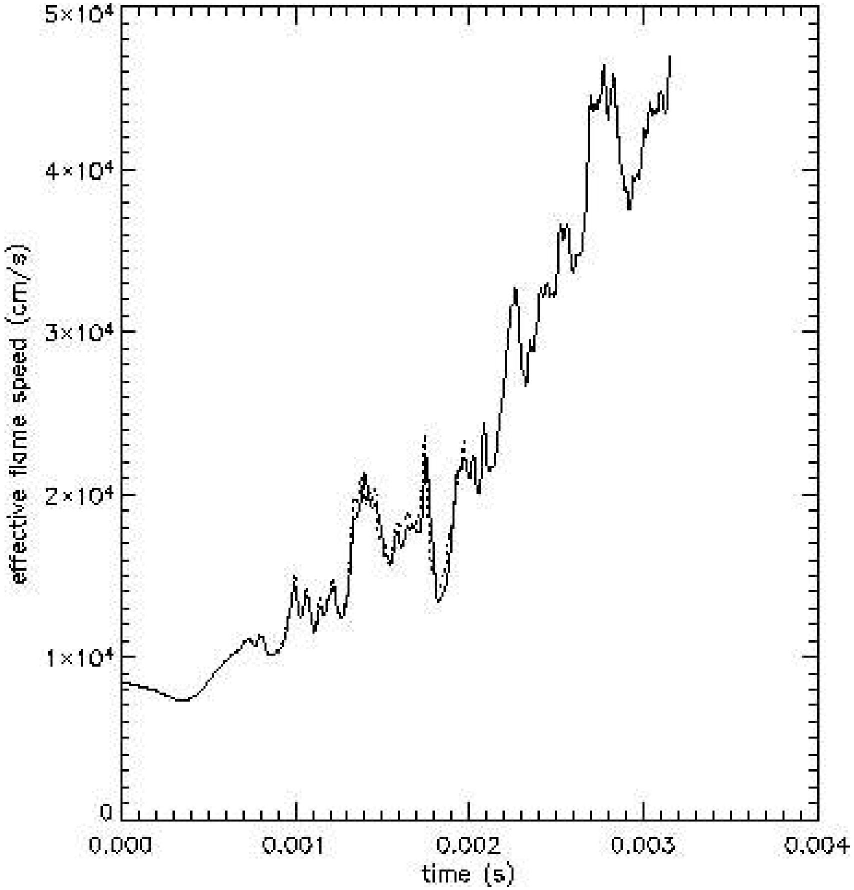
- Low Mach number hydrodynamics.
  - Advection/projection/reaction
  - Block structured adaptive mesh
  - Timestep restricted by |v| not |v| + c
  - Degenerate/Relativistic EOS used.
  - Single step $^{12}C+^{12}C$ rate

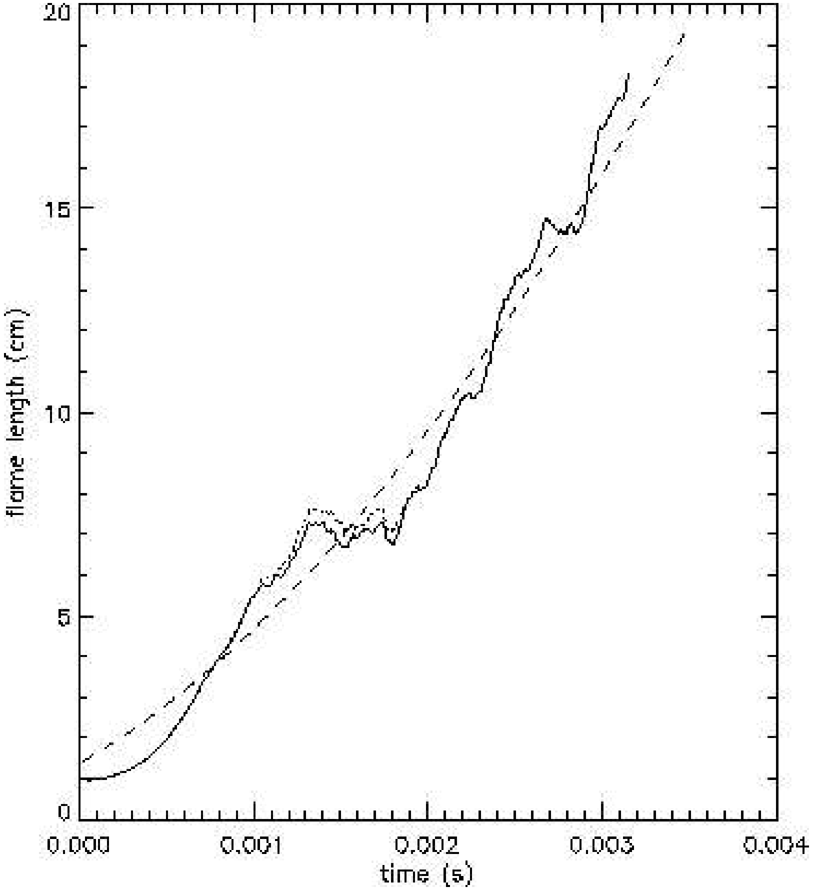


- Initialized by mapping 1-d steady-state laminar flame onto grid.
  - 5-10 zones inside thermal width.

Convergence Study

- 5 points in the thermal width yields converged integral quantities (speed, length, ...)

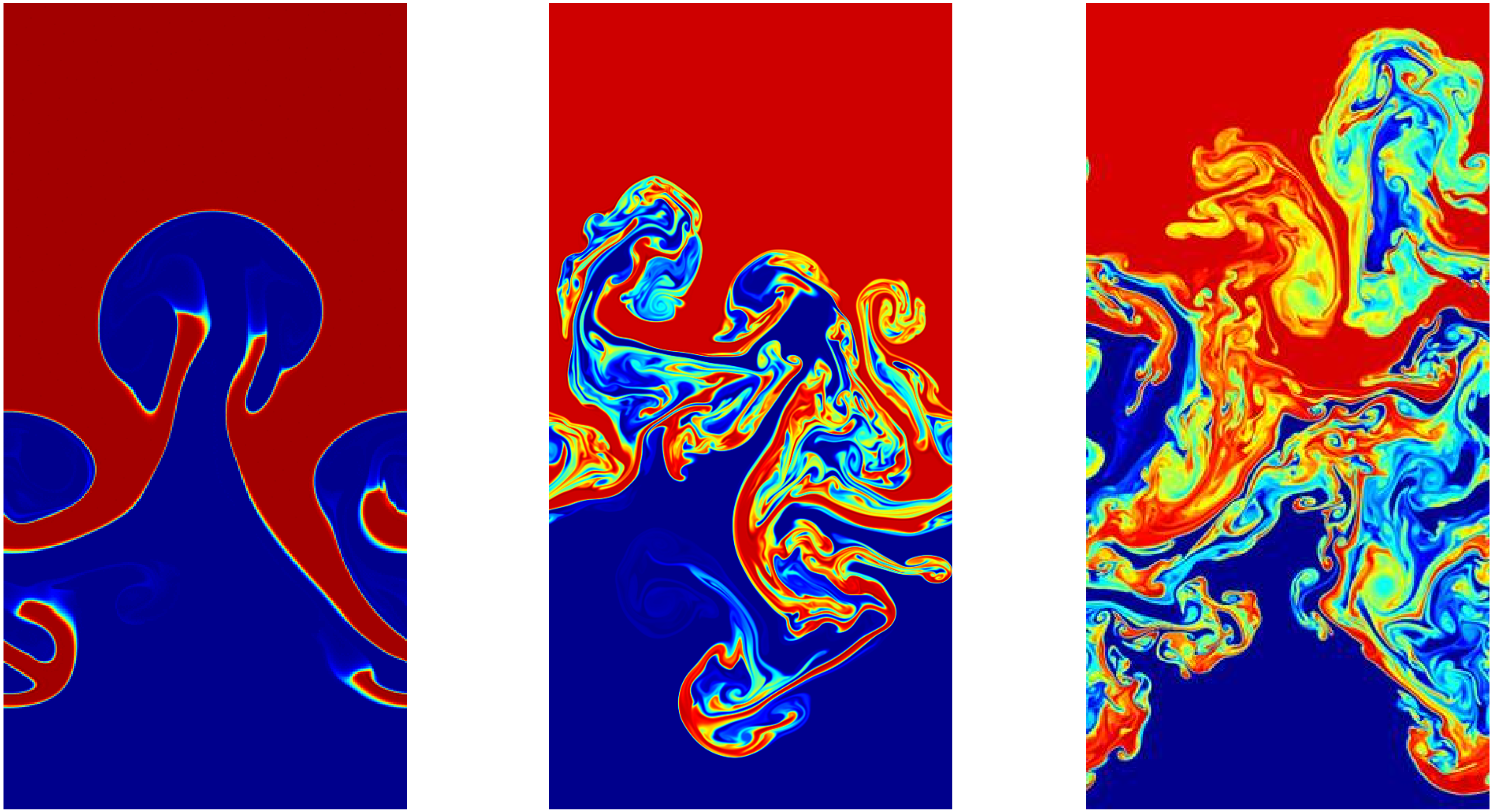




- Burning sets the small scale cutoff.

Transition to Distributed Burning

(Bell et al. 2004, ApJ, 608, 883)



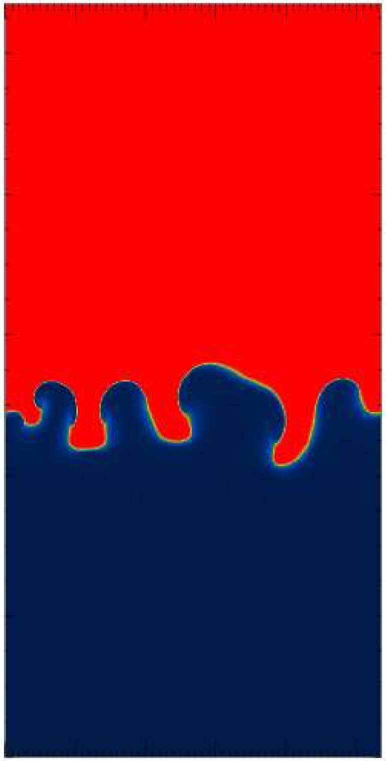
- As ρ decreases, RT dominates over burning.
- At low ρ, flame width is set by mixing scale.

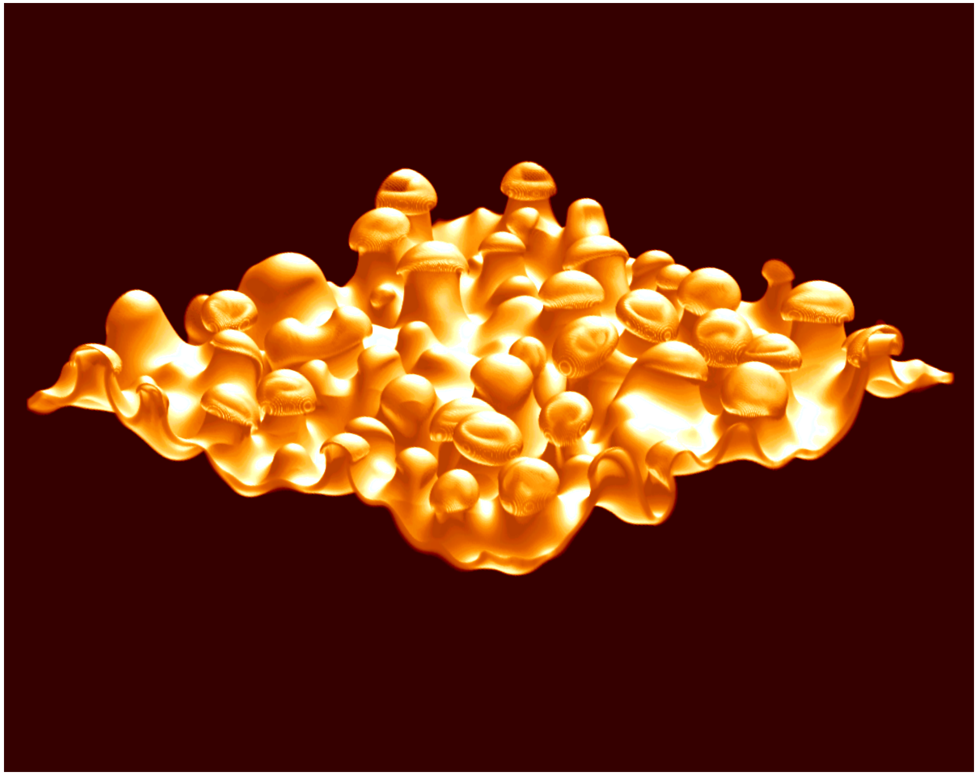
2-D Reactive RT: Transition to Distributed Burning Summary

- Accelerations to several times the laminar speed
  - Limited only by the size of the domain.
- Transition to distributed burning occurs at density of 10^7 g cm^{-3}
- Growth of reactive region scales with mixed region
  - There does not appear to be enough time for a localized transition to detonation.
- Curvature/strain effects become quite important near the transition.

3-D Reactive RT

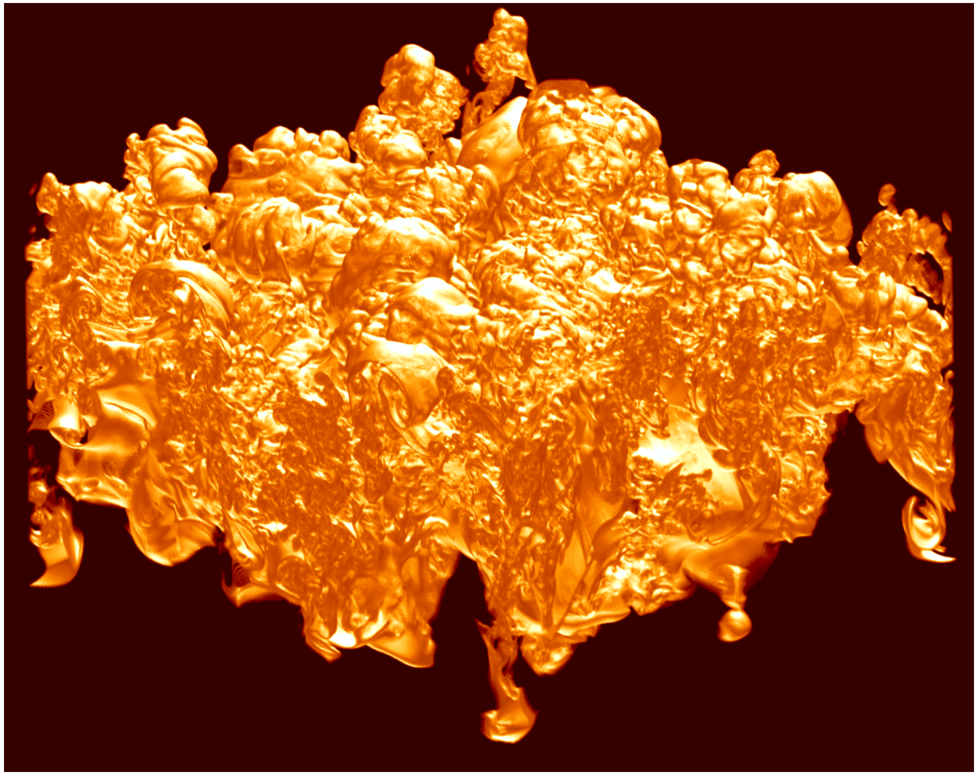
- 3-D analogue of 2-D runs previously studied
  - 512 x 512 x 1024 effective zones
  - Surface to volume is greater
  - Fire-polished RT dominates the early evolution.





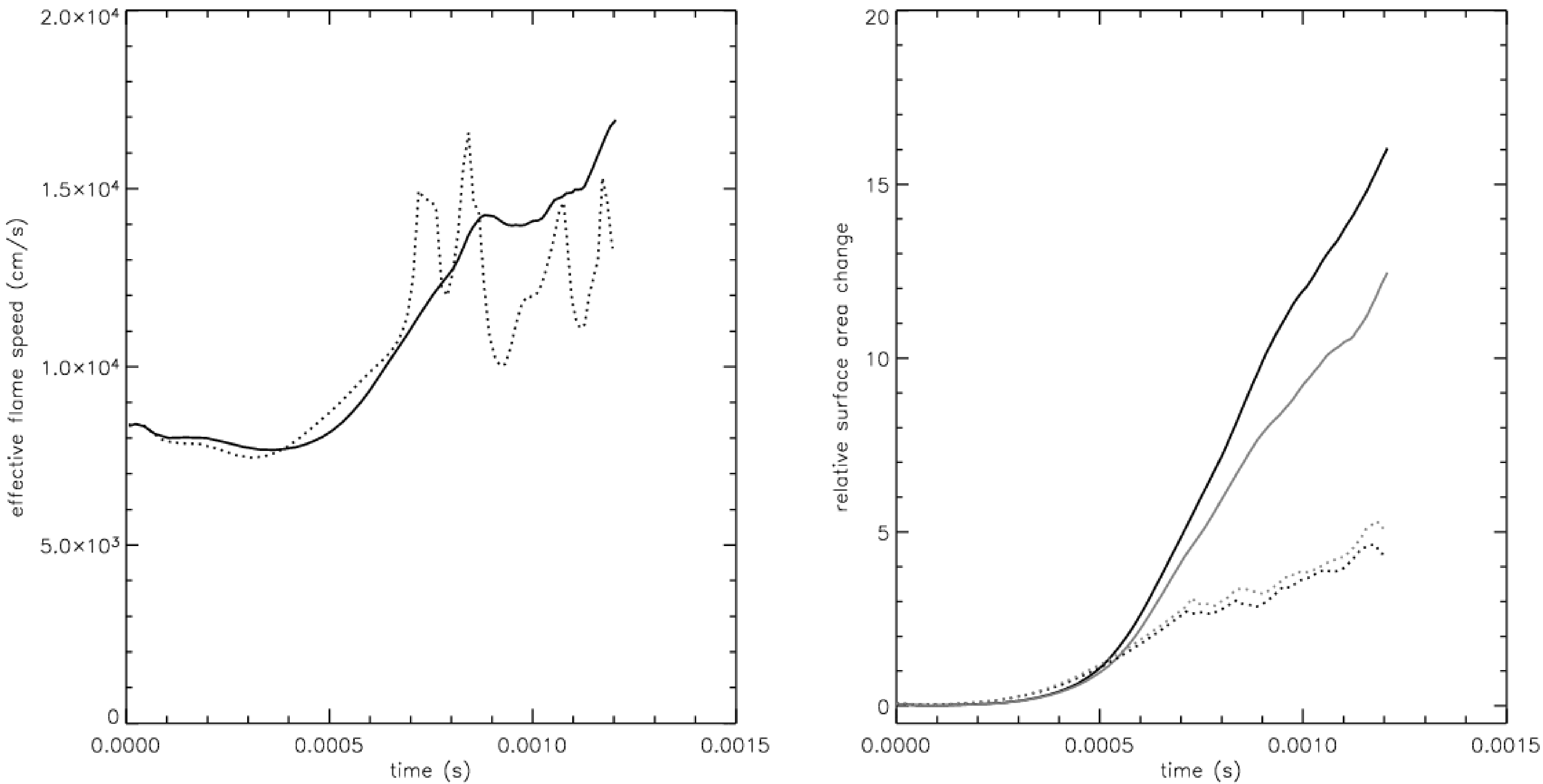
3-D Reactive RT

- At late times, a fully turbulent flame propagates
  - No analogy to the 2-D case.
  - Evolution now dominated by turbulence, not Rayleigh-Taylor.



3-D Reactive RT

- Late time acceleration in 3-d due to interaction with flame generated turbulence



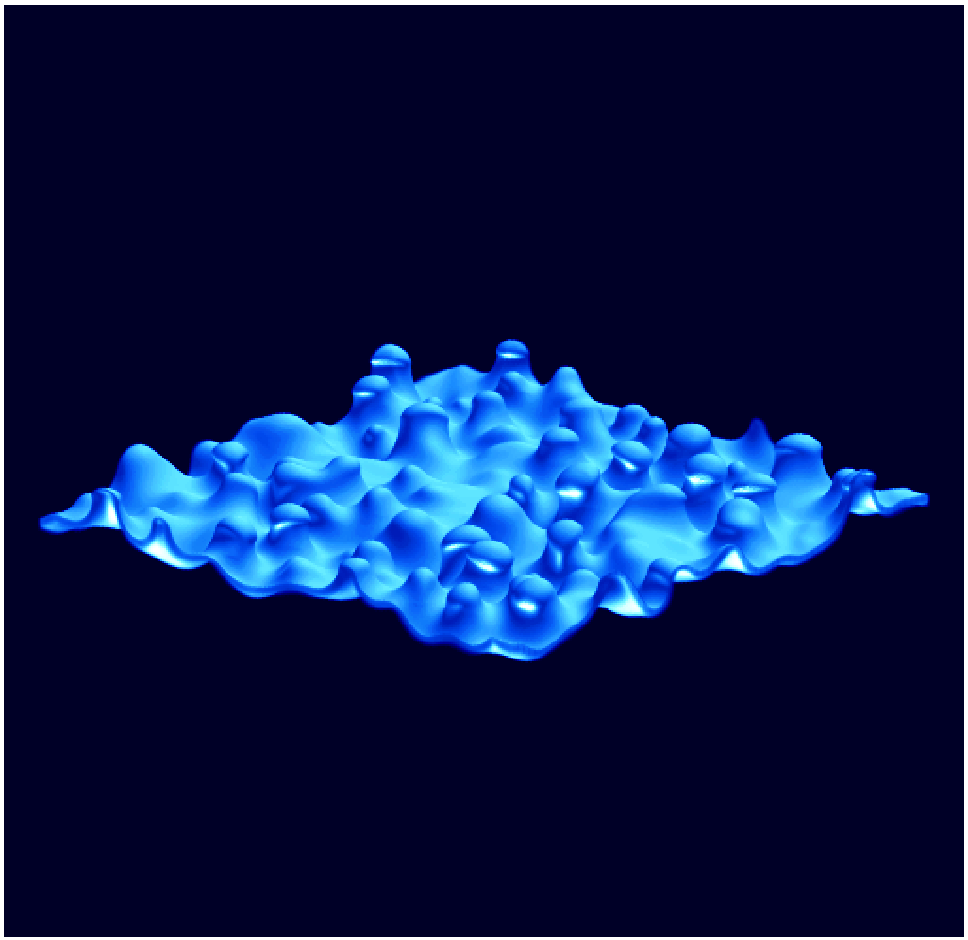
Power Spectrum

- Power spectrum can be used to determine the nature of the turbulence
  - Our domain is not periodic in all directions (inflow and outflow boundaries)
  - Velocity field is decomposed into divergence free part + effects of boundaries and compression

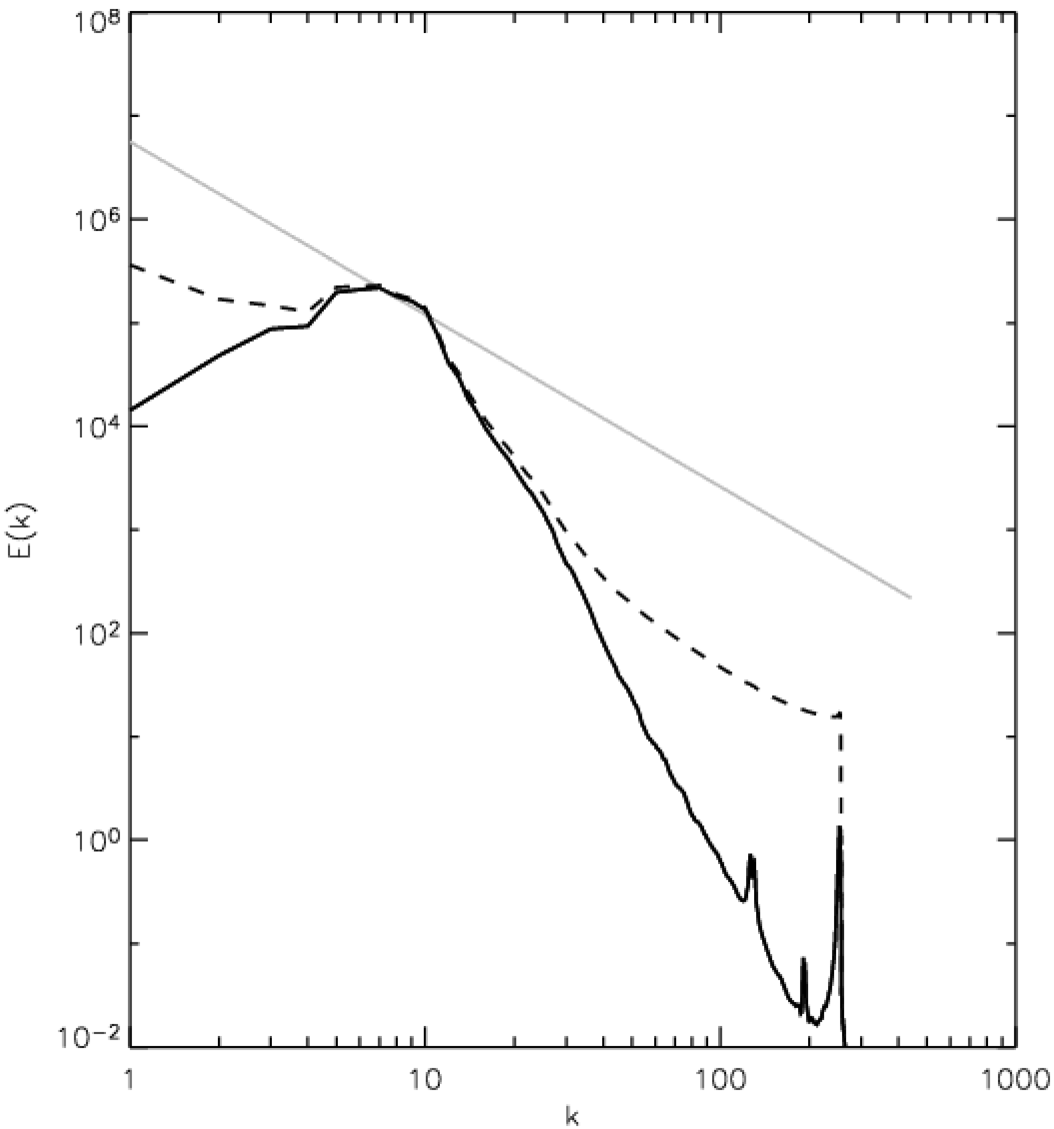
$$\mathbf{u} = \mathbf{u}_d + \nabla\phi + \nabla\psi$$

  - Divergence free part is projected out.
  - FFT is performed on divergence free field

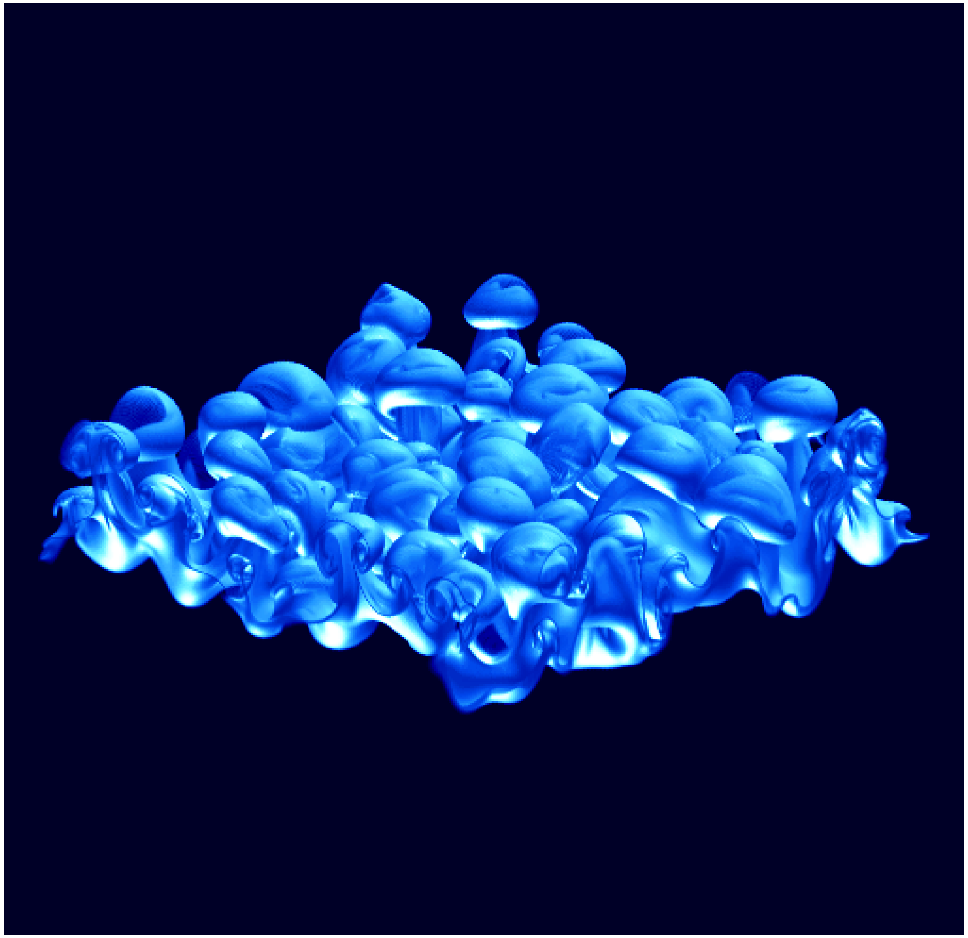
Transition to Turbulence



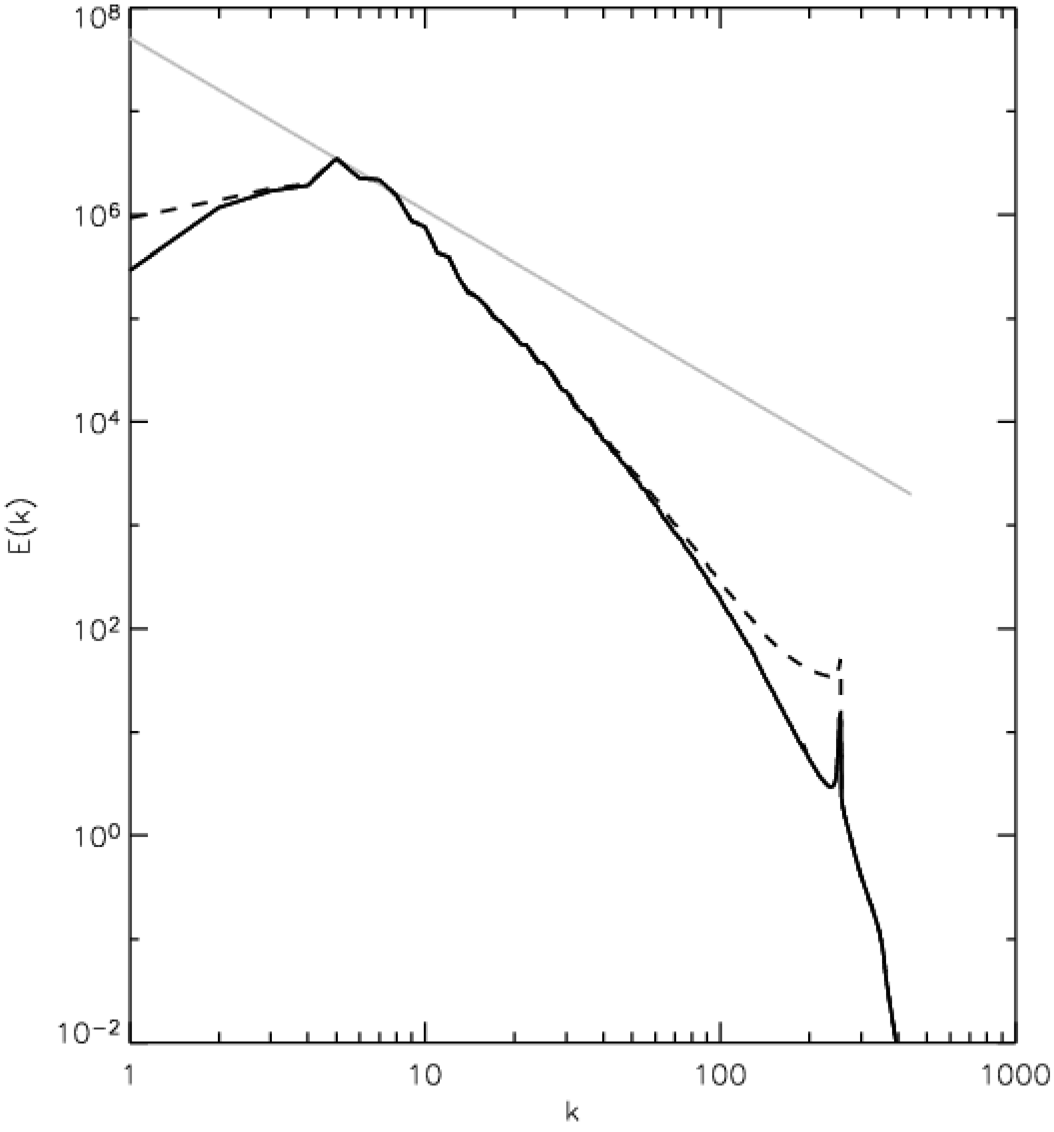
t = 4.04 x 10^{-4} s



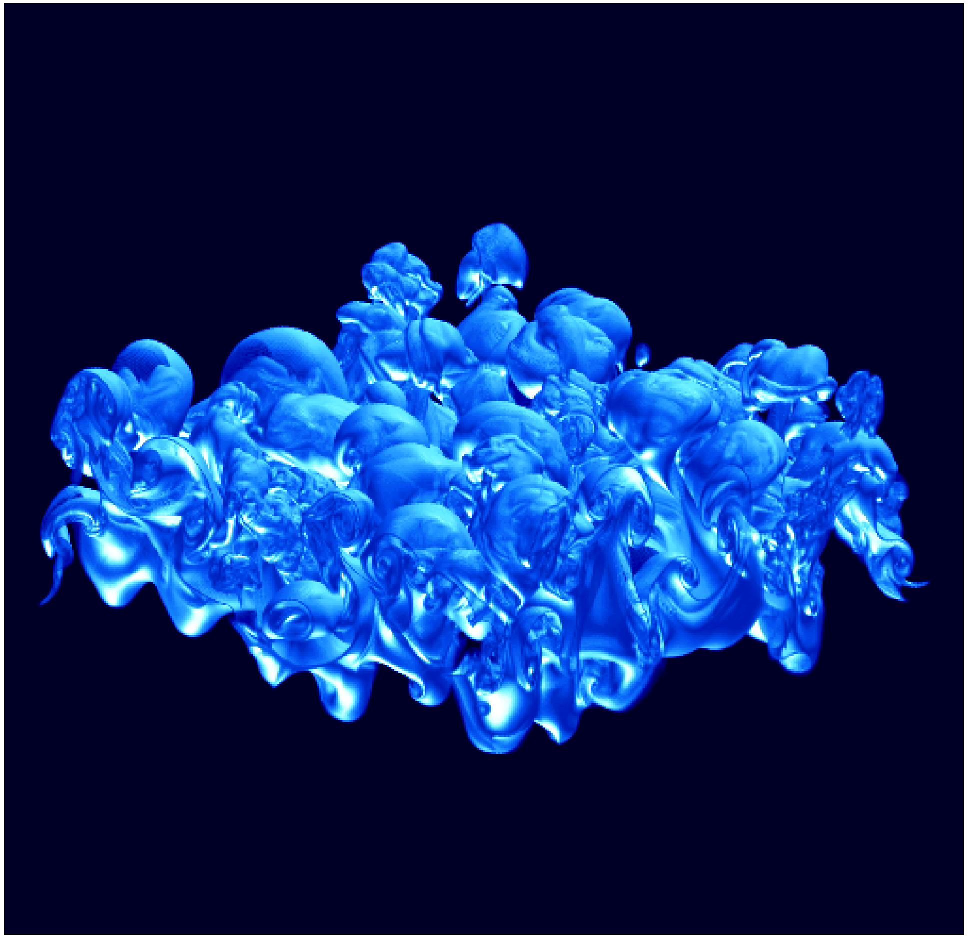
Transition to Turbulence



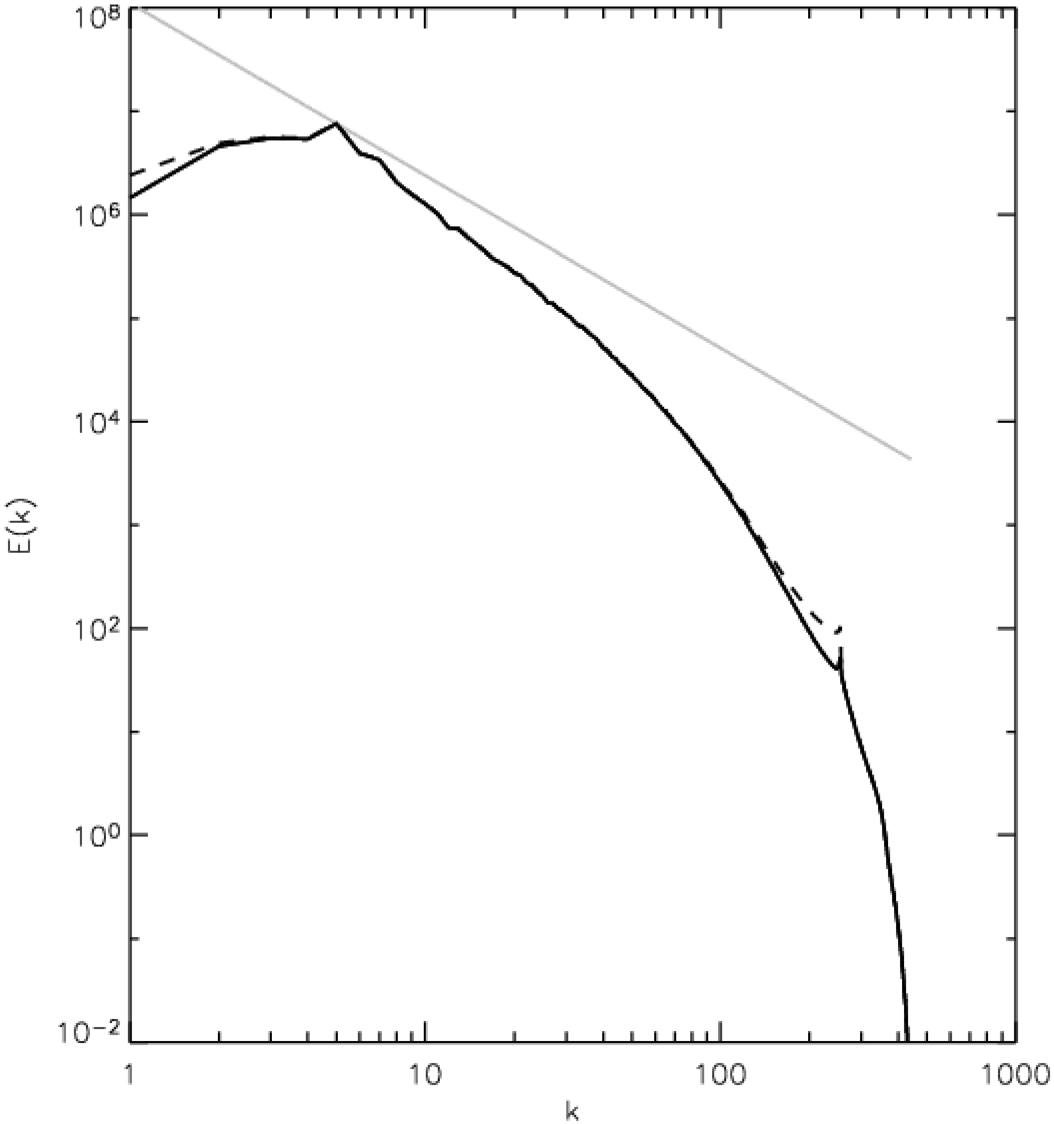
t = 6.62 x 10^{-4} s



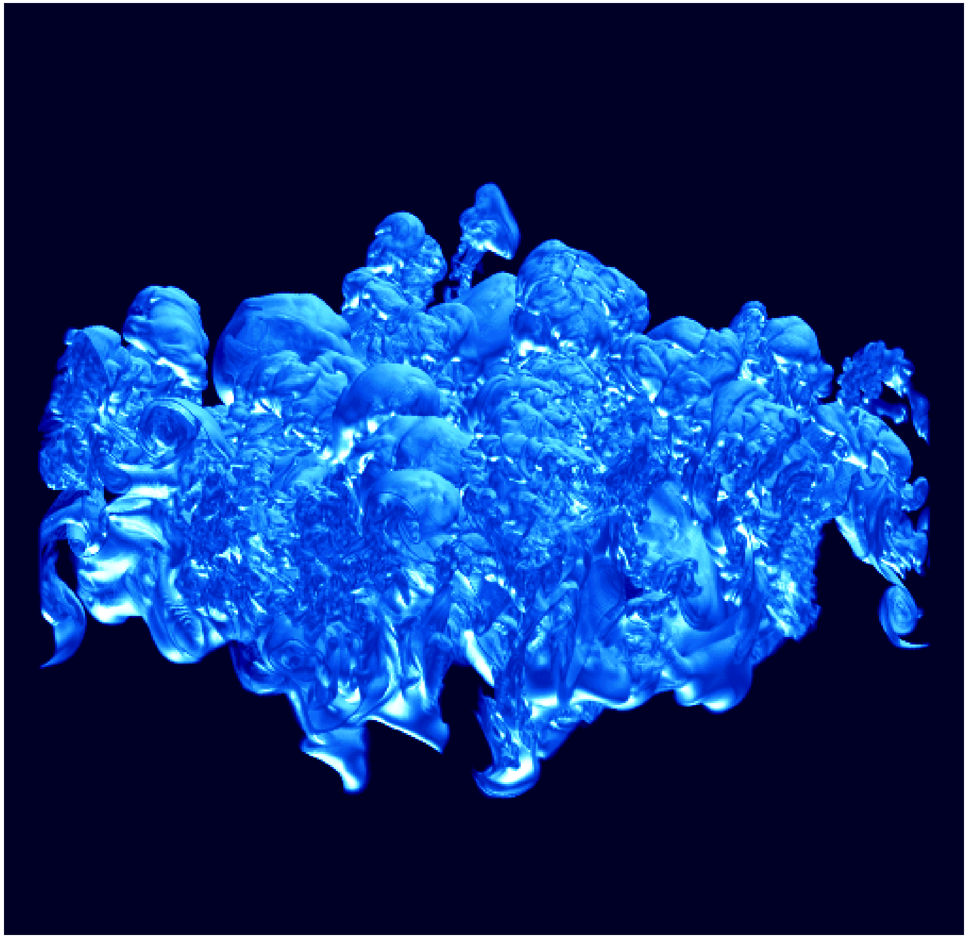
Transition to Turbulence



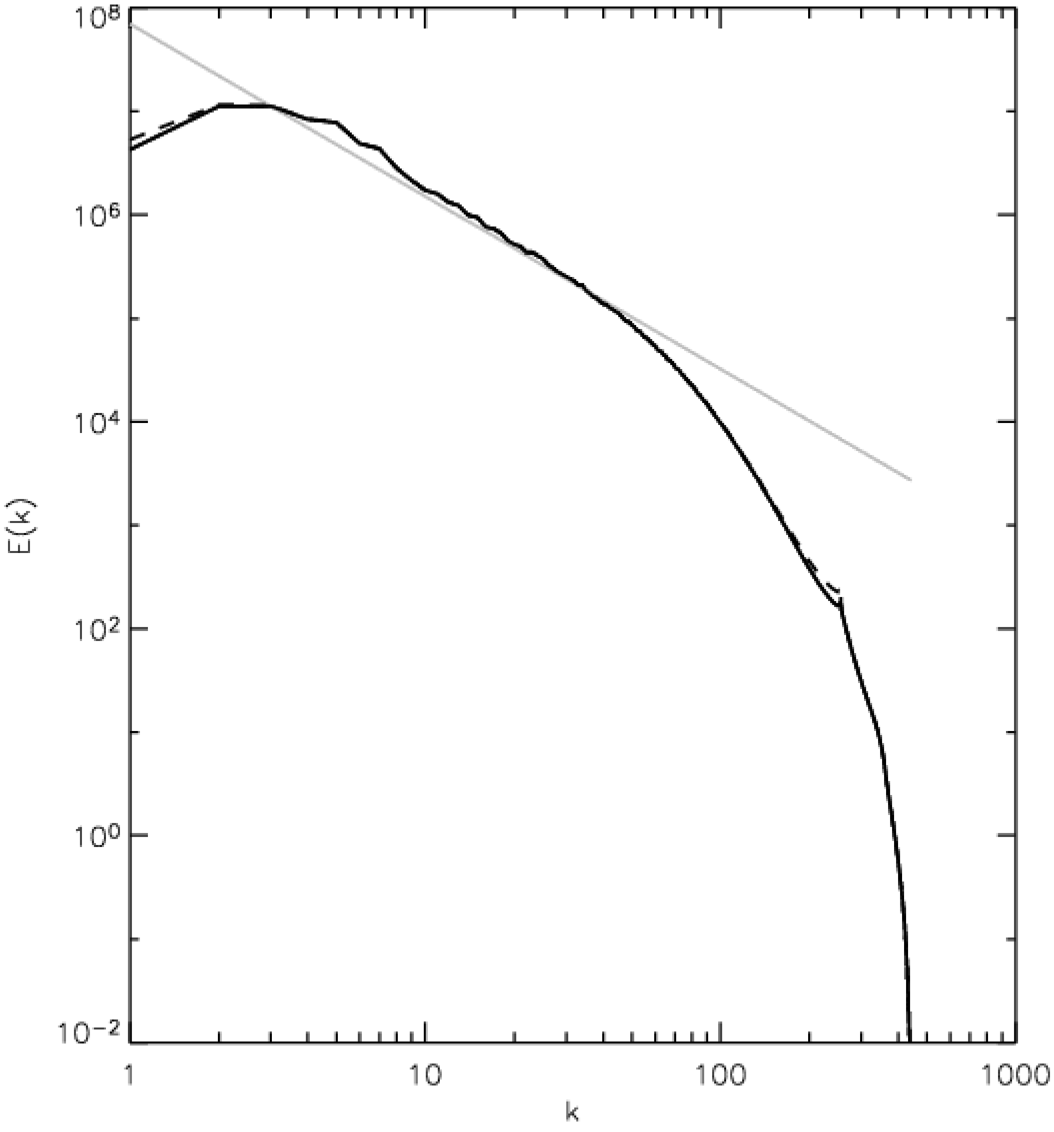
t = 8.11 x 10^{-4} s



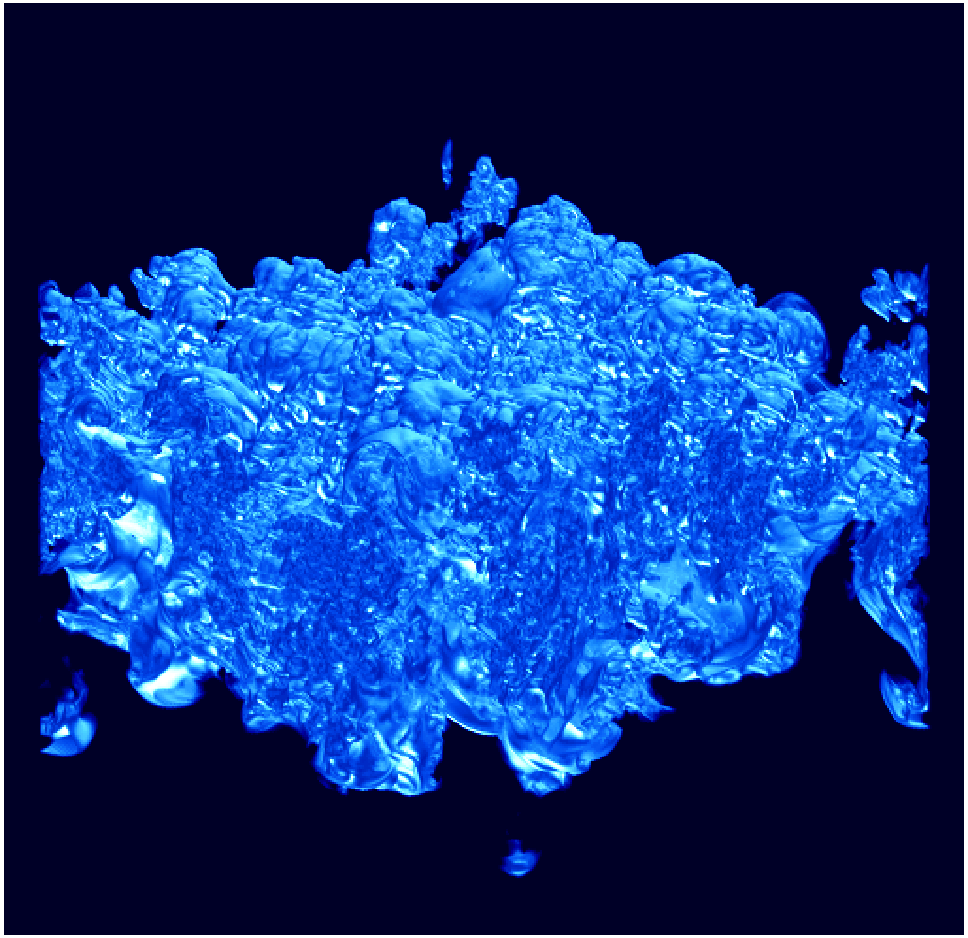
Transition to Turbulence



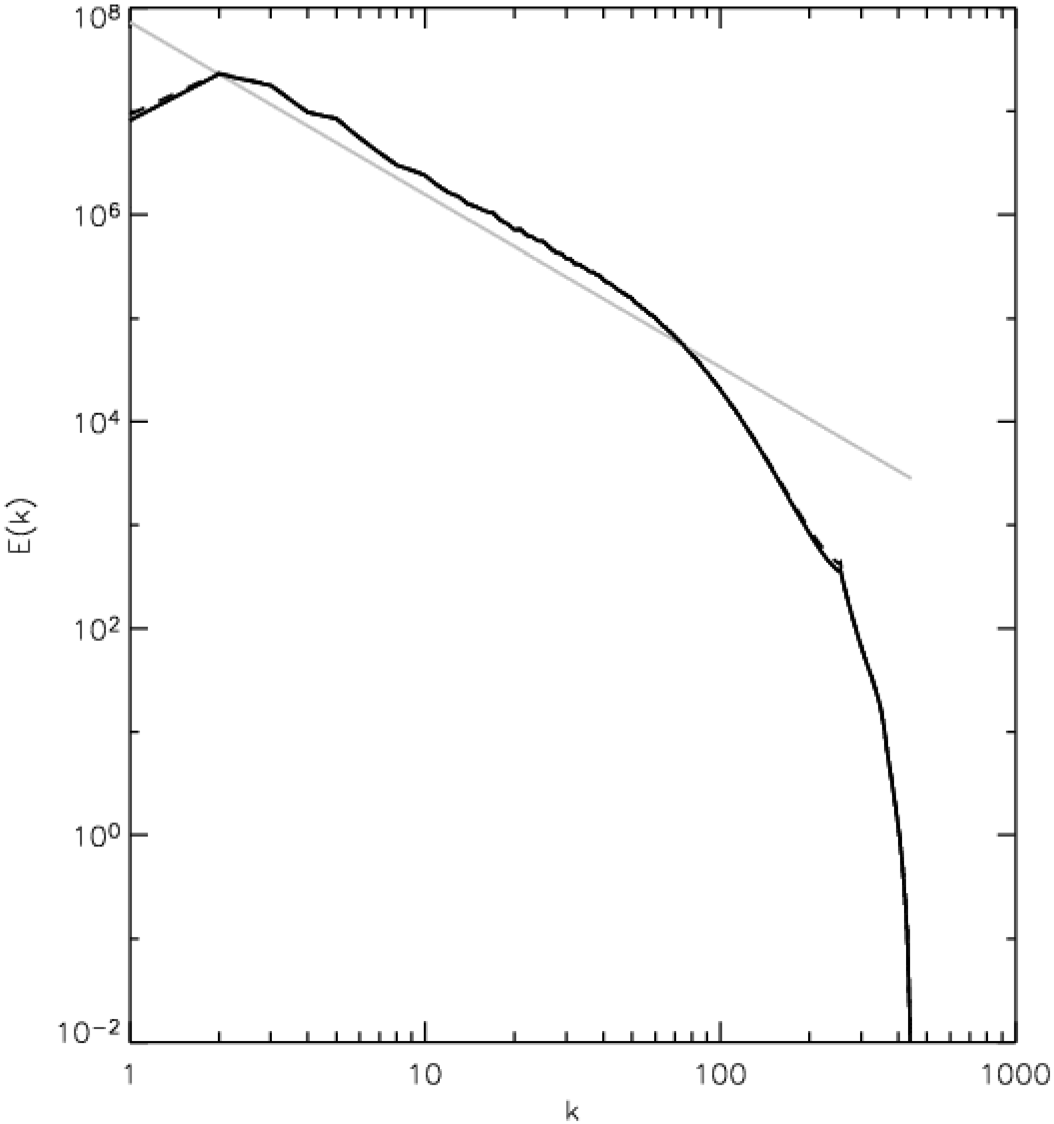
$t = 9.43 \times 10^{-4}$ s



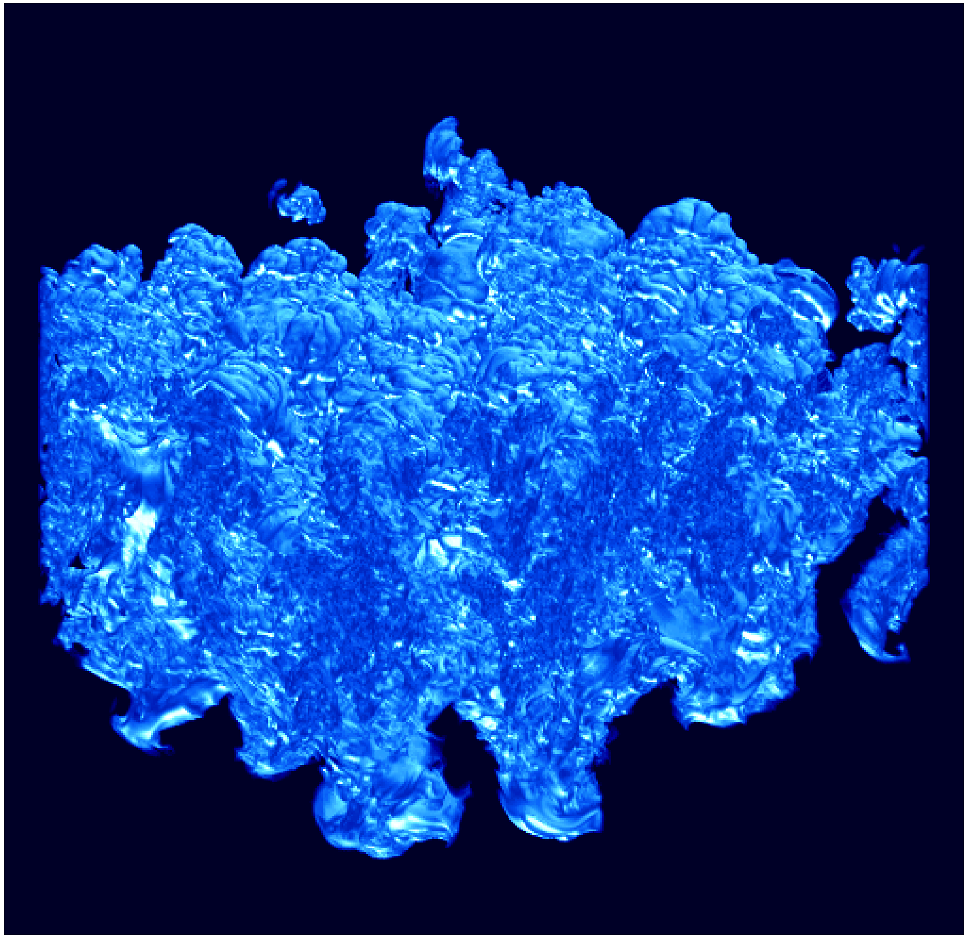
Transition to Turbulence

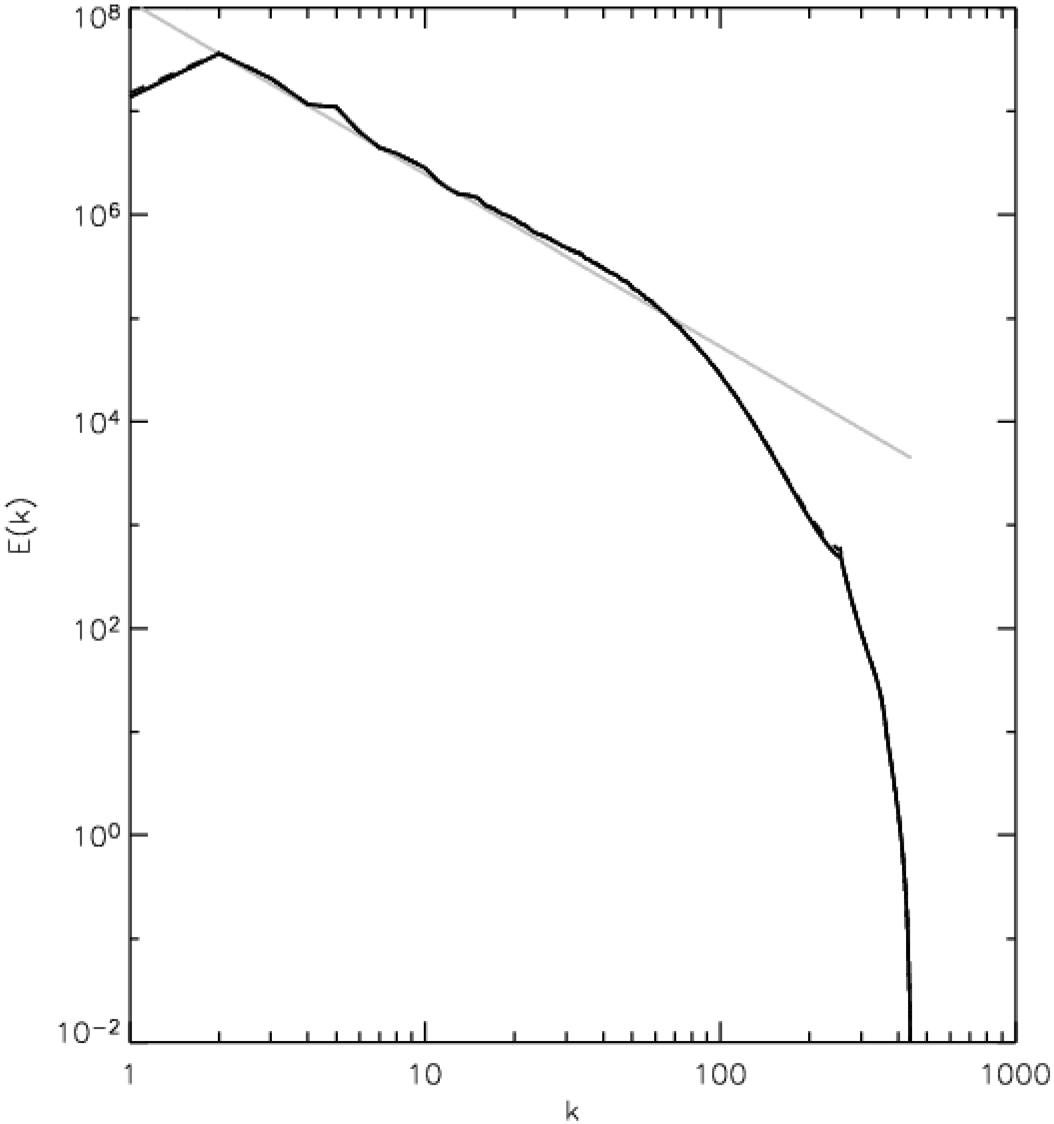


t = 1.07 x 10^{-3} s



Transition to Turbulence

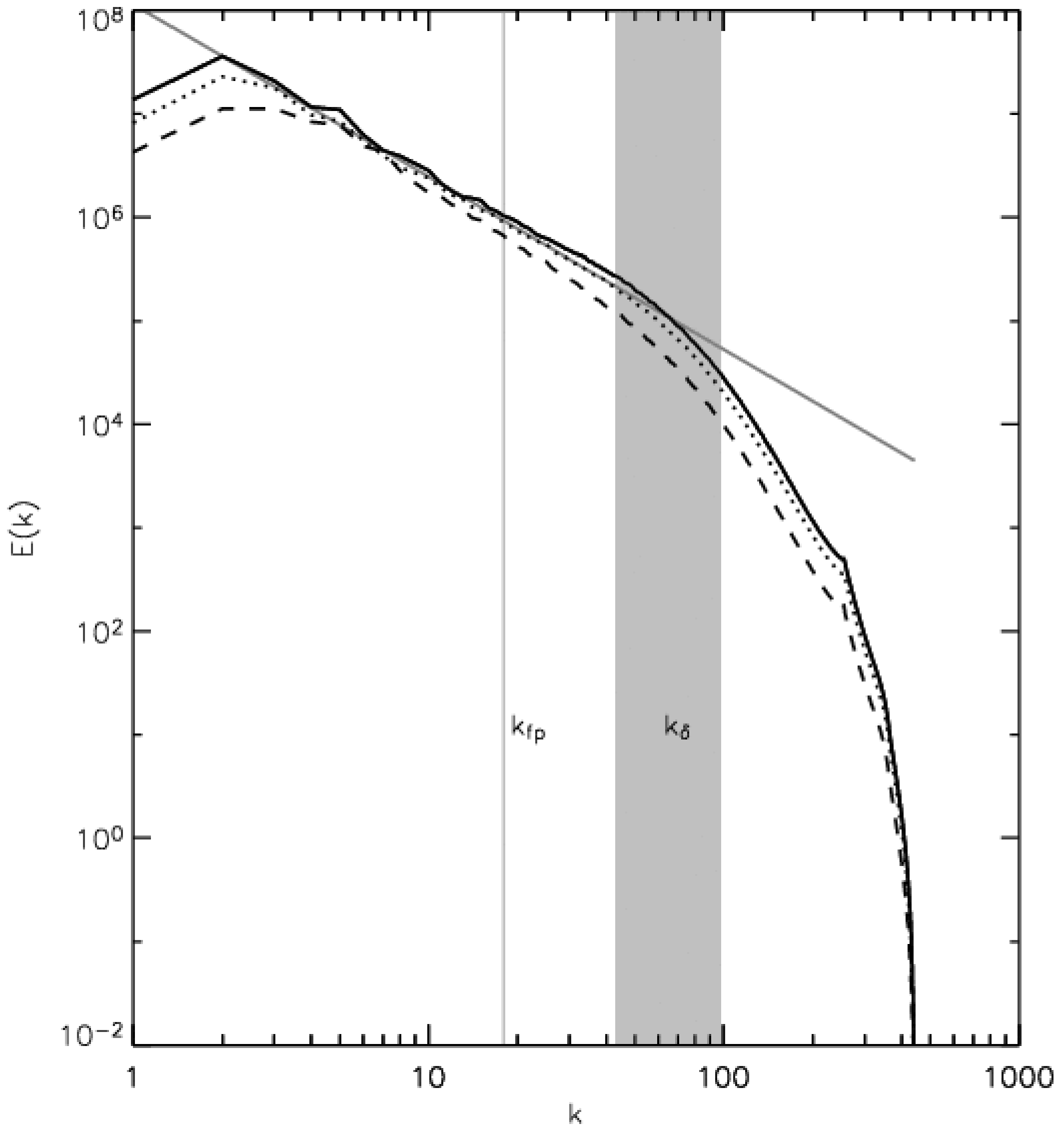




t = 1.16 x 10^{-3} s

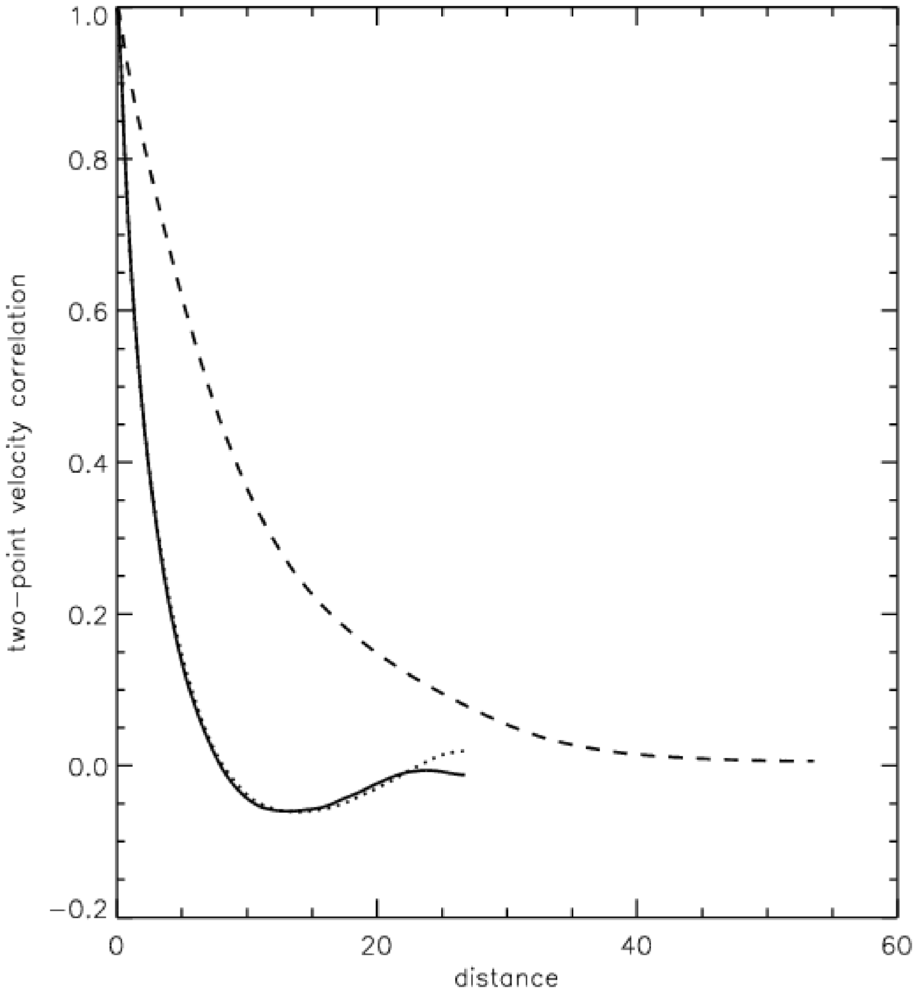
Power Spectrum

- Cutoff to power spectrum converges
  - Turbulence is fully developed
  - Inertial range of > 1.5 orders of magnitude
  - Cascade falls well below fire-polishing length



Integral Scale

$$l_t^{(x)} = \frac{1}{\int_\Omega \mathrm{d}\Omega\, u^2} \int_{\xi=0}^{L_x/2} \mathrm{d}\xi \int_\Omega \mathrm{d}\Omega\ u(x,y,z)\, u(x+\xi,y,z)$$



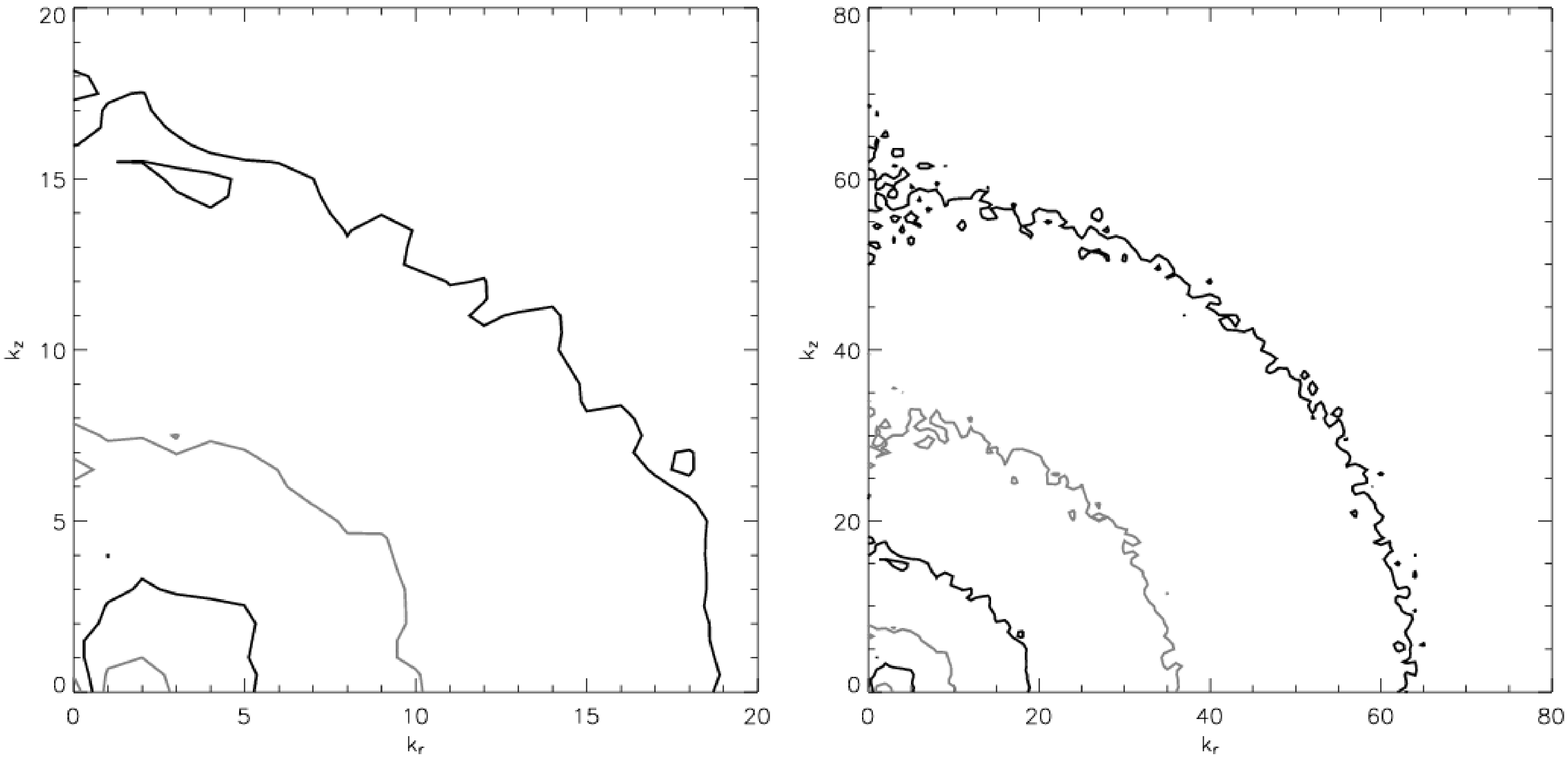
Turbulence is anisotropic

- Integral scale in z is 5x larger than in x, y
- Turbulent intensity in z is 2-3 times larger than in x,y

Gibson scale is just resolved

$$l_G = l_t \left(\frac{S_l}{u'} \right)^3$$

Turbulence on Small Scales



- Look at $E(k_x,k_y,k_z)$ to see the scales it is anisotropic
  - Average over the cylindrical angle due to symmetry
  - At the largest scales (small k) we are anisotropic
  - At small scales (large k) we get circular → isotropic.

3-D Reactive RT Summary

- Flame width, fire-polishing length, and Gibson scale are resolved on the grid.
- Flame becomes fully turbulent.
  - Anisotropic Kolmogorov spectrum becomes isotropic after a decade of turbulent cascade.
    - Turbulent flame models assuming isotropy will need to really resolve the turbulence.
  - Transition to distributed burning regime is at a higher density in 3-D.

Reacting Buoyant Bubbles

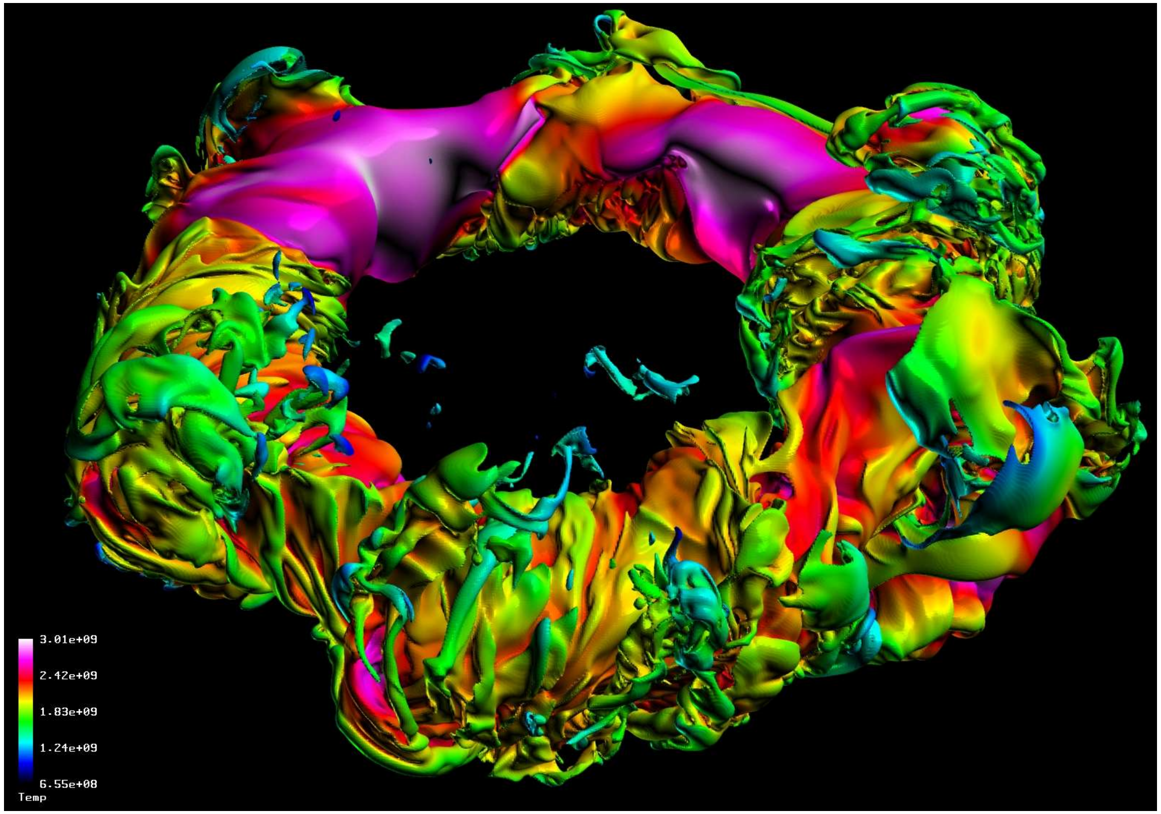
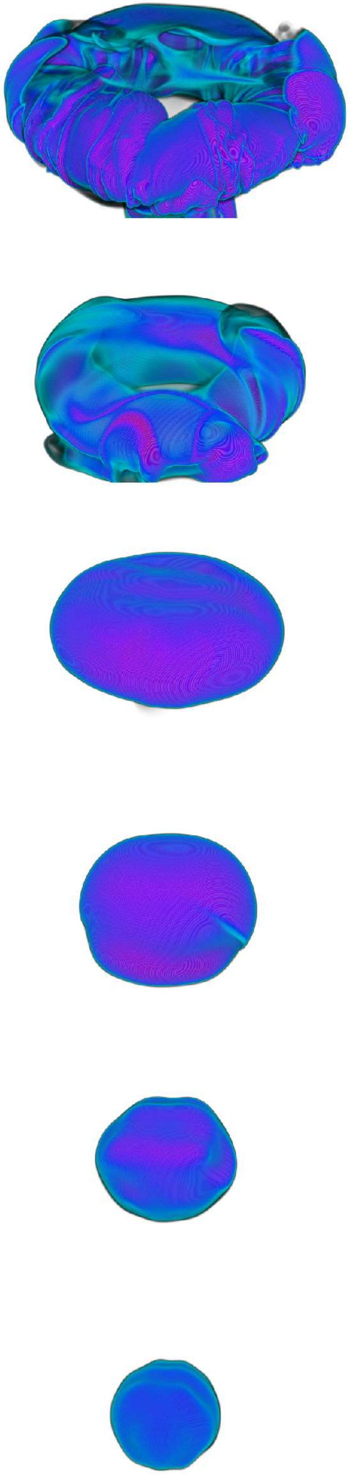


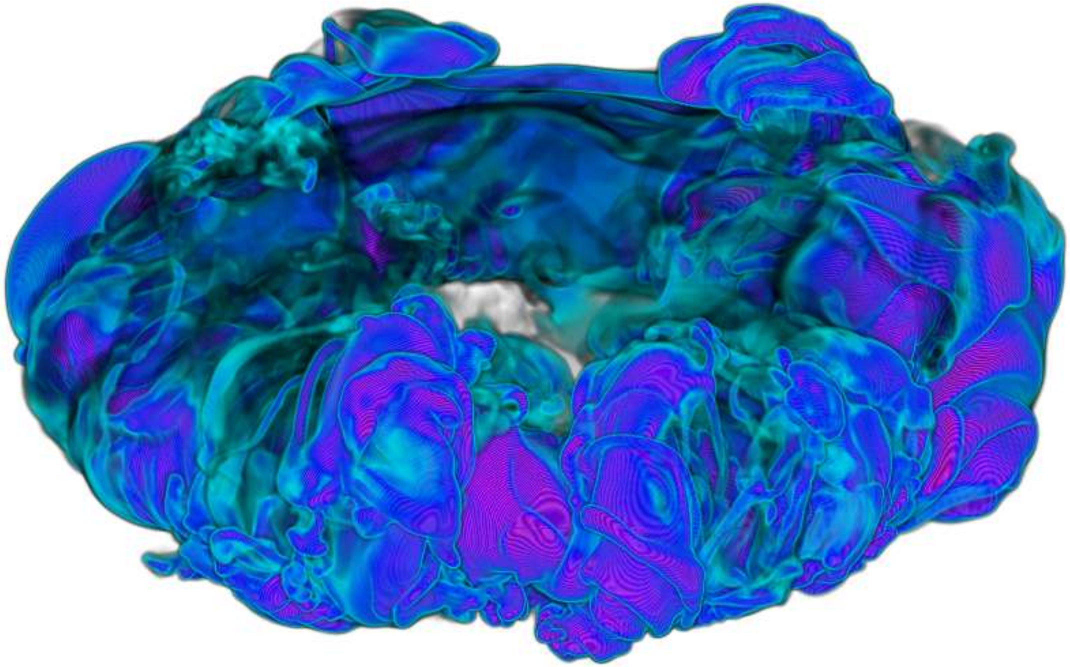
image produced by NASA/Ames

- Important to understanding the ignition process.
- 3-D, resolved studies have begun.
  - burning is non-uniform around the bubble
  - restricted to ~ 10^7 g cm^{-3}

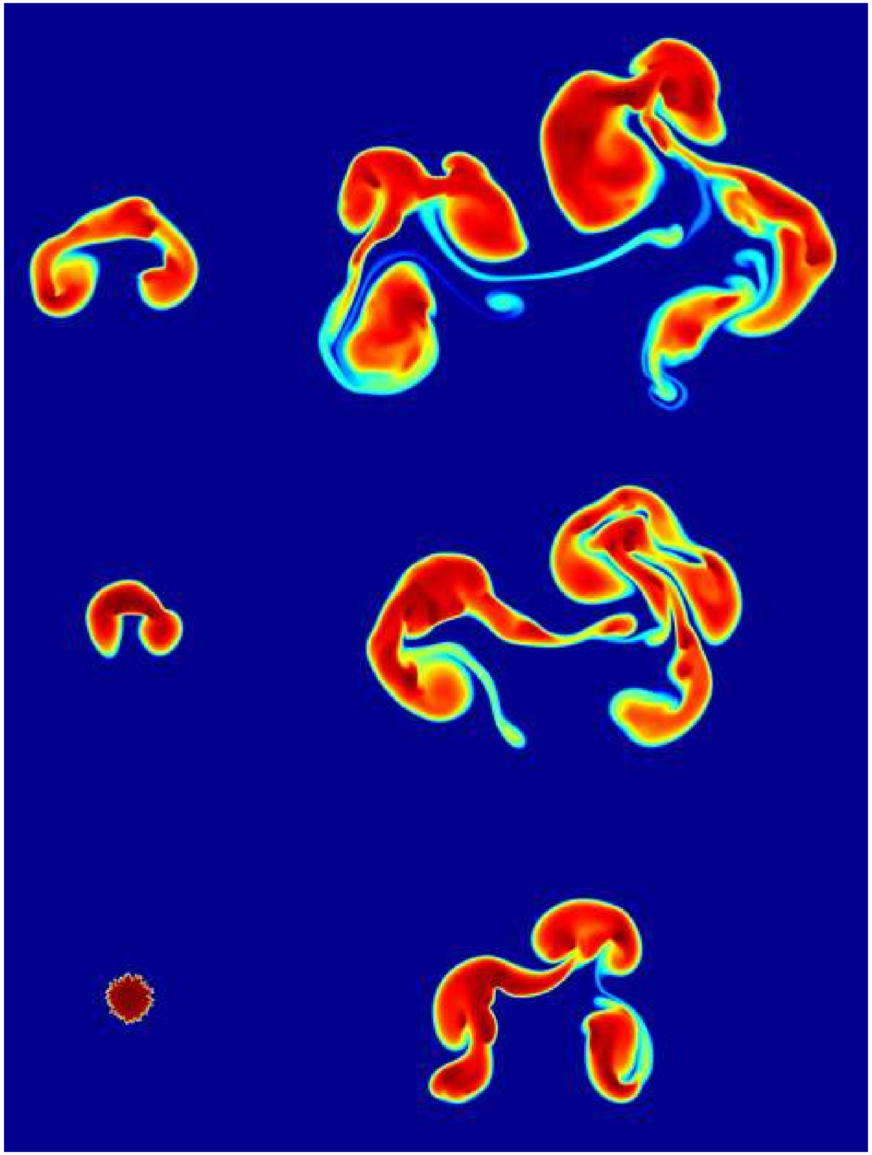
Reacting Buoyant Bubbles

- Does the bubble fragment as it evolves?
  - Initially a 7 cm radius sphere
  - It looks like we are just entering the turbulence regime.
  - Smaller bubble fragments will advect with the flow, igniting other regions of the star.





Can the Bubble Fragment?



- 2-D studies show the initial bubble quickly fragments
- Large 3-D calculations are in progress.

Flame Model Validation

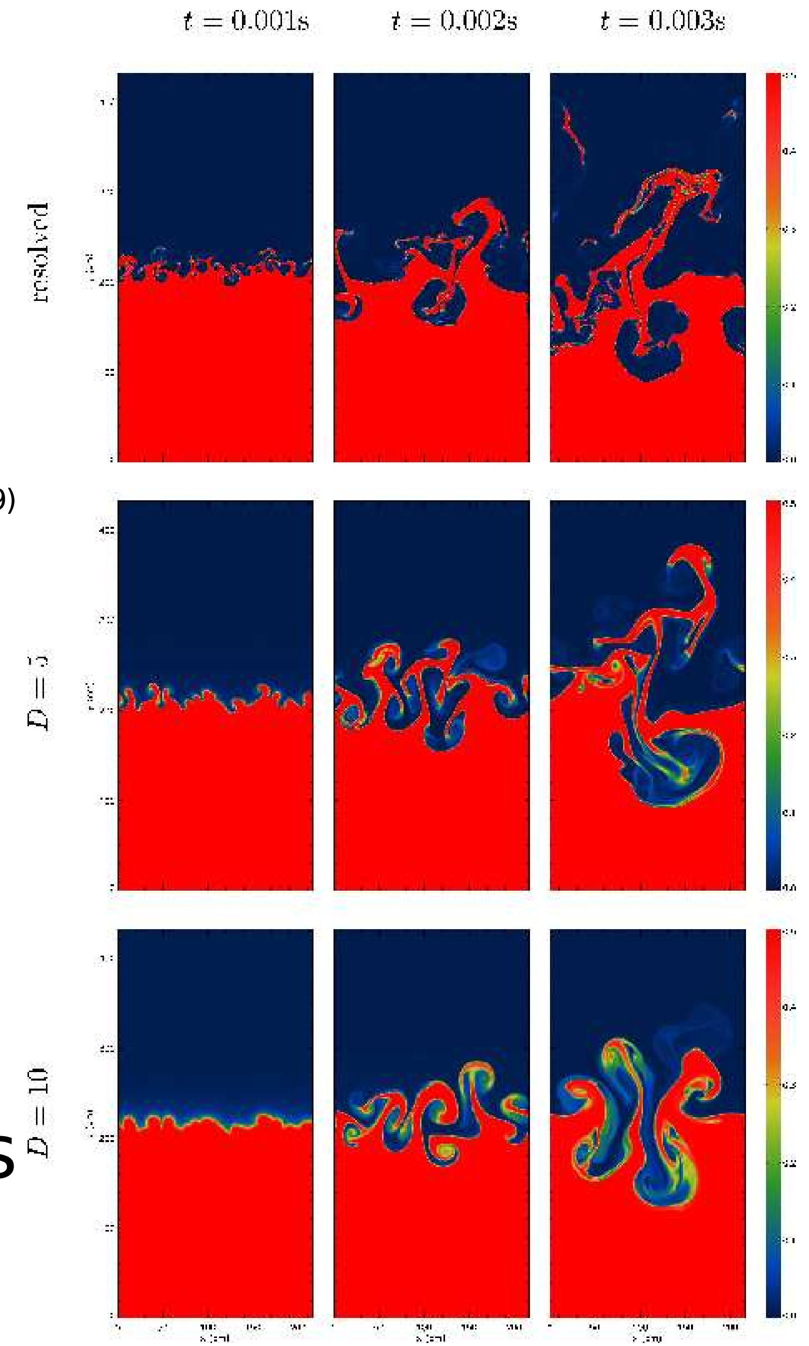
- Thickened flames

$$v \sim \sqrt{\kappa \dot{\omega}} \qquad l_f \sim \sqrt{\frac{\kappa}{\dot{\omega}}}$$

$$\kappa \rightarrow D\kappa \qquad \dot{\omega} \rightarrow \frac{\dot{\omega}}{D}$$

(O'Rourke & Bracco 1979)

  - washes out the small scale modes
  - changes the effects of curvature on the flame.
- Thickening the flame misses wrinkling on the small scales
  - Efficiency functions (Colin et al. 2000)
- Resolved calculations serve as validation for flame models.



Where Do We Go From Here?

- Lots of analysis for the 3-D bubble remains.
- Formulation of a subgrid model and flame model to advect the flame on large scales
  - Validation of thick flame approximation against the DNS flames is underway.
  - Comparison to the 3-D RT calculation is also possible.
- Modification of the algorithm to allow for multiple scale heights is underway.
  - Anelastic method
- Studies of the ignition process
  - Explicit codes cannot do this

Ignition Process

- Star convects for ~ 100 years.
- Highly screened carbon burning at the center
  - Ignition occurs when timescale for nuclear energy increase ~ convective turnover time (~10s).
  - $T \sim 7 \times 10^8$ K, $\rho \sim 2 \times 10^9$ g cm^{-3}
- Does ignition occur at a single or multiple points?
  - What is the temporal distribution?
- Studies of ignition require an implicit or anelastic hydrodynamics code.

Conclusions

- Transition to distributed burning at ~10^7 g cm^{-3}
  - Transition occurs at lower density in 2-D
- Scaling of velocity with area is not purely geometric in the flamelet regime
- Mixed region grows slower than Sharp-Wheeler model.
- Turbulence dominates in 3-D
  - Anisotropic Kolmogorov cascade
  - Isotropic on small scales
- Turbulent subgrid models assuming isotropy on small scales are a reasonable approximation.